Competitive Search Equilibrium and Moral Hazard*

Benoît Julien†    Guillaume Roger ‡

April 7, 2015

Abstract

Principals seek to enter a productive relationship with agents by posting general incentive contracts. A contract must solve both the ex post moral hazard in production and the ex ante competitive search problem (participation). Menus of contracts do not help hence (single) contract posting is optimal; the optimal contract is characterized and includes compensatory transfers to agents who meet a principal but fail to contract. To generate rents to attract agents, principals distort both the transfer function and the productive action. This implies lower welfare; the allocation is Pareto optimal conditional on the action but not constrained-efficient precisely because of this trade off. A planner is immune to principal competition and so implements the standard solution instead. The loss of efficiency is severe: no instrument can correct the market allocation. We also establish a connection between the meeting technology and the set of feasible contracts, and recover an augmented conditional Hosios condition for a subset of (suboptimal) contracts.

Keywords: moral hazard, asymmetric information, contracts, directed search, search frictions, constrained efficiency. JEL Classification: D82, D83, D86.

1 Introduction

We study optimal contracts and allocations in an economy with moral hazard and matching frictions in a competitive search model (Peters, 1991; McAfee 1993; Moen 1997; Shimer, 1996;)

*We thank Roger Myerson, Mike Peters and Randy Wright for especially helpful discussions, as well as Hector Chade, Martin Gervais, John Kennes, Ian King and Philipp Kircher for their comments.
†UNSW Business School, Australia, benoit.julien@unsw.edu.au
‡University of Sydney, guillaume.roger72@gmail.com.
Julien, Kennes and King, 2000; Burdett Shi and Wright, 2001). Principals post the terms of trade non-cooperatively and agents choose to direct their search based on the observed terms of trade. “Searching” means finding the best probability distribution over the set of contract offers – this is the directed search framework. Upon matching a principal and an agent enter into a productive relationship; the surplus depends on the effort provided by the agent. This, and directed search, are critical features. Effort is also unobservable by the principal. We believe that integrating search frictions with informational frictions capture many economic relationships such as employment or financial contracting, and may even extend to new monetarism (Lagos-Wright, 2005).

Principals may meet with many agents but form bilateral partnerships only; for this one needs to distinguish between meeting and matching (contracting). This distinction allows us to study a rich set of meeting contingent contracts that can be posted by principals. In this general framework we obtain novel results. In equilibrium a single incentive contract is posted even if allowing for a large array of meeting-contingent contracts (i.e. contracts that depend on how many agents meet a principal). This affirms the result of Selcuk (2012) whereby a fixed price (the equivalent of posting a single contract) is optimal. The reason however differs from Selcuk’s, who shows that risk-averse buyers dislike a lottery over a menu of transfers. Instead we establish that principals do not like a lottery over a menu of actions because the total cost of an action is convex.

This simplification affords us a simple characterization. The optimal contract includes a transfer function and compensatory transfers paid to the agents meeting, but not contracting with, a principal. These transfers provide insurance but have no bearing on the effort decision; they generate the rents necessary to attract agents while minimizing the distortions of the effort-inducing transfer function. Principal competition stems from bilateral contracting, which acts like a capacity constraint. We show that the equilibrium allocation is not constrained-efficient at the intensive (effort choice) and extensive (entry) margins. Constrained efficiency fails here because the social surplus increases in the agent’s action, but principal competition for agents requires rent-giving. Rent-giving is most cheaply accomplished by improving the insurance properties of the contract, which weakens incentives – in spite of compensatory transfers. We explore this trade off arising from search frictions in details and show it is common to problems with an endogenous surplus and directed search. It is a general property that generating rents is best achieved by using the two available margins: transfer and effort. In contrast, Acemoglu and Simsek (2010) show approxi-
mate efficiency in a general equilibrium model with moral hazard (so with an endogenous surplus). Crucially however there are no search frictions in their model, so no such trade off. Indeed, it is the interaction of the endogenous surplus and directed search that engenders inefficiency in our model. A planner can restore efficiency because he circumvents the directed search problem. More precisely, the planner is constrained by the equilibrium level of frictions, but not by the incentives of directed search.

We also show that the choice of a meeting technology affects social welfare because it conditions the use of compensatory transfers. A rival meeting technology precludes their usage, so rent-giving is achieved via more distortions. In this case we recover an augmented Hosios condition that characterizes Pareto optimality but not constrained efficiency. Finally we contribute to better understand the moral hazard problem in that this work endogenizes the relevant outside option. “Participation” really means participating in the search process. That decision is made on the basis of the array of contract offers and the market conditions, not any exogenously given outside option. We discuss extensions of our results, including with regards to the principals’ choice of a rationing rule (to cope with the capacity constraint); they remain robust.

The model is useful to study several important economic situations. A primary example is the labor market; firms hire workers posting incentive contracts and output depends stochastically on labor input (effort) that is unobservable by the employer. The contract affects the unobservable hours worked (intensive margin), as well as workers’ probability to find a job (extensive margin) via free entry of firms. In the context of financial contracting, the outcome of the borrower’s project may depend on her action, which is not observable. The transfer affects that action, hence the probability of repaying the loan, as well as the probability of entering into a contract in the first place; there may be credit rationing because of search frictions.

Few papers combine competitive search and moral hazard; this work addresses this point and complements the literature on moral hazard and on search. The works closest to ours are Acemoglu and Shimer (1999) and Moen and Rosen (2011). Acemoglu and Shimer (1999) study efficient unemployment insurance in a directed search framework where firms post wages. Unemployment insurance induces workers to search for high-wage jobs; in response firms improve job attributes (wages and capital/labor ratio) but unemployment risk is also higher. Output is maximized with a measure of unemployment insurance: it induces the best capital/labor ratio. In our model compen-
satory transfers act like unemployment insurance, contracting with the principal is the high-risk exercise, and the agent’s action, rather than the capital/labor ratio, governs output (stochastically). This action, however, is left to the agent (not the firm) and is taken ex post. Absent compensatory transfers, attracting agents requires more distortion of the (incentive) transfer function because the gamble is worse. This induces a lower productive action and so is not optimal, analogously to the capital/labor ratio in Acemoglu and Shimer (1999). Hence insurance (in the form of compensatory transfers) is good because it assists in attracting agents without distorting the incentives for effort. Moen and Rosen (2011) adapt the Laffont-Tirole (1986) model to a search framework. It is a model “false moral hazard”: effort is first-best conditional on the agent’s information rent (no direct distortion), so the model reduces to one of adverse selection. Moen and Rosen (2011) find the equilibrium to be constrained efficient, unlike in this work. In their model search is not directed; matching is random instead (and pairwise, hence rival), and so orthogonal to contracts.

We contribute to the theory of competitive search along several dimensions. Peters and Severinov (1997) let agents draw a valuation from an exogenous distribution before or after meeting; their focus is on equilibrium with auctions. Albrecht, Gautier and Vroman (2014) extend Peters and Severinov by allowing free entry; entry is always efficient, whether buyers’ valuations are drawn before or after meeting. In Lester, Visschers and Wolthoff (2014) sellers compete in asking prices. Buyers draw valuations from a distribution after meeting a seller, and are allowed to make counter offer to the asking price. Asking prices can be an equilibrium allowing for sellers to choose over a large set of general mechanisms. Rocheteau and Wright (2005) consider a competitive search model in which the quantity traded is posted along with a fixed price. None of this literature considers endogenous surplus generated by unobservable effort.

Second, we contribute to the connection between contract space and the meeting technology. Eeckhout and Kircher (2010) consider the sorting effects of mechanisms under different level of rivalry associated with the meeting technology; price posting dominates auctions in sorting. Lester, Visschers and Wolthoff (2015), in a model related to their aforementioned paper, establish the properties of the meeting technology under which an auction with a zero reserve price is optimal. We observe that the meeting technology determines which contracts may be used, out of an otherwise unconstrained contract space: they interact in defining the set of feasible contracts. We show this has welfare consequences. Third, we contribute to the implications of risk aversion in competitive
search. Jacquet, Kennes and Tan (2014) study competitive search with a fixed surplus and fixed posted price, but like us, allow for transfers to the unmatched buyers in multilateral meetings. In a large economy their equilibrium is unique, as ours, and efficient, unlike ours. This difference owes to the endogeneity of the social surplus. In Acemoglu and Shimer (1999) risk aversion justifies unemployment insurance.

Fourth, we contribute to competitive search under informational asymmetries. Guerrieri (2008) considers adverse selection with frictions, and shows the dynamic equilibrium is inefficient because the outside option changes over time. Guerrieri, Shimer and Wright (2010) study competitive search under adverse selection and find that the competitive equilibrium is not constrained-efficient because the joint surplus is affected by allocative distortions. So informational frictions rather than search frictions, lie at the source of this inefficiency; but thanks to the interaction with search frictions, informational frictions have implications at the extensive margin. When the surplus depends on an effort instead, search frictions induce effort distortion even under symmetric information because principals trade off transfer and action. These distortions are emphasized under moral hazard because inducing effort is more costly.

In the next section we specify the model and re-state the benchmark model. Sections 3 and 4 are the heart of the paper. We analyse the search problem and characterize the equilibrium; then we derive welfare implications and connect to the Hosios condition. We suggest a discussion in Section 5. All proofs and some supplementary material can be found in the Appendix.

2 Model

The economy is populated by a measure $N$ of homogenous agents and a measure $M$ of homogenous principals, with aggregate market tightness $\Theta = \frac{N}{M} < \infty$. Principals seek to form bilateral relationships with agents subject to moral hazard. An agent’s utility is $u(t) - c(a)$, with $u(\cdot)$ increasing and concave; $t$ is the transfer received and $a \in A \subset \mathbb{R}$ the chosen action at cost $c(a)$ increasing and convex (for example, hours worked). Action $a$ governs the distribution $F(x|a)$ of outcomes $x \in \mathcal{X} \equiv [\underline{x}, \overline{x}] \subset \mathbb{R}$, with density $f(x|a) > 0$. The likelihood ratio $f_a/f$ is increasing, concave in $x$, hence $F(x|a) < F(x|a')$ for $a' > a$. The function $t(x)$ is taken to be equicontinuous as in
Holmström (1977, 1979).\textsuperscript{1}

**Market interaction.** Principals compete by offering terms of trade (contracts) to attract agents, and agents select over principals after observing the terms of trade. Each agent can work for at most one principal, and each principal only needs one agent. Equivalently, principals are capacity constrained in their ability to contract. This restriction implies the use of a rationing rule. Under contract posting there is no loss in restricting attention to uniform rationing.\textsuperscript{2} Principals can fully commit to these contracts.

Bilateral contracting is standard in competitive search; it is also not very restrictive. Search is directed by the principals’ offers, with two effects. First, any one contract affects the others, however only through the search process. Competition directly affects the participation problem only, not the incentive problem (it does indirectly). Bilateral contracting rules out team formation and any incentive provision based on teams. Second, agents formulate a probability distribution over principals but are not ubiquitous; they can only visit one. So the extensive form rules out common agency. This stands in contrast to the works of Attar et al (2006, 2007a, 2007b), Martimort (2004), Aubert (2005) or Célérier (2012), for example. In each of these, an agent may be party to more than one contract at once; this decision interferes with both the participation and the moral hazard constraint.

Contracts cannot depend on the identity of agents – this is innocuous since agents are homogeneous. Otherwise they are as general as can be. Denote one such contract by $C_j = (t_j(x), a_j, h_j) $, $j \in M$, with $t_j(x) = (t_{j1}(x),...,t_{jN}(x))$, $a_j = (a_{j1},...,a_{jN})$, $h_j = (h_{j1},...,h_{jN})$. $C_j$ offers arrays of transfers $t_n^j(x)$, $h_j$ and prescribes an array of actions $a_j$ that may depend on the number $n$ of agents showing up; these are *meeting contingent contracts*. The transfer functions $t_j(x)$ depend on the outcome $x$ and $h_j$ are transfers paid to (from) agents approaching $j$ but unable to contract with him. It is evident they must be independent of $x$. Throughout we suppose that the first-order approach to the agency set up is valid, see Jewitt (1988) for details.

We adopt the competitive search version of submarkets akin to the one in Moen (1997) and Mortensen and Wright (2002)). However principals, and not some “market makers”, post the terms

\textsuperscript{1}These are, essentially, smooth functions. Examples include Lipschitz continuous functions and $C^1$ functions. This technical condition guarantee existence of a maximum; details can be availed from the authors upon request.

\textsuperscript{2}Details of this claim available upon request. We also discuss an alternative in some detail in Section 5.
of trade in any submarkets. The timing is as follows:

1. principals posting similar contracts form a submarket;

2. agents observe all contracts (all submarkets) and select a submarket to participate in;

3. in a submarket principals and agents meet according to some meeting technology;

4. upon meeting, if an agent accepts a contract, she selects an action;

5. payoffs are realized. Agents not meeting anyone receive \( u_0 \); agents not contracting receives the payoff \( u(h^j + y) \) where \( y = u^{-1}(u_0) \) and \( u_0 \) is an exogenous outside option; principals not matching receive 0.

**Meeting technology.** We consider general meeting technologies; this degree of generality requires making a distinction between *meeting* and *matching* (contracting). The technology is defined by the extent of rivalry between agents in the spirit of Eeckhout and Kircher (2010); we add in precision by also including the degree of rivalry in matching. At the two extremes, a meeting technology can be purely *non-rival* or *rival*. Because principals are capacity-constrained, rivalry extends to the overall meeting and matching process.

A non-rival meeting technology allows for multilateral meetings, and a meeting by any one agent with one principal does not affect the probability for other agents to meet that principal. A Poisson distribution of agents over principals is an adequate example. A principal selects only one agent so the matching probability for an agent is rival. At the other extreme the meeting technology can be purely rival – for example, pairwise meeting. However then the matching probability is purely non-rival as only one agent meets a principal and so contracts with probability one. Importantly, restricting the meeting technology to be purely rival rules out meeting contingent contracts. Then compensatory transfers, auctions or bidding cannot be part of the contract space. Meeting rivalry also applies to principals; however matching is always non-rival for them because of the extensive form we consider. So the feasible set of contracts is characterized by the combination of all feasible contracts with the meeting and matching technologies.\(^3\)

\(^3\)As raised by Eeckhout and Kircher (2010), there may be a continuum of meeting technologies between purely rival and non-rival. We stay with either extremes; so non-rival meeting means that agents meet a principal for sure but the matching is rival; rival meeting refers to meeting being rival, but matching non-rival.
Let \( \mu_N(C) \) and \( \mu_M(C) \) be positive measures of agents and principals active in a submarket, and let \( \theta = \mu_N(C)/\mu_M(C) \) be the local market tightness prevailing in that submarket under contract \( C \). Let \( p_n(\theta) \) be the probability that \( n \) agents meet a particular principal in submarket \( k \). Similarly let \( q_n(\theta) \) be the probability an agent meets a principal with \( n \) agents, including herself. Then \( q_0(\theta) \) is the probability for an agent not to match. Under rival meeting this is the probability to meet no principal; under non-rival meeting it is the combination of the probability to meet (which is 1) but not be selected to trade. Under either meeting technologies \( q_0(\theta) > 0 \). By definition

\[
\sum_{n=0}^{\infty} p_n(\theta) = \sum_{n=0}^{\infty} q_n(\theta) = 1 
\]

and \( q_1(\theta) \) is the probability an agent is in a pairwise meeting with a principal.

We follow the standard assumption that \( p'_0(\theta) < 0 \): more agents reduces the probability for principals to not meet, and \( p''_0(\theta) > 0 \). Similarly, \( q'_1(\theta) < 0 \): more agents reduces the probability for an agent to be alone meeting a principal, and \( q'_0(\theta) > 0 \): more agents reduces the probability of contracting (either by not meeting under rival, or meeting but not contracting under non-rival, technologies). With a rival technology the meeting rate for a principal is \( p_1(\theta) = 1 - p_0(\theta) \): the probability to meet one agent. With a non-rival technology this is the probability to meet at least one agent, \( \sum_{n=1}^{\infty} p_n(\theta) = 1 - p_0(\theta) \). Similarly, for agents \( q_0(\theta) \) is the probability an agent does not trade with a principal. Under a rival technology the meeting (and matching) rate is \( q_1(\theta) \); when agents select over principal, they always meet a principal (probability one), but because of the capacity constraint they may not trade. The meeting and matching rate for an agent is \( \sum_{n=1}^{\infty} q_n(\theta) \frac{1}{n} \), including the agent. The meeting technology is homogenous of degree zero so a consistency requirement links \( p_n(\theta) \) and \( q_n(\theta) \). Given \( N \) agents and \( M \) principals in a submarket for all \( n > 0 \),

\[
\forall \ n > 0, \quad \mu_M p_n(\theta) = \frac{\mu_N q_n(\theta)}{n} \Rightarrow np_n(\theta) = \theta q_n(\theta).
\]

For rival meeting, this holds only for \( n = 1 \), so \( p_1(\theta) = \theta q_1(\theta) \) or \( q_1(\theta) = \frac{1-p_0(\theta)}{\theta} \). With non-rival meetings,

\[
\sum_{n=1}^{\infty} p_n(\theta) = \theta \sum_{n=1}^{\infty} q_n(\theta) \frac{1}{n}.
\]

Thus, the meeting and matching rate for a buyer is

\[
\sum_{n=1}^{\infty} q_n(\theta) \frac{1}{n} = \sum_{n=1}^{\infty} \frac{p_n(\theta)}{\theta} = \frac{1-p_0(\theta)}{\theta}.
\]

8
Common examples of this construction include Poisson: 
\[ p_n(\theta) = \frac{\theta^n e^{-\theta}}{n!} \]  
and \[ q_n(\theta) = \frac{\theta^{n-1} e^{-\theta}}{(n-1)!}, \]
Cobb-Douglas: 
\[ p_1(\theta) = \frac{M^n N^{1-\alpha}}{M} = \theta^{1-\alpha} > 0 \]
and \[ p_n(\theta) = 0, \forall n > 1. \]

**Payoffs and equilibrium.** Upon contracting with one of the \( n \) agents he meet, principal \( j \) receives
\[
\pi(t^j_n, a^j_n) \equiv \int_X [z - t^j_n(z)] dF(z|a^j_n).
\]
For an agent, upon meeting principal \( j \) with \( n-1 \) other agents, and contracting with that principal,
\[
U(t^j_n, a^j_n) \equiv \int_X u(t^j_n(z))dF(z|a^j_n) - c(a^j_n).
\]
Prior to the meeting process, the ex ante payoff for principals is given by
\[
\Pi(C^j) \equiv \sum_{n=1}^{\infty} p_n(\theta) [\pi(t^j_n, a^j_n) - h^j_n], \quad h^j_1 = 0
\]
while for an agent selecting
\[
V(C^j) \equiv \sum_{n=0}^{\infty} q_n(\theta) \left[ \frac{1}{n} U(t^j_n, a^j_n) + \left( 1 - \frac{1}{n} \right) u(h^j_n + y) \right],
\]
An agent selecting principal \( j \) faces a probability distribution over the number of other agents visiting the same principal. Given bilateral agency formations and uniform rationing, with probability \( 1/n \) the agent matches (gets the contract). Otherwise he gets the outside option \( u(h^j_n + y) \).

**Definition 1** An equilibrium is:-

- A vector of transfers and actions \((t, h, a)\) offered by principals in each submarket, as best response to each other and the agents selection strategies;

- A vector of selection strategies as best response to observed contracts \((t, h, a)\), leading into a market tightness \(\theta(t, h, a)\) in each submarket.

Without loss we look for symmetric, subgame-perfect equilibria of this game – as is implicitly assumed in the definition of \(\Pi(C^j)\). This too we discuss further in Section 5.
The standard agency problem. Throughout we will refer to the canonical model, which features a single agent and a single principal. The payoff to the principal and the agent are respectively:

\[ \pi(t, a) \equiv \int_X [z - t(z)] dF(z|a) \]
\[ U(t, a) \equiv \int_X u(t(z)) dF(z|a) - c(a). \]

The principal maximizes \( \pi(t, a) \) by choice of the contract \((t(x), a)\), subject to \( U(t, a) \geq u_0 \) and \( U_a = 0 \) and with \( u_0 \) known. The solution \((t^{SB}(x), a^{SB})\) is characterized by the conditions

\[ \frac{1}{u'} = \lambda + \mu \frac{f_a}{f} \quad (2.1) \]

and

\[ \pi_a + \mu U_{aa} = 0 \quad (2.2) \]

where \( \lambda, \mu > 0 \) are Lagrange multipliers, together with the two aforementioned constraints (see Holmström, 1979; Jewitt, 1988). We note that \( \lambda \equiv \lambda(u_0) > 0 \) means that the participation cost is determined in terms of the agent’s outside option only.

3 Competitive search and moral hazard

The game on hand features a large strategy space; the first order of business is to simplify the analysis. In a large economy when principals deviate in a submarket the deviation does not affect the maximum expected utility agents receive by participating in any contracts offered by non-deviating principals. This is the *market utility property* (MUP), as used by McAfee (1993), Shimer (1996) and Moen (1997). This property allows us to focus on the equilibrium in one submarket and sellers must offer contracts that yield at least \( \tilde{V} \). Let \( \mathcal{C} \) be the symmetric contract posted in all other submarkets, \( \tilde{V} \) is defined as

\[ \tilde{V} = \max_{\mathcal{C}} V(\mathcal{C}). \]

With this, and the restriction to symmetric equilibria, we can formulate the problem as one of constrained optimization. Since all principals \( j \) in a submarket post the same contract, we now refer to submarket \( j \). The principals in submarket \( j \) then solve:

---

4In the Appendix we provide the finite market set up along with the limit equilibria as a foundation for the large market construct. See also Peters (2000), Burdett, Shi and Wright (2001), Julien, Kennes and King (2000), Galenianos and Kircher (2012) and Norman (2015) for more details.
Problem 1

\[
\max_{C^j, \theta} \Pi(C^j) \quad s.t.
\]

\[
U(t^j_n, a^j_n) \geq u(h^j_n + y) \quad (3.1)
\]

\[
U_a^j(t^j_n, a^j_n) = 0 \quad (3.2)
\]

\[
V(C^j) \geq \tilde{V} \quad (3.3)
\]

\[
h^j_n \geq 0. \quad (3.4)
\]

Constraints (3.1) and (3.4) together imply \(U(t^j_n, a^j_n) \geq u_0\); (3.1) requires the agent should prefer accepting the contract rather than just receiving the compensatory transfer. Condition (3.3) is the Market Utility Property; it is also a participation constraint, where participation is meant as participating in the search process. (3.2) is the moral hazard constraint.\(^5\)

**Proposition 1** All principals use a unique set of transfers that induce a unique effort level; that is, \(\forall \ j, t^j_n(x) = b^j(x), h^j_n = h^j\) and \(a^j_n = a^j\).

When the meeting technology is rival the result is obvious because \(n \equiv 1\) (and of course \(h^j_1 = 0\)). Under a non-rival meeting technology, Proposition 1 is substantive and simplifying. It is substantive in that it claims the optimal contract is independent of the actual number \(n\) of agents meeting a principal. Selcuk (2012) shows that using \(n\)-contingent transfers amounts to exposing agents (buyers) to a lottery over payoffs, which is costly with risk-averse agents. We show that the principals prefer a single contract because it minimizes the cost of implementing their preferred action. That cost is convex and increasing in the action (equivalently, the principals’ payoffs are concave decreasing), so principals are better off avoiding a lottery over actions. This result does not invalidate Selcuk’s, which may well apply here. Rather it complements it and is sufficient to limit attention to \(n\)-invariant contracts.

**Remark 1** Coles and Eeckhout (2003) show that meeting-contingent prices induce indeterminacy of equilibrium. This indeterminacy disappears here. The reason is that the distribution of rents matters because of risk aversion; this is also true in Selcuk (2012) – but he does not mention it.\(^5\)

\(^5\)It is technically easier to solve the problem with constraint (3.4), which we show to be slack.
Proposition 1 is also simplifying. With a unique transfer function and uniform rationing, the only relevant events are whether the agent matches with any principal. Then the meeting probabilities simplify to

$$\sum_{n=1}^{\infty} p_n(\theta) = 1 - p_0(\theta)$$

for the principals, thanks to the consistency requirement, and for the agent to

$$\sum_{n=1}^{\infty} \frac{\theta q_n(\theta)}{n} = 1 - p_0(\theta).$$

Our formulation can accommodate any meeting and matching technology. Under non-rival meetings, $q_n(\theta) = 0$ for all $n > 1$; from the submarket consistency condition $q_1(\theta) = \frac{1 - q_0(\theta)}{\theta}$. When there are no multilateral meetings $q_1(\theta)$ is also the probability to match in a submarket.\(^6\) An agent’s expected utility is now independent of $n$ and reads

$$V^j(C^j) = \frac{1 - p_0(\theta)}{\theta} U(t^j, a^j) + \left(1 - \frac{1 - p_0(\theta)}{\theta}\right) u(h^j + y). \quad (3.5)$$

Observe also that the number of agents receiving $h^j$ (do not contract is)\(^7\)

$$\sum_{n=2}^{\infty} p_{n-1}(n-1) = \sum_{n=1}^{\infty} p_n \theta = \frac{\theta}{\theta}.$$

For now we focus on non-rival meetings, that is, $h > 0$ may be optimal; we explore rival meetings further below. Since we focus on symmetric equilibria, we drop the superscript $j$; the program is

**Problem 2**

$$\max_{t(x), h, a} \left(1 - p_0(\theta)\right) \int [x - t(x)] dF(x|a) - \theta h$$

\(^6\)Rival meetings implies $h = 0$, so with probability $\left(1 - \frac{1 - p_0(\theta)}{\theta}\right)$ an agent receives $u(y) = u_0$.

\(^7\)This property is used in large Poisson game: with a large population it does not matter when the observing agent is in or out. In the Poisson example, $\sum_{n=2}^{\infty} p_{n-1}(n-1) = \sum_{n=2}^{\infty} \frac{\theta^{n-2} e^{-\theta}}{(n-2)!} (n-1) = \sum_{n=2}^{\infty} \frac{\theta^{n-2} e^{-\theta}}{(n-2)!} = \theta.$
s.t.

\[ \int_X u(t(z))dF(z|a) - c(a) \geq u(h + y) \tag{3.6} \]
\[ \frac{1 - p_0(\theta)}{\theta} U + \left[ 1 - \frac{1 - p_0(\theta)}{\theta} \right] u(h + y) \geq \tilde{V} \tag{3.7} \]
\[ \int_X u(t(z))dF_a(z|a) - c'(a) = 0 \tag{3.8} \]
\[ h \geq 0. \tag{3.9} \]

**Proposition 2** Under a non-rival meeting technology, the solution \((t^S(x), a^S, h^S)\) to Problem 2 is characterized by the necessary and sufficient first-order conditions

\[ \int_X [z - t(z)]dF_a(z|a) + \hat{\mu} \left[ \int_X [z - t(z)]dF_{aa}(z|a) - c''(a) \right] = 0 \tag{3.10} \]
\[ \frac{1}{w'(t)} = \frac{1}{u'(h + y)} \frac{\theta}{\theta - 1 + p_0} + \frac{\hat{\mu}}{f} \tag{3.11} \]
\[ h = -p_0(1 - p_0 + \theta p_0' U - u(h + y) \theta u'(h + y)) > 0 \tag{3.12} \]

with \( \hat{\mu} = \frac{\mu}{1 - p_0(\theta)} > 0 \) and \( \theta = \Theta \). The constraints (3.6) and (3.9) are slack but (3.7) binds.

Expression (3.10) is standard in a moral hazard problem. It results from subgame perfection: for any transfer \( t(x) \) a principal offers, the agent chooses the action that is optimal for her – after contracting and knowing the transfer function \( t(x) \). Thus search does not (directly) enter this equation. Condition (3.11) shows that the slope of the transfer is related to the likelihood ratio \( f_a/f \), as we know from Holmström (1977, 1979). This is what generates the incentives for effort. The first term includes \( h \), which is jointly determined with the transfer \( t(x) \). We note that

\[ \mathbb{E}_X \left[ \frac{1}{w'(t)} \right] = \frac{1}{u'(h + y)} \frac{\theta}{\theta - 1 + p_0} \]

by integrating (3.11) over \( X \) and where \( \frac{\theta}{\theta - 1 + p_0} = \frac{1 - \frac{p_0}{1 - p_0}}{1 - p_0} > 1 \). On average the agent contracting always receives a higher gross utility than if receiving the transfer \( h \) only. Her cost of effort must be defrayed.

Proposition 2 shows that principal competition affords the agent some form of effective bargaining power, i.e. \( U > u(h + y) > u_0 \) in equilibrium; that bargaining power attracts compensatory

\[ \text{Expression (3.10) is standard in a moral hazard problem. It results from subgame perfection: for any transfer } t(x) \text{ a principal offers, the agent chooses the action that is optimal for her – after contracting and knowing the transfer function } t(x). Thus search does not (directly) enter this equation. Condition (3.11) shows that the slope of the transfer is related to the likelihood ratio } f_a/f, \text{ as we know from Holmström (1977, 1979). This is what generates the incentives for effort. The first term includes } h, \text{ which is jointly determined with the transfer } t(x). \text{ We note that} \]

\[ \mathbb{E}_X \left[ \frac{1}{w'(t)} \right] = \frac{1}{u'(h + y)} \frac{\theta}{\theta - 1 + p_0} \]

by integrating (3.11) over \( X \) and where \( \frac{\theta}{\theta - 1 + p_0} = \frac{1 - \frac{p_0}{1 - p_0}}{1 - p_0} > 1 \). On average the agent contracting always receives a higher gross utility than if receiving the transfer \( h \) only. Her cost of effort must be defrayed.

Proposition 2 shows that principal competition affords the agent some form of effective bargaining power, i.e. \( U > u(h + y) > u_0 \) in equilibrium; that bargaining power attracts compensatory

\[ \text{Expression (3.10) is standard in a moral hazard problem. It results from subgame perfection: for any transfer } t(x) \text{ a principal offers, the agent chooses the action that is optimal for her – after contracting and knowing the transfer function } t(x). Thus search does not (directly) enter this equation. Condition (3.11) shows that the slope of the transfer is related to the likelihood ratio } f_a/f, \text{ as we know from Holmström (1977, 1979). This is what generates the incentives for effort. The first term includes } h, \text{ which is jointly determined with the transfer } t(x). \text{ We note that} \]

\[ \mathbb{E}_X \left[ \frac{1}{w'(t)} \right] = \frac{1}{u'(h + y)} \frac{\theta}{\theta - 1 + p_0} \]

by integrating (3.11) over \( X \) and where \( \frac{\theta}{\theta - 1 + p_0} = \frac{1 - \frac{p_0}{1 - p_0}}{1 - p_0} > 1 \). On average the agent contracting always receives a higher gross utility than if receiving the transfer \( h \) only. Her cost of effort must be defrayed.

Proposition 2 shows that principal competition affords the agent some form of effective bargaining power, i.e. \( U > u(h + y) > u_0 \) in equilibrium; that bargaining power attracts compensatory

\[ \text{Expression (3.10) is standard in a moral hazard problem. It results from subgame perfection: for any transfer } t(x) \text{ a principal offers, the agent chooses the action that is optimal for her – after contracting and knowing the transfer function } t(x). Thus search does not (directly) enter this equation. Condition (3.11) shows that the slope of the transfer is related to the likelihood ratio } f_a/f, \text{ as we know from Holmström (1977, 1979). This is what generates the incentives for effort. The first term includes } h, \text{ which is jointly determined with the transfer } t(x). \text{ We note that} \]

\[ \mathbb{E}_X \left[ \frac{1}{w'(t)} \right] = \frac{1}{u'(h + y)} \frac{\theta}{\theta - 1 + p_0} \]

by integrating (3.11) over \( X \) and where \( \frac{\theta}{\theta - 1 + p_0} = \frac{1 - \frac{p_0}{1 - p_0}}{1 - p_0} > 1 \). On average the agent contracting always receives a higher gross utility than if receiving the transfer \( h \) only. Her cost of effort must be defrayed.
transfers $h$. Of course they are not required in Holmström’s model (1979). Here they determine the cost of participating to the contract as they represent a lower bound on the agent’s utility. We explore the role of these transfers $h$ in more detail in Section 5.

Constraint (3.6) is not active in Problem 2 (therefore $U > u_0$) because the principal has to contend with a different problem than in the standard model. Here, attracting at least one agent (away from other principals) occurs with probability $1 - p_0(\theta)$ only. Hence principals face a trade-off between incentives and participation probability – $h$ and $t(x)$ are jointly determined. That (3.6) be inactive does not mean it is neutral or irrelevant; indeed the rent level $U$ is some kind of markup above $u_0$. As in JKS, a slack participation constraint still does matter for the equilibrium outcome: while $U - u_0$ is determined by market tightness and the severity of the moral hazard problem, it is anchored by $u_0$.

Let $k > 0$ be the entry cost faced by principals. In this model we can link action $a$ and tightness $\theta$ very simply through the principals’ free entry condition

$$[1 - p_0(\theta)]\pi(a, t) = k.$$  

By simple differentiation

$$\frac{da}{d\theta} = \frac{\pi(a, t)}{\pi_a(a, t)\pi_0(\theta)} < 0$$

(3.13)  

since $\pi_a(a, t) > 0$ when the multiplier $\mu > 0$. The function $a(\theta)$ is well defined by the Implicit Function Theorem ($\pi$ is differentiable in $a$ and $p_0(\theta)$ is also differentiable), and is non-trivial when search is directed, that is, when $p'_0(\theta) \neq 0$.\footnote{Equation (3.13) is not a statement of comparative statics; it computes the slope of the indifference curve in the $(A, \theta)$ space.}

Here, as the queue length $\theta$ increases (in response to some other exogenous change), the total cost $\theta h$ of compensatory transfers increases. By Condition (3.11) principals trade off $h$ and $t(x)$; the latter has to be further distorted in response.

4 Welfare considerations

The agents holding implicit bargaining power has welfare consequences in this model because their action is productive: it matters for the total surplus to be shared. We begin with some some intermediary results. First,
Lemma 1 Under search with friction, the optimal action \( a^S \) solving (3.10) is lower than the standard optimal action \( a^{SB} \) solving (2.2).

Principals compete to attract agents. Even if using compensatory transfers, attracting risk-averse agents is most efficiently achieved by reducing the variability of the transfer \( t(x) \): offering better insurance. As a result contracting agents face weaker incentives to exert effort.\(^{10}\) Next, social welfare reads

\[
W(a) \equiv M[1 - p_0(\theta)] [\pi(t, a) + U(t, a)] + N \left[ 1 - \frac{1 - p_0(\theta)}{\theta} \right] [u(h + y) - h],
\]

where the planner is constrained by the search frictions, and of course by moral hazard. The first term is the aggregate matching surplus of a contracting relationship. The second one is the aggregate meeting surplus: this is the total utility of paying \( h \) the unsuccessful agents. Our second intermediary result is

Lemma 2 Social welfare increases with the agents’ action.

Because a higher action shifts the distribution \( F(x|a) \) of the output in a first order sense, it is obvious that the social surplus of a dyad is increasing in the agent’s action – all things otherwise equal. It is also true in equilibrium: although a higher action is more expensive, it remains preferred by the principal.

4.1 Efficiency

In this section we establish results showing that combining search frictions with moral hazard matters for social welfare, and explain precisely the root cause of this problem.

4.1.1 The equilibrium is not constrained-efficient

A planner would direct principals to post contracts and let principals and agents match randomly. For a given number of principals and agents, the planner would choose a different allocation, even if constrained by moral hazard.

\(^{10}\)Indeed the reciprocal of \((3.13)\) is \(d\theta/da < 0\): a higher action means lower rents, and therefore a shorter queue at that principal.
Proposition 3. Fix $M$ and $N$. The competitive search equilibrium under moral hazard does not implement the utilitarian planner’s allocation. Decentralized matching induces less effort than the planner’s solution, which is the standard second-best solution.

The utilitarian planner seeks to maximize the surplus to be shared, which amounts to implementing the second-best action, and leaves no rent to the agents; this follows from Lemma 2. The planner is constrained by frictions but not subject to the MUP because he is agnostic as to which principal fails to contract. So he needs not attract agents by giving away rents. The planner can ignore this competition and select $U$ so as to maximize welfare: $U = u_0$ and $h = 0$. That is, the negative externality on principals that distorts the transfers and stems from their competing for the agents completely disappears, and principals become “efficient monoposonists” (as in Diamond, 1971). Furthermore,

Proposition 4. The competitive search equilibrium induces insufficient entry of principals. More precisely, market tightness $\Theta$ is too high in the competitive search equilibrium.

Decentralized matching induces excessive agent entry, who are attracted by the rents they can collect. These rents are generated by principal competition to attract agents. The planner is free of the externality generated by principal competition; he can direct the principals to post the standard second-best contract (Proposition 3). These generate no rents for the agents, which curbs entry.

We conclude this section with an important remark. The loss of efficiency is severe in this model in the sense that there exist no instrument (such as taxes) that can be used to correct a market allocation. This is akin to a failure of the second welfare theorem (in a second-best world).

Proposition 5. There exists no corrective instrument to restore constrained efficiency. More precisely, in any decentralized equilibrium $a^{SB} < a^S$.

The source of this result is the interaction of directed search and the endogenous nature of the surplus $E[x|a]$. While competing principals always face the incentive to trade off transfer and action, the planner does not. Instead the planner is constrained by the equilibrium level of frictions, but cares not as to who is unmatched. Hence matching is random, and orthogonal to contracts,
in the planner’s problem. It mutes the interaction between action and search, which is the only avenue to restore efficiency.

4.1.2 Action, directed search and social surplus

Both these results stand in contrast to much of the directed search literature (especially under symmetric information), where the competitive search equilibrium is second-best efficient. They hold regardless of the information structure and production technology. The fundamental reason is the interaction of directed search with the endogeneity of the social surplus – that is governed by the action.

When the surplus is fixed (no action) the only problem on hand is that of distribution. When the surplus depends on the action $a$, the first-best allocation is not replicated by the competitive search equilibrium, even under symmetric information and even with a deterministic output. Search frictions alone induce the effort choice to decrease because a principal is no longer the residual claimant of the agent’s action. Principals trade off the transfer they must give to attract agent with the benefit of a higher action; they use their two margins $t$ and $a$ to generate the rents they must give to agents to attract them.

The table below represents a taxonomy of information structure and technology. When social surplus depends on the agent’s action (and search is directed) the competitive equilibrium is never efficient.\(^{12}\)

<table>
<thead>
<tr>
<th>Technology/Information</th>
<th>Symmetric</th>
<th>Asymmetric</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deterministic</td>
<td>CS inefficient</td>
<td>n/a</td>
</tr>
<tr>
<td>Stochastic</td>
<td>CS inefficient</td>
<td>CS inefficient</td>
</tr>
</tbody>
</table>

Under search with moral hazard maximizing social welfare always conflicts with both the sharing rule – this is at the heart of the moral hazard problem – and rent-giving – at the heart of the search frictions. The interaction of moral hazard and search affects that sharing rule, which adds to the distortion in the action.

This differs from a problem with heterogenous agents (but no action $a$), where under full information principals have no second margin $a$ to use. In such a case there is no trade off and transfers are welfare neutral. Under adverse selection inefficiency arises because of the interaction

\(^{12}\)Details available in the online Appendix or from the authors.
with search frictions. Indeed incentive compatibility requires distorting the allocation, and principal competition requires further rent giving – further distortions.

4.2 Compensatory transfers.

Here we show that restricting the contract space worsens the outcome and that the meeting technology also impacts welfare. We also relate our results to the Hosios condition and clarify its interpretation when the social surplus depends on the agent’s action.

4.2.1 Characterization and welfare

Proposition 2 establishes that optimal compensatory transfers $h^S$ are always positive. These transfers not only alleviate the search problem, they also matter for efficiency. Suppose that $h ≡ 0$ exogenously so $u(h + y) = u_0$; constraint (3.6) disappears and the program becomes.

**Problem 3**

$$\max_{t(.),h} [1 - p_0(\theta)] \int_X [z - t(z)]dF(z|a)$$

s.t. (3.7), (3.8) and

$$\int_X u(t(z))dF(z|a) - c(a) \geq u_0$$

(4.1)

where (3.7) is rewritten to reflect $u(h + y) = u_0$. The characterization below departs from that of Proposition 2.

**Lemma 3** The solution $(t^N(x), a^N)$ to Problem 3 is characterized by the necessary and sufficient first-order conditions

$$\int_X [z - t(z)]dF_a(z|a) + \hat{\mu} \left[ \int_X [z - t(z)]dF_{aa}(z|a) - c''(a) \right] = 0$$

(4.2)

and

$$\frac{1}{w} = -\frac{\theta p'_0(\theta)}{1 - p_0(\theta) + \theta p'_0(\theta) U - u_0} + \hat{\mu} \frac{f_a}{f}$$

(4.3)

with $-p'_0 > 0$, $U > u_0$, $\hat{\mu} = \frac{\mu}{1 - p_0(\theta)} > 0$ and $\theta = \Theta$.

Equation (4.2) is as before; it is an envelop condition that arises from subgame perfection. Condition (4.3) is a functional equation that defines a fixed-point problem in the space of transfer
functions $T$. It is simpler than it first appears as soon as one notices that $\pi$ and $U$ are not just linear functionals, but expected values. That is, for a given function $t$, $\pi \in \mathbb{R}$ and $U \in \mathbb{R}$. Thus (3.11) rewrites

$$\frac{1}{u'} = \alpha + \hat{\mu} \frac{f_0}{f}, \quad \alpha \in \mathbb{R}_+,$$

which mimics Homlström’s standard condition (1979). Hence the first-order conditions (4.2), (4.3) and the constraints (3.8) and (3.7) completely identify the solution (for details of this approach see Roger, 2014).

**Remark 2** Restricting $h \equiv 0$ follows immediately from using a rival meeting technology, so Lemma 3 characterizes the contract when meeting is purely rival.

As before there is a trade-off between participation and incentives: a principal would like to attract more agents but he also needs to present the contracting agent with effort incentives. This trade-off is now handled by a single instrument: the transfer function $t(x)$, which must therefore be suitably distorted. Not using these compensatory transfers has welfare consequences too.

**Proposition 6** The optimal action $a^N$ solving (4.2) (when $h \equiv 0$) is always lower than $a^S$ solving (3.10) (using compensatory transfers).

Combining with Lemma 2 and Proposition 3 therefore

**Corollary 1** Welfare is higher when using compensatory transfers.

Compensatory transfers are used exclusively to attract agents. This allows the principals to reduce the rent $U$. This rent reduction is achieved by exposing the agent to more risk, which increases her action. Directly following from Proposition 6,

**Corollary 2** Entry is inefficient in the competitive equilibrium without compensatory transfers.

Constrained-efficiency fails for two reasons. First it is not induced by an optimal contract (except if the meeting technology is rival), and second a planner selects a higher action (intensive margin) but a lower market tightness (extensive margin); here the action is distorted away because of the search problem. It is also interesting to note that the adoption of a rival meeting technology has (negative) welfare consequences.
Corollary 3 Welfare is higher (under the optimal contract) when the meeting technology is non-rival.

The proof follows from Proposition 6 and Remark 2. A non-rival meeting technology allows principals to use compensatory transfers $h$. These are a form of insurance for the agents, so the rent $U$ can be reduced. If the meeting technology is a choice variable, principals or planner choose a non-rival technology.

In summary we can rank the equilibrium action for a given information structure. Under moral hazard

$$a^{SB} > a^S > a^N$$

and welfare follows the same ranking

4.2.2 The augmented Hosios condition

When principals do not use the compensatory transfers $h$ (either because the meeting technology is rival, or because the contract space is constrained) it is easy to recover an augmented Hosios condition. The elasticity of the meeting rate for principals is defined as

$$\eta(\theta) \equiv \frac{\partial (1 - p_0(\theta))}{\partial \theta} \frac{\theta}{1 - p_0(\theta)},$$

and Condition (4.3) may be re-arranged as

$$E\left[\frac{1}{u'(t)}\right] = \frac{\eta(\theta)}{1 - \eta(\theta)} \frac{\pi}{U - u_0}.$$

Then $\eta(\theta) = \frac{E[1/u'(t)](U - u_0)}{\pi + (U - u_0)E[1/u'(t)]}$ and we can write a modified Hosios condition $s(\theta; a^N) = 1 - \eta(\theta)$

$$s(\theta; a^N) = \frac{\pi(t, a^N)}{\pi(t, a^N) + (U(t, a^N) - u_0)E[1/u'(t)]}$$

(4.4)

which adjusts for risk-sharing and, most importantly, is conditional on the action $a$. That is, whether this modified Hosios condition characterizes constrained optimality depends on the equilibrium action $a^N$. So while the modified Hosios condition (4.4) affirms Pareto optimality (in the restricted class of contracts without compensatory transfers), as is known in many search problems, the allocation fails to be constrained efficient. The link between efficiency and the Hosios condition is broken here because the social surplus is endogenous to the action (production economy). Under competitive search that action is always suboptimal, and moral hazard adds to that distortion.
Pareto optimality and welfare maximization are in conflict in this model. Welfare maximization requires the agents to be pushed to their reservation utility while the equilibrium allocation reflects the implicit bargaining power search frictions afford them. That bargaining power implies a welfare-inefficient action. If the action $a^N$ were efficient (4.4) would characterize both Pareto optimality and welfare optimality.

5 Discussion and applications

In light of our results thus far we raise three items for discussion; we also briefly layout two applications.

5.1 Discussion of results

First we investigate the effect of a cap on compensatory transfers $h$; then we briefly discuss introducing an ex post auction as an alternative mechanism to contract posting. Last we discuss the restriction to symmetric equilibria.

Constraint on transfers. The analysis of the optimal contract takes transfers $h$ to be unconstrained. While the optimality condition (3.12) characterizing $h^S$ makes it plain it is always bounded, the total value $\theta \cdot h$ may be large. Here we seek the understand what happens when the sum of the transfers are bounded. The answer, not surprisingly, lies between $h \equiv 0$ and unconstrained transfers. What is more surprising is that, even when capped, compensatory transfers go a long way in unwinding distortions in $t(x)$

**Proposition 7** Insert the constraint $\theta h \leq H \in \mathbb{R}_{++}$ in Problem 2 and attach multiplier $\varphi \geq 0$. Whenever $\varphi > 0$ in equilibrium, the solution is characterized by

$$h = \frac{H}{\Theta} \quad (5.1)$$

$$\int_X [z - t(z)]dF_a(z|a) + \tilde{\mu} \left[ \int_X [z - t(z)]dF_{aa}(z|a) - c''(a) \right] = 0 \quad (5.2)$$

$$\frac{1}{u'(t)} = \frac{1 + \varphi}{u'(h + y) + \theta} \frac{\theta}{1 + p_0} + \tilde{\mu} \frac{f_a}{f} \quad (5.3)$$

with $\tilde{\mu} = \frac{\mu}{1 - p_0(\theta)} > 0$ and $\theta = \Theta$. Furthermore,

$$\nu(h^S) < \nu(H/\Theta) < \nu(h \equiv 0)$$
So we see from (5.3) that capping transfers reintroduce some distortion, as measured by the multiplier $\varphi$. But that distortion is limited and the transfer function resembles that characterized by (3.11). In consequence the multiplier $\nu$ is larger than with unconstrained transfer – and lower than without. By extension of our earlier result, the optimal action is also intermediate, and therefore so is welfare. So compensatory transfers always help.

**Ex post bidding.** Suppose the principal is able to solicit bids from agents if more than one is present at the contracting stage. This is a form of renegotiation, however it can happen only with the probability that more than one agent be present. We call this *ex post bidding*; it is often used in multistage procurement problems (e.g. bidders qualification followed by actual bidding).

There are two immediate consequences to ex post bidding. First the transfers $h$ have no credibility: they will be renegotiated away as soon as more than one agent shows up. Second, the only credible bid under moral hazard is the one that keeps the agent at her outside option – for any number $n \geq 2$ of bidding agent. This is simply Bertrand competition.

Hence ex post bidding is inefficient. First, compensatory transfers cannot be used, so by extension of Proposition 6 and Corollary 1 this mechanism underperforms compared to the optimal mechanism. Second, and less obviously, the rent $U$ offered to the agents has to be even higher than under price posting because the lottery they face changes from

$$\frac{1 - p_0(\theta)}{\theta} U + \left[ 1 - \frac{1 - p_0(\theta)}{\theta} \right] u(h + y), \ h \geq 0$$

to

$$V^j_A \equiv q_1(\theta)U^A + (1 - q_1(\theta))u_0$$

which is worse than the first one. As noted earlier (Lemma 1), increasing $U$ is achieved at the cost of decreasing the action.

**Restriction to symmetric equilibria.** In a finite economy limiting the scope to symmetric equilibria amounts to imposing that agents use mixed strategies off the equilibrium path in the continuation game. This is not restrictive: Bland and Loertscher (2012) provide a refinement that selects mixed strategies in the buyers’ continuation game. They show that all equilibria other than directed search equilibria violate a monotonicity property of agent’ strategies because they require at least one agent to visit a principal with higher probability after this principal increases
his price. Their result provides a rationale for focusing on directed search equilibrium with agents mixing off the equilibrium path. Furthermore, Galenianos and Kircher (2012) show in a canonical directed search models that the symmetric equilibrium is also unique. In other words there are no asymmetric equilibria if one focuses on agents playing mixed strategies off the equilibrium path.

5.2 Applications

**Labor market.** Take principals to be employers and agents to be workers seeking jobs. The measure of unemployed workers is fixed but free entry determines the measure of employers. Firms pay wage $w(x)$ contingent on realized output. In this static environment, the number of unemployed workers is $N_{q_0}(\theta)$, so $q_0(\theta)$ is the unemployment rate. Noting that $q_0'(\theta) > 0$, the probability to be unemployed increases with decreased measure of employers. Since $da/d\theta < 0$, there is excessive entry of workers (unemployment) in the competitive search equilibrium, and too little effort being exerted.

Much of the literature on unemployment focuses on moral hazard associated with the search effort of unemployed workers (e.g. Hopenhayn and Nicolini, 1997), or on the benefit and costs of unemployment insurance (e.g. Acemoglu and Shimer, 1999). Our model generates observationally equivalent outcomes to many of these papers: (i) employed workers receive rents and (ii) there is excessive (involuntary) unemployment. However the policy implications are very different. Offering unemployment insurance in this model amounts to increasing $u_0$, which we know worsens the search problem. That search problem is at the heart of inefficiency. Our model suggests that altering the search process is the necessary condition to restore efficiency, and so should be the object of policy intervention.

In particular, taxing employers and redistributing this revenue to workers is counterproductive in this model. It decreases the surplus $\pi$, which stifles principal entry – so the queue length $\theta$ increases – and simultaneously increases $u_0$, which worsens the search problem.

**Financial contracting.** Consider a financial contracting model as in the books of Tirole (2006) or Freixas and Rochet (2008) but with a continuous action and probability of success, say $\rho(a)$, continuous, increasing and concave. Financial institutions post contracts and borrowers search over
them; let $h \equiv 0$ for ease of exposition. The moral hazard constraint is

$$a \in \arg\max_{a' \in A} U = \rho(a')R_b - c(a')$$

where $R_b \geq 0$ is the borrower’s rent to be determined in equilibrium; let also $I$ be the investment required and $R$ the gross return of the project. Absent the search problem, a lender solves

$$\max_{R_b, a'} \rho(a')(R - R_b) - I$$

subject to the aforementioned constraint and $U \geq u_0$ as well as $\rho(a)(R - R_b) - I \geq 0$. With directed search, the problem becomes

$$\max_{\theta, R_b, a'} (1 - p_0(\theta)) \left[ \rho(a')(R - R_b) - I \right]$$

subject to the same constraints and

$$\frac{1 - p_0(\theta)}{\theta} U + \left( 1 - \frac{1 - p_0(\theta)}{\theta} \right) u_0 \geq \tilde{V}$$

It is easy to verify from the first-order conditions that the solution induces a lower action $a$, which clearly decreases the social surplus $\rho(a)R - I$. There are more borrower failures in equilibrium ($\rho(a^S) < \rho(a^{SB})$), and the condition $\rho(a)(R - R_b) - I \geq 0$ may fail (because of the search frictions); then there is credit rationing because of search.

6 Conclusion

In this paper we show that the interaction of competitive search and moral hazard has real implication for the nature of the optimal contract and for efficiency at both the intensive and the extensive margin. We explore thoroughly the source of inefficiency of the competitive search equilibrium and also clarify the distinction between meeting and matching.

First, although principals are allowed to offer menus of contracts contingent on the number of agents present at the time of contracting, these menus are dominated by a single contract that the principals commit to. A menu exposes the principals to a lottery over actions, the cost of which is convex. So principals are risk averse over actions and prefer a single contracts that guarantees them a known action. In turn this result affords us a simple characterization of the optimal contract.
We allow for directed search and moral hazard where the social surplus depends on the action of the agent(s). Precisely because search is directed, search frictions affect the social surplus to be shared through the level of action, and not just the sharing rule. The reason is that principals compete to attract agents by increasing their rents. They do so using the two margins available to them: the transfers and the action. So markets and frictions matter a great deal, even in bilateral contracting decisions. In this paper search frictions restore some bargaining power on the side of the agents – at a cost, because the social surplus is endogenous to the agents’ actions.

We believe that combining agency with competitive search creates a natural environment for a model of competitive agency. First solving the inefficiency problem requires different policy intervention. Second, search models of monetary policy may benefit of this innovation. It is already known that paying with debt is not the same as paying with cash, not because of record-keeping problems but because of ex post moral hazard (see DeMarzo, Kremer and Skrzypacz (2005) in the context of auctions). That is, there may be reasons that traders have to hold money balances beyond the standard credit rationing explanation. This question is left to future work.
Appendix

This Appendix has two parts. The first one contains the proofs. In the second one the reader one can find additional material that supports some claims in the text.

A Proofs

Proof of Proposition 1: By inspection of $V(C)$ the concavity of $u(\cdot)$ implies that $h_n^j = h^j$ for any $n$. Suppose the contract $C$ is offered in the other submarkets. Attach multipliers $\gamma, \mu, \nu$ and $\epsilon$, with $\gamma, \nu, \epsilon \geq 0$, to each of the constraints of Problem 1 and fix the action $a_n^j$ for each $n$; we solve the cost minimisation by selecting $t_n^j(x), h^j$ for each $n$. The FOC are

\[
-p_n(\theta)f(x|a_n^j) + \gamma u'(x|a_n^j) + \mu f_n(x|a_n^j) + \nu q_n(\theta)fu(x|a_n^j) = 0 \quad (A.1)
\]

\[
-p_n(\theta) - \gamma u'(h + y) + \epsilon + \nu q_n(\theta) \left[1 - \frac{1}{n}\right] u'(h + y) = 0 \quad (A.2)
\]

\[
\sum_n p_n'(\theta) \left[\int [z - t_n^j(z)]dF(z|a_n^j) - h\right] - \nu \left[\sum_n q_n'(\theta) \left(U_n^j + (n - 1)u(h + y)\right)\right] = 0 \quad (A.3)
\]

For each $n$ the complementary slackness conditions are

\[
\gamma [U_n^j - u(h + y)] = 0
\]

\[
\nu [V(C^j) - \bar{V}] = 0
\]

\[
\epsilon h = 0
\]

Condition (A.3) immediately implies $\nu > 0$. Next suppose $\gamma > 0$; re-arrange (A.1) as

\[
\gamma = \frac{p_n(\theta)}{u'(t_n^j(x))} - \left[\mu f_n(x|a_n^j) + \nu q_n(\theta)\right]
\]

and (A.2)

\[
\gamma = -\frac{1 - p_n(\theta) + \epsilon}{u'(h + y)} + \nu q_n(\theta) \left(1 - \frac{1}{n}\right)
\]

and so $\forall x \in X$ we have

\[
\frac{p_n(\theta)}{u'(t_n^j(x))} - \left[\mu f_n(x|a_n^j) + \nu q_n(\theta)\right] = -\frac{1 - p_n(\theta) + \epsilon}{u'(h + y)} + \nu q_n(\theta) \left(1 - \frac{1}{n}\right) \in \mathbb{R}^+$
\]

which is impossible; hence $\gamma$ and the first-order condition (A.1) simplifies to

\[
\frac{1}{u'(t_n^j(x))} = \nu \frac{q_n(\theta)}{p_n(\theta)} + \tilde{\mu} \frac{f_n(x|a_n^j)}{f(x|a_n^j)}, \quad n = 1, 2, ..., N
\]
where $\tilde{\mu} = \frac{\mu}{p_n(\theta)}$. Last suppose $\epsilon > 0$, (A.2) implies
\[
\nu q_n(\theta) \left[ 1 - \frac{1}{n} \right] u'(h + y)_{|h=0} = p_n(\theta) - \epsilon < \nu q_n(\theta) \left[ 1 - \frac{1}{n} \right] u'(h + y)_{|h>0} = p_n(\theta),
\]
which contradicts the fact that $u(\cdot)$ is concave. From Jewitt, Kadan and Swinkels (2008, now JKS) we know that fixing action $a^j_n$ the solution $t^j_n$ to this equation is unique for each $n$. To see why here, fix $a^j_n$, then $\tilde{\mu}$ is fixed, so by (A.4) and monotonicity of $u$, $t^j_n$ must be unique. By the first-order condition $\frac{\partial U_j}{\partial a^j_n} = 0$ (under the conditions of the FOA), we also know that the (agent-) optimal action $a^*$ is unique for a given transfer function $t^j_n$.

To show the equilibrium contract is a fixed transfer and action independent of the matching states, let $v(x) \equiv u(t(x))$, $\tilde{\nu} = \nu \frac{q_n(\theta)}{p_n(\theta)}$ and $L(\cdot)$ denote the Lagrangian of the cost-minimization problem. The effective cost of implementing action $a^j_n$ defined as
\[
C(a^j_n) = \max_{\tilde{\nu}, \tilde{\mu}} \min_v L(v)
\]
\[
= \max_{\tilde{\nu}, \tilde{\mu}} \tilde{\nu} \left[ V(C^j) \right] + \tilde{\mu} c'(a^j_n) - \int \rho \left( \tilde{\nu} u + \frac{f(a^j_n)}{f(z)} \right) dF(z|a^j_n)
\]
is convex in $a^j_n$ (see JKS). The first line is an application of the Lagrange duality theorem. In the second line $\rho(y) \equiv \max_v (yv - u^{-1}(v))$ is a convex function for any $y$ and we make use of the agent’s first-order condition $U_{a^j_n} = 0$ in the $V(\cdot)$ term. Take any $a_1 \leq a_2 \leq ... \leq a_N$ (w.l.o.g.) induced by $t_1 \neq t_2 \neq ... \neq t_N$ and define
\[
E[C(a^j_n)] \equiv \sum_{n=1}^N \Pr(a^j_n) C(a^j_n)
\]
This is a convex function for it is necessarily bounded (below and above). Furthermore, there also exists a convex function
\[
C(E[a]) \equiv C \left( \sum_{n=1}^N \Pr(a^j_n) a^j_n \right)
\]
For each $n$, let $a^j_n^*$ denote the optimal action; then $E[a^{j*}] \equiv \sum_{n=1}^N \Pr(a^j_n) a^j_n^*$ and
\[
E[C(a^j_n^*]) \geq C \left( E[a^{j*}] \right)
\]
which contradicts the premise that $t_1 \neq t_2 \neq ... \neq t_N$ are optimal given $E[a^{j*}]$. ■
Proof of Proposition 2: Form the Lagrangean of Problem 2 with the same multipliers. The FOC w.r.t. \( t(x), h, \theta \) are

\[-[1 - p_0(\theta)]f + \gamma u' f + \mu u' f_a + \nu \frac{1 - p_0(\theta)}{\theta} u' f = 0 \quad (A.5)\]

\[-\theta - \gamma u'(h + y) + \epsilon + \nu \left[ 1 - \frac{1 - p_0(\theta)}{\theta} \right] u'(h + y) = 0 \quad (A.6)\]

\[-p'_0(\theta) \int [x - t(x)]dF(x|a) - h - \nu \left[ \frac{1 - p_0 + \theta p'_0}{\theta^2} \right] [U - u(h + y)] = 0 \quad (A.7)\]

Recall that \( \pi = \int [z - t(z)]dF(z|a) \). Suppose first that \( \nu = 0 \), Condition (A.7) implies

\[ h = -p'_0 \pi > 0 \Rightarrow \epsilon = 0 \]

and Condition (A.6) becomes a contradiction since \( \theta > 0 \), \( \gamma \geq 0 \) and \( u'(h + y) > 0 \). Second, suppose \( \gamma > 0 \), (A.7) yields the same condition for \( h \) and so implies \( \epsilon = 0 \); then equate Conditions (A.5) and (A.6): \( \forall t(x), \forall x \) and any given \( h > 0 \),

\[ \frac{1 - p_0}{u'(t(x))} - \left( \frac{\mu f_a}{f} + \nu \frac{1 - p_0}{\theta} \right) = \gamma = \nu \left[ 1 - \frac{1 - p_0}{\theta} \right] - \frac{\theta}{u'(h + y)} \in \mathbb{R}^+ \]

which is impossible. Therefore \( \gamma = 0 \) too. Last \( \epsilon = 0 \) by the same argument as in the proof of Proposition 1. Hence the FOC (A.5) always reads

\[ \frac{1}{u'} = \frac{\nu}{\theta} + \frac{\mu f_a}{f} \quad \text{where} \quad \hat{\mu} = \frac{\mu}{1 - p_0}. \]

From Condition (A.6) again one has

\[ \nu = \frac{\theta}{u'(h + y) \theta - 1 + p_0} \]

To compute \( h \) re-arrange Condition (A.7) and substitute:

\[ h = -p'_0 \pi - \frac{1 - p_0 + \theta p'_0}{\theta - 1 + p_0} U - u(h + y). \]

Finally we have

\[ \frac{1}{u'(t)} = \frac{1}{u'(h + y)} \frac{\theta}{\theta - 1 + p_0} + \frac{\mu f_a}{f} \]

as claimed and indeed by integrating over \( \mathcal{X} \)

\[ \mathbb{E} \left[ \frac{1}{u'(t)} \right] = \frac{1}{u'(h + y)} \frac{\theta}{\theta - 1 + p_0} \]
because $\int f_a dF(z|a) = 0$. ■

**Proof of Lemma 1:** The statement we want to prove is this: consider a rent level $U \in [u_0, \overline{U})$, where $\overline{U}$ is such that $[[1 - p_0(\theta)]/\theta][\overline{U}] + [1 - (1 - p_0(\theta))/\theta]u_0 = \overline{V}$; to the rent level $U$ must correspond a higher action.

First note that for each action $a$, there exists a unique $x'$ such that $f_a(x')/f(x') = 0$ since the likelihood ratio is increasing and $\int f_a(z|a)dz = 0$. Let $U(t^S, a^S) = \overline{U}$ be such that the MUP (3.7) binds. Fix the action $a^S$ so that the distribution $F(\cdot|a^S)$ is fixed. For any $U < U(t^S, a^S)$, construct an alternative transfer scheme $t$ such that $U(t, a^S) = U$ (this needs not be an optimal scheme). Given $a^S$ here is some $\tilde{x} \in \mathcal{X}$ such that

$$t(x) = \begin{cases} < t^S(x), & \text{for } x < \tilde{x}; \\ = t^S(x), & \text{for } x = \tilde{x}; \\ > t^S(x), & \text{for } x > \tilde{x}. \end{cases}$$

that is, $t(\cdot)$ single-crosses $t^S(\cdot)$ from below at the point $\tilde{x}$. The action $a^S$ remains fixed and to the left of $\tilde{x}$, $f_a(x)/f(x) < 0$ while to its right $f_a(x)/f(x) > 0$. Now,

$$\int_{\tilde{x}}^{x} u(t(z))dF_a(z|a^S) + \int_{\tilde{x}}^{x} u(t(z))dF_a(z|a^S)$$

$$= \int_{\tilde{x}}^{x} u(t^S(z))dF_a(z|a^S) - \int_{\tilde{x}}^{x} [u(t^S(z)) - u(t(z))] dF_a(z|a^S)$$

$$+ \int_{\tilde{x}}^{x} u(t^S(z))dF_a(z|a^S) - \int_{\tilde{x}}^{x} [u(t^S(z)) - u(t(z))] dF_a(z|a^S)$$

$$= \int_{\tilde{x}}^{x} u(t^S(z))dF_a(z|a^S) - \int_{\tilde{x}}^{x} [u(t^S(z)) - u(t(z))] \frac{f_a}{f} dF(z|a^S)$$

$$+ \int_{\tilde{x}}^{x} u(t^S(z))dF_a(z|a^S) - \int_{\tilde{x}}^{x} [u(t^S(z)) - u(t(z))] \frac{f_a}{f} dF(z|a^S)$$

$$= c'(a^S) - \left(\int_{\tilde{x}}^{x} [u(t^S(z)) - u(t(z))] \frac{f_a}{f} dF(z|a^S) + \int_{\tilde{x}}^{x} [u(t^S(z)) - u(t(z))] \frac{f_a}{f} dF(z|a^S)\right)$$

$$> c'(a^S)$$

since $\int_{\tilde{x}}^{x} u(t^S(z))dF_a(z|a^S) + \int_{\tilde{x}}^{x} u(t^S(z))dF_a(z|a^S) = c'(a^S)$ by the moral hazard constraint and the facts that $u(t^S(x)) - u(t(x)) > 0$ and $f_a/f < 0$ to the left of $\tilde{x}$, and conversely to its right. So under $t(x)$ the moral hazard constraint (3.8) is slack and by construction $U = U(t, a^S) < U(t^S, a^S) = \overline{U}$;
that is,
\[
\int u(t(z))dF(z|a^S) - c(a^S) < \int u(t^S(z))dF(z|a^S) - c(a^S)
\]
\[
\int u(t(z))dF(z|a^S) < \int u(t^S(z))dF(z|a^S)
\]
\[
\Leftrightarrow \int t(z)dF(z|a^S) < \int t^S(z)dF(z|a^S)
\]

So the transfer \(t(x)\) is cheaper to the principal. 

**Proof of Lemma 2:** Given that \(M[1-p_0(\theta)] = N\frac{1-p_0(\theta)}{\theta}\), Social welfare rewrites
\[
W(a) \equiv M (1-p_0(\theta)) \left[ \int_X zdF(z|a) - T(a) + U(t,a) \right] + N \left[ 1 - \frac{(1-p_0(\theta))}{\theta} \right] [u(h+y) - h]
\]
where
\[
T(a) \equiv \int_X t(z)dF(z|a)
\]
is known to be an increasing, concave function (Conlon, 2008). Differentiate with respect to the action
\[
\frac{dW}{da} = -Mp'_0(\theta) \frac{d\theta}{dU} U_0 p_0(\theta) \left[ \int_X zdF(z|a) - T(a) + U(t,a) \right]
\]
\[
+ (1-p_0(\theta)) \left[ \int_X zdF_a(z|a) - T_a + U_a \right]
\]
\[
= (1-p_0(\theta)) \left[ \int_X zdF_a(z|a) - T_a \right],
\]
since \(U_a = 0\). Therefore \(\frac{dW}{da} > (\langle 0 \Leftrightarrow \int_X zdF_a(z|a) - T_a > (\langle 0. \) Because the multiplier \(\mu\) is known to be positive, the first-order condition (3.10) immediately tells us that \(\int x F_a(x|a) - T_a > 0\).

**Proof of Proposition 3:** Fix \(M, N\). In light of Proposition 1 one can use a unique tariff \(t(x)\) and a unique \(h\). To simplify consider the welfare per agent \(W(a)/N\); the utilitarian planner solves

**Problem 4**
\[
\max_{t,h,a} \frac{1-p_0(\theta)}{\theta} [\pi(t,a) + U(t,a)] + \left( 1 - \frac{1-p_0(\theta)}{\theta} \right) [u(h+y) - h]
\]
s.t. \(U(t,a) \geq u(h+y), U_a = 0, h \geq 0\)
The MUP (3.7) does not enter the planner’s problem because the planner can dictate the terms of trade to the principals. Attach multipliers $\gamma, \mu, \epsilon$ to each of these constraints. The first-order condition with respect to $a$ displays the standard envelop property. The one with respect to $t$ reads

$$[1 - p_0(\theta)]f[u'(t) - 1] + \mu u'f_a + \gamma u'f = 0,$$

which rewrites

$$\frac{1}{u'} - 1 = \hat{\gamma} + \hat{\mu} \frac{f_a}{f}$$

(A.8)

where $\hat{\gamma} = \gamma \frac{1}{1 - p_0(\theta)}$ and $\hat{\mu} = \mu \frac{1}{1 - p_0(\theta)}$ are just re-scalings. The point of interest is the complementary slackness condition

$$\gamma [U(t, a) - u(h + y)] = 0,$$

which establishes whether any rent is distributed to agents. Integrate (A.8) over $X$

$$\mathbb{E} \left[ \frac{1}{u'(t)} \right] - 1 = \hat{\gamma}$$

Case 1: $\gamma = 0$. Then $\mathbb{E} \left[ \frac{1}{u'(t)} \right] - 1 = 0$ and by the intermediate value theorem, there exist some $\tilde{x}$, $u'(t(\tilde{x})) = 1$ and so at that point $\hat{\gamma} + \hat{\mu} \frac{L_a}{f} = 0$ too. Then necessarily $\tilde{x}$ is associated to some action $a_1$ such that $f_a(\tilde{x}|a_1)/f(\tilde{x}|a_1) = 0$.

Case 2: $\gamma > 0$. In this case the same point $\tilde{x}$ is associated to some action $a_2$ such that $f_a(\tilde{x}|a_2)/f(\tilde{x}|a_2) = -\hat{\gamma} < 0$, which means that the likelihood ratio $f_a/f$ crosses 0 to the right of $\tilde{x}$. That is to say $F(x|a_2)$ first-order stochastically dominates $F(x|a_1)$. By application of Lemma 4, it is preferred by the planner. Hence we always have $\gamma > 0$.

The FOC with respect to $h$ is

$$\left(1 - \frac{1 - p_0(\theta)}{\theta} \right) [u'(h + y) - 1] + \epsilon - \gamma u'(h + y), \text{ with } \epsilon h = 0$$

Since we want $\gamma$ as large as possible, $\epsilon > 0$, that is, $h = 0$ and $u(h + y) = u_0$. Hence the planner’s preferred solution is indeed the second-best solution.

Proof of Proposition 4: Let $k$ denote the principals entry cost. By Proposition 3 a planner selects the standard second-best contract $t^{SB}(x), a^{SB}$ solving (2.1) and (2.2) with $h = 0$. With this he maximises

$$\frac{W(a; \theta)}{N} - \frac{M}{N} k = \frac{W(a; \theta)}{N} - \frac{k}{\theta}$$
by choice of the market tightness $\theta$, with condition

$$-p'_0(\theta) + 1 - p_0(\theta) \left[\left(\pi(a^{SB}) + U(a^{SB}) - u_0\right) + \frac{k}{\theta^2}\right] = 0$$

Because $\pi(a^{SB}) + U(a^{SB}) > \pi(a^{S}) + U(a^{S})$ and $u_0 \leq u(h^{S} + y)$, the market tightness $\theta$ is necessarily lower when selected by the planner. 

**Proof of Proposition 5:** Under the characterization of Proposition 2,

$$E_X \left[\frac{1}{u'(t^{SB})}\right] = \frac{1}{u'(h + y)} \frac{\theta}{\theta - 1 + p_0}$$

$h = -p'_0(\theta) \pi - \frac{1 - p_0 + \theta p'_0 U - u(h + y)}{\theta - 1 + p_0} u'(h + y)$

while efficiency requires $h = 0$, $U = u_0$ and $E_X \left[\frac{1}{u'(t^{S})}\right] = \lambda$. Consider any redistribution possible, and after redistribution, suppose there exists a transfer function $t(x)$ such that

$$\lambda = \frac{1}{u'(h + y)} \frac{\theta}{\theta - 1 + p_0},$$

then $t^{SB} = t^{S}$ since $t(\cdot)$ is uniquely defined. Hence $a^{SB} = a^{S}$ and indeed $U = u_0$, and $h = 0$. But then $-p'_0(\theta) = 0$, which contradicts our assumption that search is directed (i.e. $-p'_0(\theta) > 0$). Indeed for any corrective transfer, that is, any argument entering $\pi(t, a), U(t, a)$ or $u(h + y)$, $h = 0$ violates the optimality conditions (A.6) and (A.7). Similarly if considering the characterization of Lemma 3. 

**Proof of Lemma 3:** Contrast this to the case where $h \equiv 0$ exogenously. Then $u(y) = u_0$ and the constraint $U \geq u(h + y)$ becomes

$$U \geq u_0$$

with multiplier $\lambda \geq 0$. The FOC are

$$-[1 - p_0(\theta)] f + \lambda u' f + \mu u' f_a + \nu \frac{1 - p_0(\theta)}{\theta} u' f = 0$$

$$-p'_0(\theta) \int [x - t(x)] dF(x|a) - \nu \left[\frac{1 - p_0 + \theta p'_0}{\theta^2}\right] [U - u_0] = 0$$

with complementary slackness conditions

$$\nu [V - \tilde{V}] = 0$$

$$\lambda [U - u_0] = 0$$
Immediately we must have $\nu > 0$ and $\lambda = 0$ by direct inspection of the second condition. So again

$$\frac{1}{u'(t)} = \frac{\nu}{\theta} + \hat{\mu} \frac{f_a}{f}$$

where however

$$\nu(h \equiv 0) = -\frac{\theta^2}{1 - p_0 + \theta p'_0} \frac{p'_0 \pi}{U - u_0}$$

and the equilibrium transfer function clearly entails a distortion when compensatory transfers cannot be used. ■

**Proof of Proposition 6:** By Constraint (3.7) – the MUP,

$$U(h > 0) < U(h = 0)$$

necessarily (since $\bar{V}$ if fixed for any $h$) – the contracting agent receives a lower rent in equilibrium when using compensatory transfers. We’d like to know what that means in terms of equilibrium action. The comparison to the Lagrange multiplier $\nu(h^S)$ is not quite direct since

$$\nu(h^S) = -\frac{\theta^2}{1 - p_0 + \theta p'_0} \left[ \frac{p'_0 \pi}{U - u(h + y)} + \frac{h}{U - u(h + y)} \right]$$

from (A.7), with $h$ positive but $U - u(h + y) < U - u_0$. In addition, the equilibrium transfer functions $t(x)$ are not the same in both problems. To tackle this problem we must understand which contract is cheaper. Once again we turn to the representation of the cost of effort given by Jewitt (1997) and reproduced in JKS.

$$C(a) = \max_{\hat{\nu}, \hat{\mu}} \left[ \hat{\nu}(h)[u(h + y) + c(a)] + \hat{\mu} c'(a) - \int \rho \left( \hat{\nu}(h) + \hat{\mu} \frac{f_a}{f} \right) dF(z|a) \right]$$

where $\hat{\nu} \equiv \nu/\theta = \nu/\Theta$. Consider

$$\left. \frac{\partial \nu(h)}{\partial h} \right|_{h=0} = -\frac{\theta^2}{(1 - p_0 + \theta p'_0)(U - u(h + y))} \left[ 1 - \frac{(p'_0 \pi + h)u'(h + y)}{U - u(h + y)} \right] < 0$$

since $(p'_0 \pi + h) < 0$ as $\nu(h) > 0 \ \forall h$. So the principal always wants to offer at least some $h$, hence

$$\nu(h^S) < \nu(h \equiv 0).$$

Therefore the cost of effort is always lower when using compensatory transfers. ■
Proof of Proposition 7: Add the constraint $\theta h \leq H$ to Problem 2 with new multiplier $\varphi \geq 0$. The FOC w.r.t. $t(x), h, \theta$ are

$$-[1 - p_0(\theta)] f + \gamma u' f + \mu u' f_a + \nu \frac{1 - p_0(\theta)}{\theta} u' f = 0 \quad (A.9)$$

$$-\theta - \gamma u'(h + y) + \epsilon - \varphi \theta + \nu \left[1 - \frac{1 - p_0(\theta)}{\theta}\right] u'(h + y) = 0 \quad (A.10)$$

$$-p_0'(\theta) \int [x - t(x)] dF(x|a) - h - \nu \left[1 - p_0 + \theta p_0' \right] \frac{[U - u(h + y)] - h}{\theta^2} = 0 \quad (A.11)$$

The only new terms are $-\varphi \theta$ in (A.10) and $-h$ in (A.11). As before the complementary slackness conditions are

$$\nu [V - \tilde{V}] = 0$$

$$\epsilon h = 0$$

$$\gamma [U - u(h + y)] = 0$$

By arguments that are now standard, $\nu > 0, \gamma = 0$ so

$$\frac{1}{w'(t)} = \frac{\nu}{\theta} + \hat{\mu} \frac{f_a}{f}$$

The case of interest is when $\varphi > 0$, then $h = \frac{H}{\theta}$ and the last condition gives

$$\nu(H/\theta) = -\left[p_0' \pi + \frac{2H}{\theta}\right] \frac{\theta^2 (1 - p_0 + \theta p_0')}{U - u(H/\theta + y)}$$

and since $\frac{2H}{\theta} > 0$ we have $\nu(H/\theta) < \nu(h \equiv 0)$. Meanwhile Condition (A.11) yields

$$\nu(H/\theta) = \frac{\theta (1 + \varphi)}{w'(H/\Theta + y)} \frac{\theta^2}{\theta - 1 + p_0}$$

Substituting in $\frac{1}{w'(t)} = \frac{\nu}{\theta} + \hat{\mu} \frac{f_a}{f}$ gives (5.3), and since $\gamma > 0$, $\nu(H/\theta) > \nu(h^S)$ as claimed.  

B Material supporting the Taxonomy

Under symmetric information the action $a$ is observable and so can be used to condition the contract. Absent friction this implies that

$$u(t) - c(a) = u_0$$

$$\int z dF_a(z|a) = \frac{c'(a)}{u'(t)}$$
where $t \in \mathbb{R}$; let the solutions be denoted $a^{FB}$ and $t^{FB}$. This allocation is not replicated by the competitive search equilibrium under symmetric information.

**Lemma 4** Under symmetric information the competitive search equilibrium does not implement the first-best solution. It is characterized by the conditions

$$
\int zdF_a(z|a) = \frac{c'(a)}{w'(t)} \quad (B.1)
$$

$$
h = -p'_0(\theta)\pi - \frac{1 - p_0 + \theta p'_0}{\theta - 1 + p_0} \frac{u(t) - c(a) - u(h + y)}{w'(h + y)} \quad (B.2)
$$

with $u(t) - c(a) > u(h + y)$ and where $1/w'(t) = \nu/\theta$. The action is lower than $a^{FB}$. The planner’s solution implements the first best action.

**Proof:** A principal’s problem is

**Problem 5**

$$
\max_{t,a,\theta} \left[ 1 - p_0(\theta) \right] \left[ \int zdF(z|a) - t \right] - \theta h
$$

s.t.

$$
\begin{align*}
   u(t) - c(a) & \geq u(h + y) \\
   \frac{1 - p_0(\theta)}{\theta} [u(t) - c(a)] + \left[ 1 - \frac{1 - p_0(\theta)}{\theta} \right] u(h + y) & \geq \bar{V} \\
   h & \geq 0
\end{align*}
$$

Attach multipliers $\gamma, \nu$ and $\epsilon$. The optimality conditions read

$$
\begin{align*}
   -(1 - p_0) + \gamma u'(t) + \nu \frac{1 - p_0}{\theta} u'(t) & = 0 \quad (B.3) \\
   -\theta - \gamma u'(h + y) + \epsilon + \nu \left[ 1 - \frac{1 - p_0}{\theta} \right] u'(h + y) & = 0 \quad (B.4) \\
   -p'_0\pi - h - \nu \frac{\theta p'_0 + 1 - p_0}{\theta^2} [u(t) - c(a) - u(h + y)] & = 0 \quad (B.5) \\
   (1 - p_0) \int zdF_a(z|a) & = c'(a) \left[ \gamma + \nu \frac{1 - p_0}{\theta} \right] \quad (B.6)
\end{align*}
$$

with the usual complementary slackness conditions. Suppose $h = 0$ (i.e. $\epsilon > 0$), (B.5) implies a contradiction, so $\epsilon = 0$ and (B.4) implies $\nu > 0$. Re-arranging (B.3) and (B.4) and simplifying yields

$$
(1 - p_0) \frac{1}{w'(t)} = \nu - \frac{\theta}{w'(h)}, \forall t, h
$$
which is a contradiction. Hence $\gamma = 0$ and (B.6) rewrites
\[
\int z dF_a(z|a) = c'(a)\frac{\nu}{\theta} \quad \text{where}
\]
\[
\frac{1}{w'(t)} = \frac{\nu}{\theta}
\]
From (B.5) compute $h$. The allocation is not first-best because $\gamma = 0$ and $h > 0$. The proof of the last statement follows that of Proposition 3. ■

C Small market: validation

This section validates the use of a large market in the main paper. For concreteness we focus on a non-rival meeting technology, and for conciseness on the case of $h \equiv 0$ – which is also the norm in the literature.

There is a finite number $N$ of agents and a finite number $M$ of principals. The rest of the environment is as described in the main text. To solve for an equilibrium in finite market, we postulate a candidate (symmetric) equilibrium contract $C^* = \{t^*_n(x), a^*_n\}_{n=1}^N$, posted by all Principals $k \in \mathcal{M}\setminus j$, and consider the benefit of Principal $j$ deviating to contract $C^j = \{t^j_n(x), a^j_n\}_{n=1}^N$.

To simplify notation let $\sigma^j = \sigma$. Furthermore, let $B_n(N, \sigma) = \binom{N}{n} \sigma^n (1 - \sigma)^{N-n}$ the probability of $n$ agents selecting the deviating Principal. Similarly, let $B_n(N - 1, \sigma) = \binom{N-1}{n} \sigma^n (1 - \sigma)^{N-n-1}$ be the probability of $n$ other agents selecting the deviating Principal from the perspective of an agent considering selecting that Principal. Both these functions are clearly continuous and differentiable in $\sigma$. Noting that $B_n(N - 1, \sigma) = \frac{B_n(N, \sigma)}{\sigma N}$, the deviating Principal solves:

\[
\max_{\{t_n, a_n\}_{n=1}^N, \sigma \in (0,1)} \sum_{n=1}^N B_n(N, \sigma) \pi(t_n(x), a_n)
\]
\[\text{s.t. } U(t_n, a_n) \geq u_0 \quad \text{(C.1)}\]
\[a_n \in \arg\max U(t_n, a_n) \quad \text{(C.2)}\]
\[V_P(\sigma) \geq V_P(\sigma^*) \quad \text{(C.3)}\]
\[\sigma + (M - 1)\sigma^* = 1 \quad \text{(C.4)}\]

where
\[V_P(\sigma) \equiv \sum_{n=1}^N \frac{B_n(N, \sigma)}{\sigma N} (U(t_n, a_n) - u_0) + u_0,\]
and
\[ V_P(\sigma^*) = \sum_{n=1}^{N} B_n(N, \sigma^*) (U^*(t_n^*, a_n^*) - u_0) + u_0. \]

\( \sigma^* \) represent agents selection strategy for any non-deviating Principals. Note that it is responsive to \( \sigma \) so the Market Utility Property may not be applied. The Lagrangian for the problem is:

\[
\max_{\{t_n, a_n\}_{n=1}^{N}, \sigma \in (0,1)} \sum_{n=1}^{N} B_n(N, \sigma) \pi(t_n(x), a_n) + \mu U_{a_n} + \lambda(U - u_0) + \nu[V_P(\sigma) - V_P(\sigma^*)].
\]

We optimize over \( \sigma \) as in Proposition 1 (envelope condition). The necessary conditions are:

\[
t_n : \pi t_n + \mu U_{a_n} + \lambda \mu t_n + \nu \frac{U_{t_n}}{\sigma N} = 0, \ \forall n \leq N
\]

\[
a_n : \pi a_n + \mu U_{a_n} + \lambda U_{a_n} + \nu \frac{U_{a_n}}{\sigma N} = 0, \ \forall n \leq N
\]

\[
\sigma : \sum_{n=1}^{N} \frac{\partial B_n(N, \sigma)}{\partial \sigma} [\pi + \mu U_{a_n} + \lambda(U - u_0)] + \nu \left[ \sum_{n=1}^{N} \frac{\partial B_n(N, \sigma)}{\partial \sigma} (U - u_0) - \sum_{n=1}^{N} \frac{\partial B_n(N, \sigma^*)}{\partial \sigma^*} \frac{\partial \sigma^*}{\partial \sigma} (U^* - u_0) \right] = 0
\]

(C.5)

where \( \frac{\partial \sigma^*}{\partial \sigma} = -\frac{1}{M-1} \). The complementary slackness conditions associated with (C.3) and (C.1) read

\[
\nu(V - V^*) = 0 \text{ and } \lambda(U - u_0) = 0.
\]

Since we focus on symmetric equilibrium where \( \sigma = \sigma^* = \frac{1}{M} \), \( U = U^* \), \( t_n = t_n^* \) and \( a_n = a_n^* \) for all \( n \), (C.5) becomes:

\[
\sum_{n=1}^{N} \frac{\partial B_n(N, \sigma)}{\partial \sigma} [\pi + \mu U_{a_n} + \lambda(U - u_0)] + \nu \left[ \sum_{n=1}^{N} \frac{\partial B_n(N, \sigma)}{\partial \sigma} (U - u_0) - \sum_{n=1}^{N} \frac{\partial B_n(N, \sigma^*)}{\partial \sigma^*} \frac{\partial \sigma^*}{\partial \sigma} (U^* - u_0) \right] = 0
\]

Next, suppose \( \lambda > 0 \) then \( U = u_0 \) by complementary slackness; combining with \( U_{a_n} = 0 \) generates a contradiction by the third condition. So \( \lambda = 0 \) necessarily. To show \( \nu > 0 \), rewrite (C.5) again as

\[
\nu = \frac{\sum_{n=1}^{N} \frac{\partial B_n(N, \sigma)}{\partial \sigma} \pi(t_n, a_n)}{-\sum_{n=1}^{N} \frac{\partial B_n(N, \sigma)}{\partial \sigma} \frac{M}{M-1} (U(t_n, a_n) - u_0)}.
\]

It is easy to show that \( \frac{\partial B_n(N, \sigma)}{\partial \sigma} > 0 \) and \( \frac{\partial B_n(N, \sigma)}{\partial \sigma} < 0 \) for all \( n \). Therefore \( \nu > 0 \) and is unique and independent of \( n \). Rewriting the first two necessary conditions again using \( \sigma = \frac{1}{M} \) and \( \Theta = \frac{N}{M} \),

\[
\frac{1}{u_{t_n}} = \frac{\nu}{\Theta} + \frac{f_{a_n}}{\widetilde{f}}, \ \forall n \leq N \]

(C.6)

\[
\pi_{a_n} + \mu U_{a_n} = 0, \ \forall n \leq N.
\]
Here too we can make use of Proposition 1, which is not specific to a large market. Then things simplify further:
\[ \nu = \frac{(1 - \frac{1}{M})^N \pi}{\left[1 - (1 - \frac{1}{M})^{N-1} \pi \frac{M}{M-1} (U - u_0)\right]} > 0.\]

Substituting in (C.6) yields
\[ \frac{1}{u_t} = \mu f_a + \frac{\Theta (1 - \frac{1}{M})^{N-1} \pi}{\left(1 - (1 - \frac{1}{M})^N - \Theta (1 - \frac{1}{M})^{N-1} \frac{M}{M-1} (U - u_0)\right)},\]
and taking the limit as \(N, M \to \infty\) (but maintaining \(\Theta\) finite)
\[ \frac{1}{u'} = \lambda + \mu f_a + \frac{\Theta e^{-\Theta}}{1 - e^{-\Theta} - \Theta e^{-\Theta} (U - u_0)} \]
just as in the large market.
References


[36] Roger, G. (2014)“Participation in moral hazard problem.” mimeo, the University of Sydney

