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Formal and Informal Markets: A Strategic and Dynamic Perspective

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Abstract

In formal markets, to attract buyers, sellers must publicly advertise their prices and locations. But in informal markets, sellers must remain anonymous from government authorities. Since agents’ payoffs depend on the ratio of buyers and sellers in each of these markets, all agents try to position themselves in the market which can yield them the highest possible payoff. This strategic interaction in turn critically affects the time evolution of these two markets. In our benchmark model, in which only sellers can switch between these markets, there exists a unique stable dynamic equilibrium where formal and informal markets co-exist. Sellers switch from the formal to the informal market whenever costs of trading in the informal market decrease, and vice versa. In a richer environment, where both sellers and buyers can switch between markets, and the sellers’ and buyers’ costs of trading in the formal market net of those in the informal market have opposite signs, there exists a unique stable dynamic equilibrium where formal and informal markets co-exist.

JEL Codes: C7, D49.
Keywords: Price posting, bargaining, matching, formal sector, informal sector

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1 Introduction

While the definition of informal economies is subject to some disagreement, there is never any
debate that sellers in these markets strive to remain anonymous from taxing and regulatory
authorities (see, Feige (1990), for instance). Thus, while in formal markets, sellers need to
publicly advertise their prices and locations to attract buyers, in informal markets, sellers
need to avoid providing any public information about their locations. This simple and
innocuous fact has profound implications on: (i) the viable types of trading mechanisms
in these markets, and (ii) other distinguishing features that formal and informal sellers
would face, such as presence of taxes, regulations and provision of quality assurances in
formal markets relative to theft/bribes/confiscations faced by informal sellers.\(^1\) This has
not received attention in the literature and it is the focus of this paper. In particular, we
consider a framework where buyers can buy goods in either formal and informal markets
that are characterized by different trading protocols and the aforementioned specific market
features.\(^2\) Within this environment, we explore how these characteristics affect the agents’
payoffs, which depend on the ratio of buyers and sellers in each of these markets, which in
turn affects the evolution of the formal and informal market activity.

Currently the two main trading mechanisms that are typically observed throughout the
world are bargaining and price posting.\(^3\) These trading protocols differ in various dimensions.
It is also important to note that informative advertising, which coincides with the inception
of price posting, is an essential feature for this trading mechanism to be effective. This is
the case as price-posting sellers need to send informative signals describing their product,
price and location in order to attract buyers.\(^4\) Such detailed information is observed by all
market participants, including competitors, potential buyers, and also the tax and regula-
tory authorities. Consequently, public observability of sellers’ prices and locations makes
price posting incompatible with informal activity, as informal sellers, to remain hidden from

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\(^1\)Given that the formal sellers and their fixed locations are publicly observed by buyers and authorities,
formal sellers can credibly provide warranties to their customers. (This is consistent with anti-lemon laws
enforcing certain money-back guarantees prevalent in the formal markets in real-life.) In contrast, informal
sellers cannot credibly provide any such quality assurance to their customers as they do not legally exist.

\(^2\)The vast majority of informal sector activities in real life involves goods and services whose production
and distribution are perfectly legal. As such, in this paper, we focus on legal goods and services.

Clearly, illegal goods are also an important component of the informal economy. However, we have chosen
not to incorporate them, as the literature has not yet come to an agreement on how to model them. We
refer to the reader to Feige (1990), De Soto (1989) and Portes et al. (1989), among others, for more details
of informal market activities.

\(^3\)Historically, bargaining was the predominant trading mechanism; the use of posted prices by sellers is a
relatively recent phenomenon. Posted prices’ ascent and eventual widespread use date back to 1823, when
Alexander Stewart introduced posted prices in his New York City ‘Marble Dry Goods Palace’.

\(^4\)See Bagwell (2007) for more on the evolution of advertising.
the taxing and regulating authorities, need to use a trading mechanism that disseminates minimal public information about their whereabouts. Bargaining offers such a possibility for informal sellers. Hence, in this paper we consider an environment where, after sellers and buyers are matched in the informal sector, they share the total surplus via bargaining, whereas after sellers publicly post their prices (and locations) in the formal sector, buyers are directed to sellers by observing the sellers’ prices and decide which seller to visit as in Burdett et al. (2001). The agents’ payoffs critically depend on the ratio of buyers and sellers in each of these markets. Borrowing from evolutionary game theory, we then consider best response dynamics where buyers and sellers play the market posting and bargaining game infinitely many times. Agents switch from one market to the other at a rate which is proportional to their payoff differential between the two markets.

Since price-posting in the formal market and the relative bargaining shares in the informal market depend on the ratio of buyers and sellers in each of these two markets, all agents try to position themselves in the market which can yield them the highest possible payoff. This strategic interaction in turn critically affects the the relative size and evolution of these two markets. In our benchmark model, only sellers can switch between markets, while a constant fraction of buyers remain in either market. Within this simplified environment, Theorem 1 provides conditions under which there exists a unique stable equilibrium where formal and informal markets of nontrivial size co-exist.\(^5\) Theorem 1 also provides comparative statics results, which describe how the stable equilibrium is affected by the number of buyers in each market, and also by the distinguishing features of our formal and informal markets. In particular, we find that some sellers switch from the formal to the informal market whenever: (i) the formal sellers’ warranty erodes, (ii) the government imposes higher taxes and regulations in the formal market, (iii) the risk of crime (theft) and/or confiscation decreases in the informal market, and (iv) the number of buyers in the informal market increases. Conversely, sellers will switch from the informal to the formal market whenever the opposite changes occur.

Next, we examine a situation where both sellers and buyers can switch between formal and informal markets. In contrast to the previous case, analytical results are not possible in this richer environment. Result 2 summarizes the numerical results and qualitative properties of the equilibria. In this new environment, we find that, for a broad range of parameter values, if the net lump-sum cost for a seller in the formal sector relative to that in the informal sector has the opposite sign of the net lump-sum cost for a buyer in the formal sector relative to that in the informal sector, then there exists a locally stable equilibrium in which formal and

\(^5\)Throughout this paper, we use the term “equilibrium” to mean a stationary equilibrium — that is, one which is unchanging over time.
informal markets co-exist. If the two relative net lump-sum costs are both negative, then there exists a unique locally stable equilibrium where only formal activity takes place in the long run. In contrast, if the two relative net lump-sum costs are both positive, then there exists a unique locally stable equilibrium with only informal trade.

2 Related Literature

There is a vast literature that tries to estimate the size of the informal economy, which has resorted to direct methods such as surveys, as well as indirect methods which exploit discrepancies in data from multiple sources. For instance, researchers have examined inconsistencies between wages paid versus taxes raised, data from household expenditure surveys versus retail trade surveys and expenditure data versus income reported by the taxing authorities to estimate the size of the informal economy.\(^6\)

Much less attention, however, has been paid to the development of theoretical frameworks that incorporate formal and informal activity. Notable exceptions are the pioneering work of Rauch (1991) who considers various forms of regulation avoidance and that of Nicolini (1998) where tax evasion is the main cause for informal activities to exist along side formal markets. In particular, Rauch (1991) considers an environment with a minimum wage policy that can only be enforced for sufficiently large firms. This results in smaller firms operating in the informal sector and paying lower wages. In contrast, Nicolini (1998) shows how inflation is optimal when tax evasion in the informal is widespread as income generated by cash is difficult to monitor by tax authorities.

Subsequent general equilibrium models employ existing estimates of the informal economy as targets for their quantitative experiments. For instance, Koreshkova (2006) and Aruoba (2010) consider optimal taxation models and aim at rationalizing observed income and inflation tax rates for a cross section of countries taking as given the existing empirical estimates of the informal economy. Ordonez (2013) assess the quantitative effect of incomplete tax enforcement on aggregate output and productivity for Mexico.

Typically, the general equilibrium analyses that study the co-existence of informal and formal markets share the assumption that these activities are different in nature. These differences are such that either the goods being produced in formal and informal markets are assumed to be different, as in Aruoba (2010), or the technologies used to produce the goods in both markets or the means of payment required to obtain the goods in formal and informal markets are assumed to be different as in Koreshkova (2006), Antunes and Cavalcanti (2007), Amaral and Quintin (2006), Prado (2011) and D’Erasmo and Boedo (2012).

\(^6\)See Schneider and Enste (2000) for a thorough review of this literature.
As alluded to before, the literature has so far not fully explored the importance of different trading mechanisms with their corresponding matching technologies and their implied informational requirements for the co-existence of formal and informal activity. Exceptions are Aruoba (2010) and Gomis-Porqueras et al. (2012), who consider an environment where firms produce goods in different markets while using different trading mechanisms. Agents in decentralized markets bargain, and can evade taxes as they can always settle their transactions with fiat money as in Nicolini (1998). Buyers can also consume other formal goods where the terms of trade are given by Walrasian pricing which implicitly assumes complete observability of the terms of trade by all market participants. As in Aruoba (2010), in this paper all activity in bargaining markets is informal.\footnote{In contrast, in Gomis-Porqueras et al. (2012) not all activity in bargaining markets is informal as they can decide what fraction of decentralized trade is to be made visible to the taxing authority.}

3 Benchmark Model

Consider an economy with a large number of buyers and sellers. Each period, capacity-constrained sellers produce at most one indivisible unit of a homogeneous perishable good. Thus, it is not possible for sellers to accumulate an inventory of unsold goods, resulting in “production on demand” as in Burdett et al. (2001). Buyers can purchase this good in either formal or informal markets, but any given buyer cannot coordinate his search decisions with other buyers as to which seller to visit at a given point in time.

Let $b$ denote the ratio of buyers to sellers in the economy as a whole. As previously mentioned, in this section it is assumed that buyers are exogenously split between formal and informal markets. Let $b_{fo}$ ($b_{in}$) be the ratio of buyers in the formal (informal) market vis-a-vis the total number of sellers in both markets; thus, $b_{fo} + b_{in} = b$.

In contrast to buyers, sellers can switch between formal and informal markets. Let $s_{fo}$ ($s_{in}$) denote the fraction of sellers in the formal (informal) market so that $s_{fo} + s_{in} = 1$. These fractions can then change over time depending of the relative payoffs of sellers in formal and informal markets.

The simplification that only sellers can switch between formal and informal markets allows us to obtain analytical solutions. It also reflects the fact that sellers’ predominant factor in deciding where to locate their business is to be close to buyers, while for most buyers accessibility to sellers is not of first order importance. This is the case as households can also take into account other factors, such as access to the workplace and schools and commuting costs to name a few, when making their location choices. Thus, when one takes into account these factors, then the buyer’s decision on where to locate is less affected by the
sellers’ location. Thus, we can think of buyers being streamed into one market or the other on the basis of exogenous factors, whereas the seller’s location is endogenous and strategic.\(^8\)

In the next subsection, we describe the underlying preferences of buyers and sellers, the characteristics that distinguish formal and informal markets, as well as the different trading mechanisms and matching technologies.

### 3.1 Preferences

Buyers and sellers have quasilinear utilities. A buyer in the formal (informal) market obtains a value of \(v_{bu}^{fo}(v_{bu}^{in})\) when she consumes a unit of the good.\(^9\) Thus, if the formal (informal) market price of the good is \(p^{fo}(p^{in})\), then the formal (informal) buyer obtains a total payoff of \(v_{bu}^{fo} - p^{fo}(v_{bu}^{in} - p^{in})\) for any given period.

Sellers in the formal (informal) market incur a cost of \(c_{fo}^{se}(c_{in}^{se})\) per unit of good produced, which includes both labor and other input costs. Thus, the per period total payoff of formal (informal) sellers is \(p^{fo} - c_{fo}^{se}(p^{in} - c_{in}^{se})\).\(^10\)

In order for trades to occur in both markets, individual rationality for both buyers and sellers needs to hold. This requires that \(c_{fo}^{se} \leq p^{fo} \leq v_{bu}^{fo}\) and \(c_{in}^{se} \leq p^{in} \leq v_{bu}^{in}\) for every time period. Let \(g_{fo} := v_{bu}^{fo} - c_{fo}^{se}\) be the measure of the total “gains from trade” in the formal market, while \(g_{in} := v_{bu}^{in} - c_{in}^{se}\) represent the total “gains from trade” in the informal market. Without loss of generality, we normalize the buyer and seller’s utility functions so that \(g_{fo} = 1\).\(^11\) If agents do not trade, then each buyer and each seller obtains a zero payoff.

### 3.2 Formal versus Informal Markets: Further Differences

As mentioned before, apart from their trading mechanisms, formal and informal markets differ along several other features. Formal markets have taxes, regulations and possibility of quality assurance. Meanwhile, in informal markets, sellers incur costs due to theft, bribery, fines, and confiscations. These differences are mostly implied by different requirements of formal and informal markets’ sellers regarding the public information about their locations. The next two subsections will elaborate on these other distinguishing features of formal and informal markets, with significant emphasis on provision of quality assurances.

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\(^8\)For example, illiterate buyers find it more difficult to participate in the formal market, as fine print regarding sale conditions and quality assurance does not provide any information for them.

\(^9\)In Section 3.2.2, we will explain why, in general, \(v_{bu}^{in} < v_{bu}^{fo}\).

\(^10\)In Section 3.2.2, we will explain why, in general, \(c_{in}^{se} < c_{fo}^{se}\).

\(^11\)In Section 3.2.2, we will see that, in general, \(g_{in} < g_{fo}\).
3.2.1 Taxation, Regulation, Confiscation and Theft

As in Nicolini (1998), we assume formal sellers are taxed a fraction of their profits. Let us denote the effective tax rate for a formal seller as $T_{fo}$. In the informal sector, sellers neither pay taxes nor incur any regulation cost. However, each informal seller faces confiscation risk that his profits can be taken away from the government (through a fine) or by criminals (through theft). As in Prado (2011), these costs are presumed to be proportional to the informal seller’s earnings. These payments effectively function as a form of “income tax” on informal sector earnings. Let $T_{in}$ represent the effective tax rate associated with operating in the informal economy.

3.2.2 Quality Assurance

An important distinguishing feature of formal markets relative to informal ones—and one which has not previously been emphasized by the literature—is the provision of warranties. Since formal sellers are registered and monitored by government authorities, they can credibly write contracts that provide quality assurance to their customers. In contrast, informal sellers cannot credibly offer such quality assurances, as they are unregistered, so they do not exist within the confines of the legal system.

To incorporate warranties into this framework, let us consider uncertainty regarding the quality of the product purchased by the buyer. In particular, we assume that all sellers have access to the same technology, so that the probability of faulty goods in either the formal and informal market is the same. Let $c_{in}$ be the unit cost of sellers producing in the informal market, i.e., the unit cost of sellers producing the good without any quality assurance. Let $q$ be the per unit quality assurance cost that each formal seller incurs when providing the warranty. Thus, with warranty, the unit cost of production for formal sellers becomes $c_{fo} = c_{in} + q$. To simplify exposition, we assume that formal sellers pass all of the quality assurance costs $q$ onto the buyer.

When a good is defective, it provides less utility to the buyer. Thus, if $v_{bu}^{in}$ is the expected utility to the buyer of the good purchased in the informal market, then $v_{bu}^{in} < v_{bu}^{fo}$. The total value for the buyer of purchasing the good in the formal market is denoted by $v_{bu}^{fo} = v_{bu}^{in} + \alpha(q)$, where $\alpha(q)$ is the benefit that each formal buyer receives from quality assurance. We assume

---

12 Note the profit tax is equivalent to a value added tax on the sale of the goods.
13 This can capture the probability that an informal seller’s profit can be stolen or their merchandise be confiscated by government authorities.
14 This can take many forms such as free repair/replacement, a full money-back guarantee, on-site customer service, twenty-four hour telephone customer assistance, and/or cash compensation for unsatisfactory product performance.
that \( \alpha \) is increasing, differentiable, and concave, with \( \alpha(0) = 0 \).

**Lemma A**  If \( \alpha(\cdot) \) is increasing, differentiable, and concave, and formal sellers provide the efficient level of quality assurance, then \( g^{\text{in}} \leq g^{\text{fo}} \).

The trading protocols employed in the formal and informal markets are used as mechanisms for dividing the gains from trade between the buyer and the seller. Thus, the fact \( g^{\text{in}} \leq g^{\text{fo}} \) implies that there is generally a larger surplus to be divided in the formal market so both buyer and seller can potentially be better off. This, in turn, implies that the government can tax a fraction of up to \( T_0 = g^{\text{fo}} - g^{\text{in}} \) without driving participants into the informal market.

### 3.3 Trading Mechanisms and Matching Technologies

Below we provide details regarding the formal and informal markets’ respective matching technologies and trading mechanisms.

#### 3.3.1 Formal Markets

In order to capture the informational requirements of formal sellers, we use the price-posting and directed-search framework of Burdett et al. (2001). In the formal market, each ex-ante identical sellers have a precise location at any point in time. In order to attract buyers, these formal sellers advertise the price of their good as well as their location. The information contained in these advertisements are seen by all relevant ex-ante identical uncoordinated buyers.

Since sellers compete for buyers, and these buyers cannot coordinate which seller to visit, sellers and buyers in the formal market play a strategic game of complete information composed of three stages. In the first stage, sellers simultaneously, independently and costlessly advertise a single posted price as well as their location. In the second stage, buyers costlessly observe prices, and simultaneously and independently choose which seller(s) to visit. In the third stage, matches are realized and a trade takes place. It is assumed that, after visiting a seller, for buyers it is prohibitively costly to search again within the same period. So, for any

\footnote{Concavity is a very reasonable assumption in this context, because \( \alpha \) is generally bounded above: \( \lim_{q \to \infty} \alpha(q) = v^* - v^{\text{in}} \), where \( v^* \) is the value of consuming a “perfect” commodity, with no defects upon repair or replacement. Note that given the concavity of \( \alpha \) and the linearity of \( c_{\text{fo}} \) in \( q \), there is an optimal amount of \( q \), which may lead to a less than full replacement or less than perfect repair. This may also explain why many commodities’ free warranties in the U.S. only provide a coverage of parts for a limited period of time such as 90 days and a coverage of labor for even a shorter period of time. The extended warranty is not provided for free beyond a legally determined period of time.}
given period of time buyers can visit only one seller, but a seller can be visited by multiple buyers. In that case the seller sells his product to a randomly chosen buyer.

Note that, in this environment, formal buyers are more likely to visit a formal seller with the lowest posted price. But since buyers are not coordinated, they may face more competition at these cheaper locations. If multiple buyers choose to visit the same seller, then only one of the buyers can purchase the good, while the rest of buyers receive a payoff of 0. On the other hand, if no buyers visit a seller, then he cannot sell his good, so he receives a payoff of 0. In contrast to Camera and Selcuk (2009), here prices posted by sellers cannot be renegotiated depending on market conditions so that there is no distinction between the posted list price and the sale price.\footnote{Thus, it is not possible to consider a situation where transactions in some markets are settled at a price that differs from the posted price, depending on demand conditions.}

Since all buyers are \textit{ex-ante} identical, we focus on the symmetric equilibrium where buyers use the same mixed strategy when deciding which seller(s) to visit.\footnote{This focus on the symmetric equilibrium is very common in the price posting literature.} Likewise, since all sellers are also \textit{ex-ante} identical, they all use the same pricing strategy. The next theorem summarizes the main results of Burdett et al. (2001).

\textbf{Theorem BSW} \textit{Let }$m$\textit{ be the total number of sellers in the formal market, and let }$B_f$\textit{ be the ratio of buyers to sellers in the formal market (so there are }$B_f m$\textit{ buyers). There is a unique symmetric Nash equilibrium of the formal market game where all sellers post an identical price, }$p$\textit{, and all buyers randomly visit all sellers with equal probability. Let }$\Phi$\textit{ be the probability that any given seller sells his product (i.e. is visited by at least one buyer), and let }$\Omega$\textit{ be the probability that any given buyer purchases the good. Then }$p$, $\Phi$, \textit{and }$\Omega$\textit{ are entirely determined by }$B_f$\textit{ and }$m$. \textit{Furthermore, if we let }$m \to \infty$\textit{ while holding }$B_f$\textit{ fixed, then we get:}

$$
p(B_f) := \lim_{m \to \infty} p(B_f, m) = c_{se}^f + u_{se}^f(B_f), \quad (1)
$$

$$
where \quad u_{se}^f(B_f) := 1 - \frac{B_f}{e^{B_f} - 1}. \quad (2)
$$

$$
Also, \quad P_{se}^f(B_f) := \lim_{m \to \infty} \Phi(B_f, m) = 1 - e^{-B_f}, \quad (3)
$$

$$
and \quad P_{bu}^f(B_f) := \lim_{m \to \infty} \Omega(B_f, m) = \frac{P_{se}^f(B_f)}{B_f}. \quad (4)
$$

Intuitively, $P_{se}^f(B_f)$ ($P_{bu}^f(B_f)$) is the probability at any point in time that any particular
seller (buyer) is able to sell (buy) the good. For a given period, if a seller makes a sale in the large formal market \((m \to \infty)\), then his pre-tax payoff is given by \(u^\omega(B_f)\); otherwise his payoff is 0. Thus, the seller’s pre-tax expected payoff in the large formal market game is \(U^\text{fo}_{\infty}(B_f) := P^\text{fo}_{\infty}(B_f) \cdot u^\omega(B_f)\) where \(B_f = b^\omega/s^\omega\).

Given that the proportional tax rate paid by formal sellers is \(T^\text{fo}_{\infty}\), the seller’s expected after-tax payoff in the formal market is given by:

\[
\tilde{U}^\text{fo}_{\infty}(s^\omega) := (1 - T^\text{fo}_{\infty}) U^\text{in}_{\infty}(b^\omega/s^\omega)
= (1 - T^\text{fo}_{\infty}) \left(1 - \exp\left(\frac{-b^\omega}{s^\omega}\right)\right) \left(1 - \frac{b^\omega/s^\omega}{\exp(b^\omega/s^\omega) - 1}\right).
\] (5)

Note that we write \(\tilde{U}^\text{fo}_{\infty}\) as a function of \(s^\omega\) only, because in the benchmark model, \(b^\omega\) is fixed.

### 3.3.2 Informal Markets

Informal sellers cannot publicly advertise their exact locations and prices because they are trying to avoid government detection. However, buyers would nevertheless have some general knowledge where to find these informal sellers. As in the formal sector, each informal buyer can visit only one seller per period, and buyers cannot coordinate which seller to visit. Informal buyers search informal sellers with the matching probabilities as in the directed search model of Burdett et al. (2001). Thus, if \(B_i := b^\text{in}/s^\text{in}\) is the ratio of buyers to sellers in the informal market, then equation (3) in Theorem BSW implies that the probability that any given informal seller makes a sale during any given period is given by \(P^\text{in}_{\infty}(B_i) = 1 - e^{-B_i}\).

Instead of trading at publicly posted prices, the informal seller and buyer negotiate a price through bargaining, thereby splitting the total surplus, \(g^\text{in}\). Suppose that the informal seller receives a fraction \(\eta(B_i) \in [0, 1]\) of this surplus, while the informal buyer receives the remaining fraction \(1 - \eta(B_i)\). Thus, a matched informal buyer then receives a payoff equal to \(u^\text{in}_{\infty} := \eta(B_i) g^\text{in}\). Then the resulting pre-theft/confiscation cost expected payoff for sellers in the informal market is given by:

\[
U^\text{in}_{\infty} \left(\frac{b^\text{in}}{s^\text{in}}\right) := u^\text{in}_{\infty} \cdot P^\text{in}_{\infty} \left(\frac{b^\text{in}}{s^\text{in}}\right) = g^\text{in} \eta \left(\frac{b^\text{in}}{s^\text{in}}\right) \cdot P^\text{in}_{\infty} \left(\frac{b^\text{in}}{s^\text{in}}\right).
\] (6)

---

18. Recall that \(b^\omega\) is the ratio of buyers in the formal market relative to the total number of sellers in both markets, while \(s^\omega\) is the fraction of sellers in the formal market to the total number of sellers in both markets. Thus, we have that \(B_f = b^\omega/s^\omega\).

19. Informal sellers tend to gather in certain locations — certain street corners, parking lots, parks — and buyers know that informal sellers ‘can be found’ at these locations.

20. Note that \(\eta(B_i)\) is the function of the ratio of buyers to sellers, \(B_i\), which will determine the relative bargaining power of a buyer and a seller. Later, in Section 4.2, we will present one possible model of this surplus division process, but there is no need to commit to a specific model here.
Clearly, the higher the ratio of buyers to sellers in the informal market, the stronger each seller’s negotiating position or bargaining power becomes, and the better each seller will do in bilateral bargaining. In the limit when there are infinitely many buyers for every seller, the sellers will capture all of the surplus in the informal market. Thus, we suppose that the seller’s bargaining power $\eta$ is an increasing function, such that:

$$\lim_{B_i \to \infty} \eta(B_i) = 1.$$  

(7)

Recall that in our benchmark model buyers cannot switch between informal and formal markets, thus $b^i$ is a constant. In contrast, sellers are able to switch between markets at any point in time. Since $s^i = 1 - s^o$, we can regard the informal seller’s utility as a function of $s^o$ only. Thus, the relevant expect payoff for the informal sellers after taking into account their confiscation/theft cost is given by:

$$\tilde{U}_{in}^i(s^o) := (1 - T_{in}) \tilde{U}_{in}^i\left(\frac{b^i}{1 - s^o}\right) = (1 - T_{in}) g^i \eta \left(\frac{b^i}{1 - s^o}\right) \left(1 - \exp\left(-\frac{b^i}{1 - s^o}\right)\right).$$  

(8)

### 3.4 Dynamic Equilibrium

In the previous sections we have specified the payoffs of buyers and sellers in formal and informal markets for a given point in time. To explore how informal and formal markets evolve over time, we need to specify how agents will adjust and consequently how their payoffs will change over time. Here we consider this dynamic link between periods via our main equilibrium concept, ‘best response dynamics’, where agents that can switch between markets will migrate from one market to the other at a rate which is proportional to their payoff differential between the two markets. More precisely, let $s^o(t)$, and $s^i(t)$ represent the populations of formal/informal sellers at time $t$, and let $\dot{s}^o(t)$ and $\dot{s}^i(t)$ represent the derivatives of these functions at time $t$.\(^{22}\) Then we have that the evolution of these populations is as follows:\(^{23}\)

$$\dot{s}^o(t) = -\dot{s}^i(t) = \lambda_{se} \left(\tilde{U}_{fo}^o(s^o(t)) - \tilde{U}_{in}^i(s^o(t))\right).$$  

(9)

\(^{21}\)It would also be reasonable to assume $\lim_{B_i \to 0} \eta(B_i) = 0$. But this is unnecessary for our analysis.

\(^{22}\)Formally, in a continuous-time model, where $t$ ranges over the set of real numbers, we would define $\dot{s}^o := ds^o/dt$ and $\dot{s}^i := ds^i/dt$. In a discrete-time model, where $t$ ranges over the set of integers, we would define $\dot{s}^o(t) := s^o(t + 1) - s^o(t)$ and $\dot{s}^i(t) := s^i(t + 1) - s^i(t)$. Thus, the dynamical equation (9) admits both a continuous-time and a discrete-time interpretation. The equilibrium characterization of Theorem 1 holds in both cases.

\(^{23}\)The expected utility of buyers in the informal market is irrelevant to the dynamics of this model, because we have assumed that they cannot switch from informal to informal markets.
where \( \widetilde{U}^{fo}(s^{fo}) \) and \( \widetilde{U}^{in}(s^{in}) \) are as defined in equations (5) and (8), while \( \lambda_{se} : \mathbb{R} \rightarrow \mathbb{R} \) is a strictly increasing function which modulates the speed of adjustment, with \( \lambda_{se}(0) = 0 \). If migration from the formal to informal sector is exactly as difficult and costly as migration from the informal to the formal sector, then \( \lambda_{se} \) will be an odd function. However, if migrating in one direction is more difficult than migrating in the other direction, then \( \lambda_{se} \) will not be an odd function.

An equilibrium of an economy where sellers can decide to produce in formal and informal markets is a fixed point of the dynamical system represented in equation (9). Given a fixed fraction of buyers participating in the formal and informal market (\( b^{fo} \) and \( b^{in} \) respectively), sellers will migrate between the two markets at a rate which is proportional to the payoff differential between the two markets (of which speed of adjustment is modulated by \( \lambda_{se} \)), until their payoffs from both markets are the same. Thus, the economy is in equilibrium if and only if \( s^{fo} = s^{*} \) is a value such that:

\[
\widetilde{U}^{fo}(s^{*}) = \widetilde{U}^{in}(s^{*}).
\]

There are two other frameworks that lead to the equilibrium represented by equation (10), namely those of the uncorrelated, symmetric mixed Nash equilibrium and the replicator/imitation dynamics. These additional equilibrium concepts, along with best response dynamics, and their interpretations will complement each other in covering different practical features of the structure and dynamics of formal and informal markets. We refer the reader to Appendix B for further details.

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24 Typically, \( \lambda_{se} \) is just multiplication by a positive constant.

25 That is: \( \lambda_{se}(-r) = -\lambda_{se}(r) \), for all real numbers \( r \).

26 If \( \lambda_{se} \) is an odd function, then the dynamical system converges to equilibrium just as quickly from either direction. Thus, the informal market would show a symmetric response to tax increases and tax decreases, as found by Christopoulos (2003) in Greece. To see how \( \lambda_{se} \) might not be odd, note that it might cost more for a seller to switch from the informal market to the formal market than vice versa (e.g. because of the need to acquire licenses, rent a retail location, etc.); this would be reflected by having \( |\lambda_{se}(r)| < |\lambda_{se}(-r)| \) for any given \( r > 0 \). This is consistent with empirical findings by Giles et al. (2001) and Wang et al. (2012) in Taiwan and New Zealand, respectively.

27 Throughout this paper, we use the term “equilibrium” to mean a stationary equilibrium—that is, one which is unchanging over time.

28 Best response dynamics highlights a behavioral aspect where agents play an infinitely repeated game, in which each seller decides whether to migrate from one market to the other at a rate which is proportional to their payoff differentials in these markets. Uncorrelated, symmetric mixed Nash equilibrium is appropriate in a model where each seller may divide his efforts—or perhaps randomize—between formal and informal market activities (although these activities need not take place at the same location). Replicator/imitation dynamics is appropriate in a model where sellers decide to migrate by imitating other agents; in this case, the size of each group following a strategy could be as important as their relative payoffs.
3.5 Existence and Properties of the Equilibrium

Let us define $\tilde{U}(s) := \tilde{U}_{se}^f(s) - \tilde{U}_{se}^i(s)$, which represents the sellers’ net gain from being in the formal market over being in the informal market. Note then that the equilibrium condition, given by equation (10), is equivalent to $\tilde{U}(s^*) = 0$.

We say that an equilibrium $s^*$ is *locally stable* and has formal and informal market of nontrivial size whenever we satisfy: (i) $\tilde{U}'(s^*) < 0$ and (ii) $0 < s^* < 1$. The main result of this section is the following.

**Theorem 1** If we satisfy the following conditions

$$1 - \frac{b^f + 1}{\exp(b^f)} < \frac{(1 - T_{se}^i) g^i}{(1 - T_{se}^f)} < \frac{1}{\eta(b^i) \left(1 - \exp(-b^i)\right)},$$

then there exists a locally stable mixed market equilibrium $s^* \in (0, 1)$. Furthermore, the equilibrium fraction of formal sellers, $s^*$, has the following properties: (i) $s^*$ is decreasing as a function of $g^i$, $T_{se}^f$, and $b^i$; and (ii) $s^*$ is increasing as a function of $T_{se}^i$ and $b^i$.

Recall that $b^f$ and $b^i$ are exogenous constants in this model. If $b^f = 0$, then the equilibrium described in Theorem 1 will yield $s^f = 0$. Likewise, if $b^i = 0$, then we will have $s^i = 0$. However, if $b^f$ and $b^i$ are both nonzero, then $s^f$ and $s^i$ will also be nonzero in equilibrium. Thus, for a broad range of parameters of the model, formal and informal markets of nontrivial size will co-exist in a stable equilibrium. As long as there are some buyers in both formal and informal markets, all sellers cannot find it in their interest to settle in only of one of the markets regardless of $g^i$, $T_{se}^f$, $T_{se}^i$, since some sellers in the market that would contain all sellers would find in their interest to switch to the other market with no sellers to serve the buyers there.

**Theorem 1** also highlights what makes sellers move between markets and in which direction. Part (ii) of Theorem 1 states that sellers will tend to migrate from the formal market to the informal market if the formal market’s advantage in quality assurance erodes ($g^i$ increases relative to $g^f$), or the government imposes higher taxes and regulations ($T_{se}^f$ increases), or more buyers migrate to the informal market ($b^i$ increases). Conversely, sellers will migrate from the informal market back to the formal market whenever the opposite changes occur in these parameters. Likewise, sellers will migrate to the formal market if the risk of crime and/or confiscation increases in the informal market (i.e. $T_{se}^i$ increases) or if buyers migrate to the formal market ($b^f$ increases).

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29One can also imagine circumstances where it is the buyers who are mobile, while the sellers are fixed—for instance, tourists arriving at New York City who know where to find all formal and informal sellers. In that case, it would not be difficult to see that the above result (and comparative statics) would still hold.
Both Buyers and Sellers Moving across Markets

Now we consider an environment where both sellers and buyers can switch among formal and informal markets at any point in time. In this new environment, the buyer populations, $b^i$ and $b^o$, and seller populations, $s^i$ and $s^o$, are endogenous and can change over time. As in Section 3, we also consider factors other than the trading mechanisms that make formal and informal different.

Next we describe the new equilibria resulting in formal and informal markets.

4.1 Formal Markets

As in Section 3.3.1, the behaviour and payoffs of buyers and sellers in the formal market are summarized in Theorem BSW, but the behaviour and payoffs of buyers and sellers in the informal market will need to be re-derived.

Recall that $B_f = b^o / s^o$ is the ratio of buyers to sellers in the formal market. Then a formal seller’s pre-tax expected utility is again given by $U_{se}^{fo}(B_f) := P_{se}^{fo}(B_f) \cdot u_{se}^{fo}(B_f)$, where $P_{se}^{fo}(B_f)$ and $u_{se}^{fo}(B_f)$ are defined in equations (2) and (3).

Let $p(B_f)$ be the formal market equilibrium price from equation (1). If a formal market buyer makes a purchase, then her payoff will be given by:

$$u_{bu}^{fo}(B_f) := v_{bu}^{fo} - p(B_f) = v_{bu}^{fo} - c_{se}^{fo} - u_{se}^{fo}(B_f) = g^{fo} - u_{se}^{fo}(B_f) = \frac{B_f}{e^{BF} - 1}. \quad (11)$$

If a buyer doesn’t make a purchase then her payoff is zero. Thus, her expected payoff for participating in the large formal market game is $U_{bu}^{fo}(B_f) := P_{bu}^{fo}(B_f) \cdot u_{bu}^{fo}(B_f)$, where $P_{bu}^{fo}(B_f)$ is defined in equation (4).

4.2 Informal Markets

Here, we assume that informal buyers have fixed locations (home or workplace), and informal sellers visit them. This feature tries to capture the door to door selling strategy used by informal sellers in some developing countries. As in the previous section, the matching probabilities and tie breaking rule are given by Burdett et al. (2001). Thus, if $S_i := s^i / b^i$ is the ratio of sellers to buyers in the informal market, then the probability for a given buyer to be visited by at least one seller during any given period is obtained by replacing $B_f$ with $S_i$.

\[^{30}\text{The only difference is that here the roles of buyers and sellers are reversed relative to Burdett et al. (2001).}\]
in equation (3), to obtain:

\[ P_{\text{bu}}(S_i) = 1 - e^{-S_i}. \]

Likewise, the probability of a given seller making a sale to the one buyer he visits is given by:

\[ P_{\text{se}}(S_i) = \frac{P_{\text{bu}}(S_i)}{S_i}. \]

In section 3.3.2, we assumed that the informal seller and buyer negotiated to split the surplus according to proportions \((\eta, 1 - \eta)\), where \(\eta \) depended on the ratio of buyers to sellers in this market.\(^{31}\) Now, however, we need to model the negotiation process more explicitly, as the outside options of buyers and sellers are both relevant. This is the case as both buyers and sellers can switch markets. The trading mechanisms we use are the solution concepts of the prominent Nash bargaining framework. Given our assumptions about the utility functions of the buyer and seller, the (bargaining) set of feasible utility allocations is the convex hull of the points \((0, 0)\), \((g^u, 0)\), and \((0, g^u)\), where \(g^u\) is the total gains from trade to be divided in the informal market. Thus, the Pareto frontier is the diagonal line from \((g^u, 0)\) to \((0, g^u)\). Hence, due to this linearity of the Pareto frontier, the axioms of Symmetry and Pareto Optimality imply that the Nash bargaining solution, the egalitarian bargaining solution, and the Kalai-Smorodinsky bargaining solution all coincide to yield the same outcome. Thus, it does not matter which particular bargaining solution we use; the bargaining outcome is robust.

Within the Nash bargaining framework, the bargaining outcome is determined by the disagreement payoffs of the two parties, namely the fallback positions or outside options available to buyers and sellers. Formally, let \(U_{\text{se}}^\text{in}\) be the expected payoff for a seller participating in the informal market, and let \(U_{\text{bu}}^\text{in}\) be the expected payoff for a buyer participating in the informal market. Note that \(U_{\text{se}}^\text{in}\) and \(U_{\text{bu}}^\text{in}\) are functions of \(S_i := s_{\text{in}}/b_{\text{in}}\), the ratio between informal sellers and buyers. Let \(\delta \in (0, 1)\) be a discount factor. If bargaining breaks down, then both parties must re-enter the informal market during the next period. The outside option for the seller is \(\delta U_{\text{se}}^\text{in}(S_i)\), while the outside option for the buyer is \(\delta U_{\text{bu}}^\text{in}(S_i)\). The Nash bargaining solution awards the seller a payoff of \(u_{\text{se}}^\text{in}(S_i)\) and the buyer a payoff of \(u_{\text{bu}}^\text{in}(S_i)\), where

\[
\begin{align*}
  u_{\text{se}}^\text{in}(S_i) &= \frac{\delta U_{\text{se}}^\text{in}(S_i) + g^u - \delta U_{\text{bu}}^\text{in}(S_i)}{2}; \\
  u_{\text{bu}}^\text{in}(S_i) &= \frac{\delta U_{\text{bu}}^\text{in}(S_i) + g^u - \delta U_{\text{se}}^\text{in}(S_i)}{2}.
\end{align*}
\]

\(^{31}\)It was not necessary to be more specific about the negotiation process in order to obtain Theorem 1.
However, $U_{in}(S_i) = P_{in}(S_i) w_{in}(S_i)$ and $U_{in}(S_i) = P_{in}(S_i) w_{in}(S_i)$. Once we substitute these expressions into (14), we obtain a pair of linear equations for $u_{in}(S_i)$ and $u_{in}(S_i)$. Solving these equations yields the following payoffs for the buyer and seller:\[u_{in}(S_i) = g_{in}^\delta \frac{\delta P_{in}^u(S_i) - 1}{\delta P_{bu}^u(S_i) + \delta P_{in}^u(S_i) - 2};\]
\[u_{in}(S_i) = g_{in}^\delta \frac{\delta P_{in}^u(S_i) - 1}{\delta P_{bu}^u(S_i) + \delta P_{in}^u(S_i) - 2}.
\[\text{(15)}\]

### 4.3 Lump-sum Costs in Formal and Informal Markets

As discussed in Section 3.2.1, $T_{fo}$ is the effective tax rate paid by sellers in the formal market that takes into account taxes. As in Prado (2011), in this section we also consider other per-period costs that are incurred when participants in these markets. In particular, a seller participating in the formal market must incur lump-sum costs (independent of profits) which we will denote by $L_{fo}$. These lump-sum costs involve the combined cost of having retail space and paying for licensing fees to comply with government regulations (e.g. fire safety). Formal buyers incur transportation costs that are independent of whether they acquire the product or not. These lump-sum costs are represented by $L_{fo}^b$.

When sellers operate in the informal market they face an implicit tax rate $T_{in}^s$ which incorporates the theft/confiscation risks that an informal seller may face. Informal sellers may also have to pay bribes to corrupt police officials or “protection fees” to organized crime against confiscation and theft risks; these are lump-sum payments, independent of a seller’s earnings. Thus, we further assume that each informal seller also incurs a lump-sum cost of $L_{in}^s$ dollars. Informal buyers incur an opportunity cost of waiting around for sellers to arrive; we represent this by a lump-sum cost of $L_{in}^b$ dollars.

Let us define $L_{in} := L_{fo} - L_{in}^s$ ($L_{bu} := L_{fo}^b - L_{in}^b$) which represents the net lump-sum cost for sellers (buyers) in the formal sector. For modelling purposes, it is equivalent to suppose that informal buyers and sellers face no lump sum costs, whereas formal buyers and sellers face lump sum costs of $L_{in}^b$ and $L_{in}^s$ respectively.

Let $R_{fo} := 1 - T_{fo}$ ($R_{in}^s := 1 - T_{in}^s$) denote the “residual” earnings rate of sellers in the formal (informal) markets after proportional taxes are paid. Let $R_{in} := R_{fo}^s/R_{in}^s$; this is

\[\text{If } \delta = 1, \text{ then the bargaining outcome (15) can be seen as a particular case of the abstract surplus-division model considered in Section 3.3.2. To see this, let } B_i := 1/S_i, \text{ and let } \eta(B_i) := w_{in}(S_i)/g_{in}, \text{ where } w_{in}(S_i) \text{ is defined as in Eq.(15). Then } \eta \text{ satisfies the conditions proposed in Section 3: it is an increasing function of } B_i \text{ because } w_{in} \text{ is a decreasing function of } S_i, \text{ and the limit (7) holds because } w_{in}(0) = g_{in}.\]

Clearly, these net lump sums could be negative as well, if the costs in the informal sector are higher than those in the formal sector.
effectively the “net” residual earnings rate for formal sellers, if we normalize the informal sellers’ residual earnings rate to 1. Note that this is equivalent to assuming that informal sellers capture all their earnings, while formal sellers only capture a proportion $R_{se}$. This can represent a situation where informal sellers face no risk of theft, while formal sellers pay an effective tax rate of $T_{ax} := 1 - R_{se}$. Note that if expected losses due to theft in the informal market are higher that the formal tax rate, then we will have $R_{se} > 1$, which implies that $T_{ax} < 0$.

4.4 Dynamic Equilibrium

Having specified all differential costs of trading in formal and informal markets, we can now analyze the corresponding dynamic equilibrium for this new environment. As in Section 3, we consider the dynamic equilibrium induced primarily by best response dynamics. This yields the following dynamic equations:

\[
\begin{align*}
\dot{s}_{fo}(t) &= \lambda_{se} \left( (1 - T_{ax}) U_{wo} \left( \frac{b_{fo}(t)}{s_{fo}(t)} \right) - L_{wo} - U_{wo} \left( \frac{s_{in}(t)}{b_{in}(t)} \right) \right) \\
\dot{b}_{fo}(t) &= \lambda_{bu} \left( U_{bu} \left( \frac{b_{fo}(t)}{s_{fo}(t)} \right) - L_{bu} - U_{bu} \left( \frac{s_{in}(t)}{b_{in}(t)} \right) \right);
\end{align*}
\] (16)

where $\dot{s}_{fo}(t)$, $\dot{b}_{fo}(t)$, $\dot{s}_{in}(t)$, and $\dot{b}_{in}(t)$ represent the corresponding time derivatives, and where $\lambda_{bu} : \mathbb{R} \rightarrow \mathbb{R}$ and $\lambda_{se} : \mathbb{R} \rightarrow \mathbb{R}$ are strictly increasing functions which modulate the speed of adjustment, with $\lambda_{bu}(0) = 0 = \lambda_{se}(0)$.

As in Section 3, a seller will find the formal market more attractive than the informal market if and only if

\[(1 - T_{ax}) U_{wo} \left( \frac{b_{fo}}{s_{fo}} \right) - L_{wo} > U_{wo} \left( \frac{s_{in}}{b_{in}} \right).\]

Likewise, a buyer will find the formal market more attractive than the informal market if and only if

\[U_{bu} \left( \frac{b_{fo}}{s_{fo}} \right) - L_{bu} > U_{bu} \left( \frac{s_{in}}{b_{in}} \right).\]

As a result, the necessary and sufficient condition for a population distribution $(b_{fo}, b_{in}, s_{fo}, s_{in})$ to be an equilibrium is that these shares are a fixed point of equation (16), which means

\[
(1 - T_{ax}) U_{wo} \left( \frac{b_{fo}}{s_{fo}} \right) - L_{wo} = U_{wo} \left( \frac{s_{in}}{b_{in}} \right) \quad \text{and} \quad U_{bu} \left( \frac{b_{fo}}{s_{fo}} \right) - L_{bu} = U_{bu} \left( \frac{s_{in}}{b_{in}} \right).\] (17)

Since $b_{fo} + b_{in} = b$ and $s_{fo} + s_{in} = 1$, the market is completely described by the ordered pair

\[\text{footnote 22.}\]

\[\text{See footnote 24 and 26.}\]
\((b^{in}, s^{in})\), and equation (17) reduces to:

\[
(1 - T_{se}) U_{se}^{fo} \left( \frac{b - b^{in}}{1 - s^{in}} \right) - L_{se} = U_{se}^{in} \left( \frac{s^{in}}{b^{in}} \right) \quad \text{and} \quad U_{bu}^{fo} \left( \frac{b - b^{in}}{1 - s^{in}} \right) - L_{bu} = U_{bu}^{in} \left( \frac{s^{in}}{b^{in}} \right).
\] (18)

An equilibrium \((b^*, s^*)\) is locally stable if there exists some neighbourhood \(U\) around \((b^*, s^*)\) such that, for any \((b^{in}, s^{in}) \in U\), the forward-time orbit of \((b^{in}, s^{in})\) under (16) converges to \((b^*, s^*)\). Graphically, it is easy to identify a locally stable equilibrium. To this end, let us rewrite (16) more generally as follows:

\[
\dot{b}^{in} = \beta(b^{in}, s^{in}) \quad \text{and} \quad \dot{s}^{in} = \sigma(b^{in}, s^{in}).
\]

Here, \(\beta\) and \(\sigma\) are the functions appearing on the right hand side of equation (16). Then an equilibrium is simply an intersection of the two isoclines \(B := \{(b^{in}, s^{in}) ; \beta(b^{in}, s^{in}) = 0\}\) and \(S := \{(b^{in}, s^{in}) ; \sigma(b^{in}, s^{in}) = 0\}\). Typically, \(B\) and \(S\) are smooth curves in the rectangular domain \([0, b] \times [0, 1]\) and are given by:

\[
S(T_{se}, L_{se}) := \left\{ (s^{in}, b^{in}) \in [0, 1] \times [0, b] ; \ (1 - T_{se}) U_{se}^{fo} \left( \frac{b - b^{in}}{1 - s^{in}} \right) - L_{se} = U_{se}^{in} \left( \frac{s^{in}}{b^{in}} \right) \right\},
\]

\[
B(L_{bu}) := \left\{ (s^{in}, b^{in}) \in [0, 1] \times [0, b] ; \ U_{bu}^{fo} \left( \frac{b - b^{in}}{1 - s^{in}} \right) - L_{bu} = U_{bu}^{in} \left( \frac{s^{in}}{b^{in}} \right) \right\}.
\] (19)

The equilibrium \((b^*, s^*)\) is locally stable if the following conditions are met in a neighbourhood of \((b^*, s^*)\):

(i) The absolute slope of \(B\) at \((b^*, s^*)\) is larger than the absolute slope of \(S\) at this point.\(^{37}\)

(ii) \(\beta\) is positive to the left of \(B\), and negative to the right of \(B\).

(iii) \(\sigma\) is positive below \(S\), and negative above \(S\).

If the population of informal buyers unilaterally dips below (above) \(b^*\), then Condition (ii) says that the payoff for informal buyers will be higher (lower) than the payoff for formal buyers, causing buyers to migrate into (out of) the informal market until \(b^{in} = b^*\). Likewise, if the population of informal sellers unilaterally dips below (above) \(s^*\), then Condition (iii) says that the payoff for informal sellers will be higher (lower) than the payoff for formal sellers, causing sellers to migrate into (out of) the informal market, until \(s^{in} = s^*\). Thus, a stable equilibrium is such that any point to the left (right) of \(B\) will move in a rightward (leftwards) direction and any point below (above) \(S\) will move upwards (downwards).

\(^{37}\)Heuristically, this means we can think of \(B\) as a roughly “vertical” curve near \((b^*, s^*)\), whereas \(S\) is roughly “horizontal” near \((b^*, s^*)\).
We say there is a pure formal market equilibrium if the point \((b^a, s^a) = (0, 0)\) satisfies the equilibrium condition given by (18). We say there is a pure informal market equilibrium if the point \((b^u, s^u) = (b, 1)\) satisfies equation (18). Finally a mixed-market equilibrium is a point \((b^*, s^*) \in (0, b) \times (0, 1)\) which satisfies equation (18). To establish the robust co-existence of formal and informal markets, we must show that there exists a locally stable mixed-market equilibrium.

4.5 Properties of the equilibrium

Given the complexity of the model, no closed form solutions exist, so that numerical analysis is required to characterize the equilibrium. We now examine different scenarios and their implications for the resulting equilibria.

4.5.1 No Taxes and No Quality Assurance

To isolate the implications of the trading protocol, we first consider an environment with \(T_x = 0\) and \(g^u = g^o\). In other words, we initially suppose that the formal market has no quality assurance advantage, and that neither market has a tax advantage. This would occur, for example, if the tax rate in the formal market exactly matched the rate of theft in the informal market, and if products had zero probability of defects or if \(\alpha(q) = q\) for all \(q\).

When \(T_x = 0\) and \(L_{bu} = L_{se} = 0\), the two curves \(S(0, 0)\) and \(B(0)\) characterizing the stability of the equilibrium are very close to the diagonal. Heuristically, this means that buyers and sellers are both essentially indifferent between the two markets, as long as

\[
\frac{b^a}{s^a} = b = \frac{b^o}{s^o}.
\]  

(20)

Numerical methods suggest that, in this case, buyers and sellers exhibit a very weak preference for an all-formal market equilibrium. But the difference in payoff between the all-formal market equilibrium and other points on the diagonal (20) is so small that all points on this diagonal could be regarded as “quasi-equilibria”. However, if \(L_{bu} \neq 0\) and \(L_{se} \neq 0\), then the picture becomes much clearer.

Result 2. Suppose \(T_x = 0\). Then numerical methods suggest that:

(a) if \(L_{bu}\) and \(L_{se}\) have opposite signs, and \(|L_{bu}|\) and \(|L_{se}|\) are large enough, then there is a locally stable mixed-market equilibrium.

(b) if \(L_{bu} < 0\) and \(L_{se} < 0\), and \(|L_{bu}|\) and \(|L_{se}|\) are large enough, there exists a locally stable pure formal market equilibrium.
(c) if \( L_{bu} > 0 \) and \( L_{se} > 0 \), and \( |L_{bu}| \) and \( |L_{se}| \) are large enough, there exists a locally stable pure informal market equilibrium.

In all three cases, the equilibrium appears to be unique.

To gain some deeper understanding of Result 2, notice that an equilibrium (18) is any crossing point of the isocline \( B(L_{bu}) \) (from equation (19)) and the isocline

\[
S(0, L_{se}) := \left\{ (s^{in}, b^{in}) \in [0, 1] \times [0, b] : U_{se}^{in} \left( \frac{b - b^{in}}{1 - s^{in}} \right) - L_{se} = U_{se}^{in} \left( \frac{s^{in}}{b^{in}} \right) \right\}.
\]

Let us now define the functions \( \beta, \sigma : [0, b] \times [0, 1] \rightarrow \mathbb{R} \) by setting

\[
\sigma(b^{in}, s^{in}) := U_{se}^{in} \left( \frac{s^{in}}{b^{in}} \right) - U_{se}^{in} \left( \frac{b - b^{in}}{1 - s^{in}} \right)
\]

and

\[
\beta(b^{in}, s^{in}) := U_{bu}^{in} \left( \frac{s^{in}}{b^{in}} \right) - U_{bu}^{in} \left( \frac{b - b^{in}}{1 - s^{in}} \right),
\]

for all \( b^{in} \in [0, b] \) and \( s^{in} \in [0, 1] \). Suppose \( L_{se} = 0 \); then \( \sigma \) measures how relatively attractive the informal market is for sellers. If \( \sigma(b^{in}, s^{in}) \) is positive (negative), then sellers will move into (out of) the informal market, so \( s^{in} \) will increase (decrease). Likewise, suppose \( L_{bu} = 0 \); then \( \beta \) measures how relatively attractive the informal market is for buyers. If \( \beta(b^{in}, s^{in}) \) is positive (negative), then buyers will move into (out of) the informal market, so \( b^{in} \) will increase (decrease). The isocontours of \( \beta \) are the isoclines \( B(L_{bu}) \) for various choices of \( L_{bu} \). The isocontours of \( \sigma \) are the isoclines \( S(0, L_{se}) \) for various choices of \( L_{se} \). These isocontours cross if and only if the gradient vector field \( \nabla \sigma \) is not parallel to the gradient vector field \( \nabla \beta \). So this is what we must verify to demonstrate Result 2.

If the two gradient vector fields were parallel, then we would have

\[
\phi(b^{in}, s^{in}) := \frac{\nabla \sigma(b^{in}, s^{in}) \cdot \nabla \beta(b^{in}, s^{in})}{\| \nabla \sigma(b^{in}, s^{in}) \| \cdot \| \nabla \beta(b^{in}, s^{in}) \|} = \pm 1,
\]

for all \( b^{in} \in [0, b] \) and \( s^{in} \in [0, 1] \). Using a symbolic computation package like Mathematica or Maple, it is easy to verify that \( \phi(b^{in}, s^{in}) \neq \pm 1 \), for any choice of \( (b^{in}, s^{in}) \) which is not close to the diagonal line \( \{(b^{in}, s^{in}); b^{in}/s^{in} = b/s \} \) and also to verify that

\[
\lim_{\epsilon \to 0} \phi(b - \epsilon, \epsilon) = 0.
\]

In other words, if \( (b^{in}, s^{in}) \) is close to \( (b, 0) \), then the gradient vectors \( \nabla \sigma(b^{in}, s^{in}) \) and \( \nabla \beta(b^{in}, s^{in}) \) are not only non-parallel, but nearly orthogonal, meaning that the isoclines \( S \)
A locally stable mixed-market equilibrium defined by the crossing of $S(0, 0.5)$ and $B(-0.5)$. (B) A locally stable mixed-market equilibrium defined by the crossing of $S(0, -0.4)$ and $B(0.4)$. (C) A pure formal market equilibrium $(0, 0)$ exists for $S(0, 0.1)$ and $B(0.1)$. (D) A pure in formal market equilibrium $(b, 1)$ for $S(0, -0.1)$ and $B(-0.1)$. and $B$ cross at right angles.$^{38}$

Any crossing of the isoclines $B(L_{ba})$ and $S(0, L_{\infty})$ will determine an equilibrium (18) of the economy. However, not all such equilibria are locally stable. If the slope of $S(0, L_{\infty})$ is less than the slope of $B(L_{ba})$ when they cross, then it is easy to check that conditions (i)-(iii) from Section 4.4 are satisfied, so that the equilibrium is locally stable. For example, we now consider four illustrations of Result 2 where in all cases, $\delta = 0.99$ and $g^{in} = g^{fo}$; the $S$-isoclines are the dashed curves, while the $B$-isoclines are the solid curves. In particular, Figure 1(A) shows the curves $B(-0.5)$ and $S(0, 0.5)$ intersecting in a locally stable equilibrium. Figure 1(B) shows the curves $B(0.4)$ and $S(0, -0.4)$ intersecting in a locally stable equilibrium. Furthermore, a plot of the vector fields defined by best response differential equations (16) reveals that these equilibria are in fact global attractors; see Figures 5(A,B) in Appendix D.

Thus, there is a stable equilibrium with a mixture of formal and informal markets when-

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$^{38}$Unfortunately, it is not possible to obtain a similar asymptotic result for $(b^{in}, s^{in})$ is close to $(0, 1)$, because $\phi$ has a singularity there.
ever the buyers and sellers face lump-sum costs in different markets. However, if both buyers and sellers face lump-sum costs in the same market, then the isoclines do not cross. In this case, the dynamics cause all buyers and sellers to migrate to the market without the lump-sum costs. If $\mathcal{S}(0, L_{\infty})$ is always below $\mathcal{B}(L_{\infty})$, then all buyers and sellers migrate to the formal market, as described by Result 2(b). If $\mathcal{S}(0, L_{\infty})$ is always above $\mathcal{B}(L_{\infty})$, then all buyers and sellers migrate to the informal market, as described by Result 2(c). Figures 1(C,D) illustrate these cases. (Once again, the associated vector fields reveals that these equilibria are global attractors; see Figures 5(C,D) in Appendix D.)

### 4.5.2 Crime and taxation

In the previous numerical example we have studied the case where the tax rate in the formal sector is exactly equal to the crime rate in the informal sector, so that $T_{\text{ax}} = 0$. 

Ceteris paribus, raising tax rates in the formal sector (or lowering crime rates in the informal sector) will cause some buyers and sellers to migrate from the formal to the informal market. To see this, consider the isocline

$$
\mathcal{S}(T_{\text{ax}}, L_{\infty}) := \{ (s^{\in}, b^{\in}) \in [0, 1] \times [0, b] : (1 - T_{\text{ax}}) U_{\text{sc}} \left( \frac{b - b^{\in}}{1 - s^{\in}} \right) - L_{\infty} = U_{\text{sc}} \left( \frac{s^{\in}}{b^{\in}} \right) \}
$$

for any net tax level $T_{\text{ax}}$. It is easy to numerically show that increasing $T_{\text{ax}}$ will cause this curve to shift upwards. As a result, a higher formal taxes will cause a larger fraction of both buyers and sellers to migrate to the informal sector. However, as long as the net tax is small enough, the equilibrium is such that formal and informal markets of non-trivial size exists.

We now want to examine the impact of distortionary taxation on the size of the informal sector under different lump sum costs assumptions. Let us consider a situation where $L_{\infty} = 0.4$. Figure 2 shows different scenarios. In Panel (a) we consider $b^{\in} = b/2$. If $T_{\text{ax}} = 0$, then the unique value of $s^{\in}$ such that $(s^{\in}, b/2) \in \mathcal{S}(T_{\text{ax}}, L_{\infty})$ is the value such that $U_{\text{sc}} \left( \frac{b/2}{1 - s^{\in}} \right) - 0.4 = U_{\text{sc}} \left( \frac{s^{\in}}{b^{\in}} \right)$. But if $T_{\text{ax}} = 0.3$ (so that $1 - T_{\text{ax}} = 0.7$), then $s^{\in}$ must shift to the right in order that $0.7 \cdot U_{\text{sc}} \left( \frac{b/2}{1 - s^{\in}} \right) - 0.4 = U_{\text{sc}} \left( \frac{s^{\in}}{b^{\in}} \right)$. To emphasise this point further, notice that for any fixed $b^{\in} \in [0, b]$, an increase in the net tax level $T_{\text{ax}}$ will increase the corresponding value of $s^{\in}$.

In Panel (b) we increase the net tax from $T_{\text{ax}} = 0$ to $T_{\text{ax}} = 0.3$ shifts the curve $\mathcal{S}(T_{\text{ax}}, 0.4)$ upwards, so that the intersection with $\mathcal{B}(-0.4)$ moves to the northeast. In other words, both buyers and sellers migrate from the formal to the informal market. In Panel (b) we have $L_{\infty} := 0.4$ and $L_{\text{bu}} = -0.4$, but we would get a similar picture for any $L_{\text{bu}} < 0 < L_{\infty}$.

To gain further insight, we now consider a situation where $L_{\infty} = -0.4$ while $L_{\text{bu}} := 0.4$, for
and we compare the net tax levels $T_x = 0$ and $T_x = 0.5$ which can be found in Figure 3.39

Figure 2 and Figure 3, clearly show that the effect of taxation depends on the relative lump-sum entry costs sellers and buyers face in the formal market. In particular, taxation has a stronger effect when sellers (but not buyers) must pay a net positive fee to enter the formal market. In contrast, taxation causes a weaker effect on the informal sector size when buyers (but not sellers) must pay a net fee to enter the formal market.

4.5.3 Quality assurance versus taxation

So far in all of our previous numerical analysis we have assumed that $g^{in} = g^{fo}$ so that the formal market has no advantage over the informal market due to warranties. This would be the case, for example, if the quality assurance technology has constant returns to scale as

\[ L_{bu} > 0 > L_{se}. \]

39We would get a similar picture for any $L_{bu} > 0 > L_{se}$.  

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stated in Lemma A.

In this section we examine a situation where $g^a < g^f$ so that the “quality assurance” in the formal market has an advantage and creates an extra surplus that the government can tax. For instance, let us suppose that $g^a = 0.9 g^f$. Figure 4(a) shows a market with no net taxation and no lump-sum costs (i.e. $T_x = L_{\infty} = L_{bu} = 0$). We see that $S(0,0)$ is always below $B(0)$, so all buyers and sellers migrate to the formal market. Figure 4(b) shows a market with no lump-sum costs (i.e. $L_{\infty} = L_{bu} = 0$), but with a 45% net tax rate on the formal sector (i.e. $T_x = 0.45$), we see that $S(0.45,0)$ is still below $B(0)$, so that all buyers and sellers remain in the formal market. Thus, if the formal market has even a small quality assurance advantage, then it can withstand a large amount of government taxation.

Notwithstanding the absence of an informal market in the equilibrium in Figure 4, if the burden of taxation or regulation in the formal market is high enough, then the equilibrium will involve a nonzero amount of informal activity, despite the formal market’s quality assurance advantage. And indeed, this is what we observe empirically in many real-world markets.

5 Conclusion

While in formal markets sellers need to publicly advertise their prices and locations in order to attract buyers, in informal markets sellers need to avoid providing any public information about their locations in order to avoid taxes and regulation. This basic fact, which has significant implications on viable types of trading mechanisms in these markets, as well as
on their other distinguishing features (e.g. taxes, crime, quality assurance), has not received attention in the literature. To that end, in this paper we consider an infinite-horizon setup in which the trading mechanism is bargaining in the informal sector, and price posting in the formal sector. Since agents’ payoffs depend on the ratio of buyers and sellers in each of these markets, all agents try to position themselves in the market which can yield them the highest possible payoff. This strategic interaction in turn critically affects the relative size and the evolution of these two markets. Borrowing from evolutionary game theory, we consider ‘best response dynamics’ so that agents switch from one market to the other at a rate which is proportional to their payoff differential between the two markets.

In our benchmark model, only sellers can switch between markets, so that the number of buyers in formal and informal markets is always fixed. We then analytically show that formal and informal markets of nontrivial size co-exist in a stable equilibrium. We also show that some sellers will switch from the formal to the informal market whenever the formal sellers’ warranty erodes, the government imposes higher taxes and regulations in the formal market, the risk of crime (theft) and/or confiscation decreases in the informal market, or the number of buyers in the informal market increases. Conversely, sellers will switch from the informal to the formal market whenever the contrary changes take place.

Once we relax the immobility of buyers and allow both buyers and sellers to switch between formal and informal markets, we have a less tractable environment and analytical solutions are not possible. In this richer environment we further consider additional costs that buyers and sellers face which are independent of the value of the good that is being traded. These additional characteristics provide very interesting dynamics, and highlight the importance of the underlying trading mechanisms. We illustrate that, when the net lump-sum cost for a seller in the formal sector relative to that in the informal sector and the net lump-sum cost for a buyer in the formal sector relative to that in the informal sector have opposite signs, then there exists a locally stable equilibrium in which formal and informal markets co-exist; the parameter space consistent with this equilibrium, however, is smaller compared to that in our simpler benchmark model. If the above-mentioned different relative net lump-sum costs are both negative (positive), then there exist a unique locally stable equilibrium where only formal (informal) activity takes place in the long run.

References


Appendix A: Proofs

Proof of Lemma A. The efficient value $q^*$ of investment in quality assurance is the value such that $\alpha'(q^*) = 1$ (i.e. such that one additional cent spent on quality assurance increases the buyer’s expected utility by exactly one cent). Since $\alpha'$ is nonincreasing (by concavity), we have $\alpha'(q) \geq 1$ for all $q \in [0, q^*]$. Thus, since $\alpha(0) = 0$, the Fundamental Theorem of Calculus implies that $\alpha(q^*) \geq q^*$ (i.e. the benefit of quality assurance outweighs its cost). Thus,

$$\frac{g^t}{g^m} = \frac{v_{tba}^{fio} - c_{sec}^{fio}}{v_{tba}^{fin} - c_{sec}^{fin}} = \frac{v_{tba}^{fin} + \alpha(q) - c_{sec}^{fin} - q}{v_{tba}^{fin} - c_{sec}^{fin}} = 1 + \frac{\alpha(q) - q}{v_{tba}^{fin} - c_{sec}^{fin}} \geq 1,$$

because $\alpha(q) \geq q$. Thus, $g^m \leq g^o$. \qed

Proof of Theorem 1. We must show that the interval $[0, 1]$ contains a zero for the function $\widehat{U}$. By inspecting formulae (5) and (8), we see that, for all $s^{fio} \in [0, 1]$, we have

$$\widehat{U}(s^{fio}) = \widehat{U}^{fio}_{se}(s^{fio}) - \widehat{U}^{fin}_{se}(s^{fio}) = (1 - T^{fio}_{se}) \widehat{U}(s^{fio}),$$

with $\widehat{U}(s^{fio}) := \widehat{U}_1(s^{fio}) - K \widehat{U}_2(s^{fio})$,

where $K := \frac{(1 - T^{fin}_{se}) g^{fin}}{(1 - T^{fio}_{se})}$, \hspace{1cm} (22)

while $\widehat{U}_1(s^{fio}) := \left[1 - \exp\left(-\frac{b^{fio}}{s^{fio}}\right)\right] \cdot \left(1 - \frac{b^{fio}/s^{fio}}{\exp(b^{fio}/s^{fio}) - 1}\right)$, by Eq.(5),

and $\widehat{U}_2(s^{fio}) := \eta\left(b^{fin}/1 - s^{fio}\right) \cdot \left[1 - \exp\left(-\frac{b^{fin}}{1 - s^{fio}}\right)\right]$ by Eq.(8).

Clearly, it will be sufficient to find a zero for $\widehat{U}$ instead. From equation (22), simple computations yield:

$$\lim_{s \searrow 0} \widehat{U}(s) = 1 - K \eta(b^{fin}) (1 - \exp(-b^{fin})) \quad \text{and} \quad \lim_{s \nearrow 1} \widehat{U}(s) = 1 - \frac{1 + b^{fio}}{\exp(b^{fio})} - K.$$

From here, it is easy to check that

\begin{align*}
& \left(K < \frac{1}{\eta(b^{fin}) (1 - \exp(-b^{fin}))}\right) \implies \left(\lim_{s \searrow 0} \widehat{U}(s) > 0\right) \\
\text{and} \quad & \left(K > 1 - \frac{b^{fio} + 1}{\exp(b^{fio})}\right) \implies \left(\lim_{s \nearrow 1} \widehat{U}(s) < 0\right). \hspace{1cm} (23)
\end{align*}
But $\tilde{U}$ is continuous on $[0,1]$. Thus, if $K$ satisfies both the conditions in (23), then the Intermediate Value Theorem implies that $\tilde{U}(s^*) = 0$ for some $s^* \in (0,1)$. Furthermore, $\tilde{U}$ is going from positive values (near 0) to negative values (near 1), so $\tilde{U}$ must be decreasing near $s^*$; hence $s^*$ is a stable equilibrium.

Now, it is easy to check that the function $\tilde{U}_2$ is positive everywhere on $[0,1]$. Thus, if $K$ increases, then the graph of $\tilde{U}$ will move downwards everywhere. Since $\tilde{U}$ is decreasing near $s^*$, a downwards movement of the graph will cause $s^*$ to move to the left in the interval $[0,1]$. In other words, $s^*$ will decrease when $K$ increases. By inspection of formula (22), $K$ is increasing with $g^o$ and $T^o_{se}$, while it is decreasing with $T^i_{se}$. Thus, $s^*$ is decreasing with $g^o$ and $T^o_{se}$, and increasing with $T^i_{se}$.

Meanwhile, $\tilde{U}_1$ is clearly increasing as a function of $b^o$, and independent of $b^i$. On the other hand, $\tilde{U}_2$ is independent of $b^o$, but increasing as a function of $b^i$ (because $\eta$ is an increasing function, by hypothesis). Thus, $\tilde{U}$ is decreasing as a function of $b^i$ (because $K$ is positive by inspection of formula (22)). Thus, if we increase $b^o$, then the graph of $\tilde{U}$ is will move upwards (and hence, $s^*$ will move to the right), whereas if we increase $b^i$, then the graph of $\tilde{U}$ will move downwards (hence, $s^*$ will move to the left). Thus, $s^*$ is an increasing function of $b^o$, and a decreasing function of $b^i$.

**Appendix B: Different Equilibrium Concepts**

Here we formalize two alternative frameworks/equilibrium concepts that also lead to the equilibrium represented by equation (10).

**Uncorrelated, Symmetric Mixed Nash Equilibrium.** We can think of a static environment, where each of the sellers plays a mixed strategy, randomly choosing whether to participate in the formal or informal market. Sellers cannot coordinate, so there is no correlation between their strategies. All sellers are *ex-ante* identical, so they play identical strategies, given by the probability vector $(s^o, s^i)$. Equation (10) is then equivalent to saying that this profile of mixed strategies is a Nash equilibrium, as in Camera and Delacroix (2004) or Michelacci and Suarez (2006). This equilibrium concept also allows each seller to participate in both formal and informal markets, at potentially differing rates.

**Replicator/Imitation Dynamics.** As in best response dynamics, suppose there is an infinite sequence of time periods, with trade occurring in each market during each time period. But instead of migrating between markets in response to higher payoffs, agents learn by *imitating* other agents. The more agents choose a particular strategy, and the better they
are doing relative to the average payoff, the more likely it is that other agents will imitate their behavior.

Alternatively, we can interpret the same model in terms of successive generations of agents. During each time period, some agents produce one or more children, and some agents die. Children remain in the same market as their parents. The net reproductive rate (births minus deaths) of each market type is determined by how much the payoff for that market exceeds the population average payoff. To be precise, the population average payoff for sellers at time $t$ is given by:

$$s_{fo}(t) U_{se}^{fo} \left[ b_{fo}(t)/s_{fo}(t) \right] + s_{in}(t) U_{se}^{in} \left[ s_{in}(t)/b_{in}(t) \right]$$

so the reproductive rate of the formal sellers will be:

$$\rho(t) = (1 - s_{fo}(t)) U_{se}^{fo} \left[ b_{fo}(t)/s_{fo}(t) \right] - s_{in}(t) U_{se}^{in} \left[ s_{in}(t)/b_{in}(t) \right].$$

The population of formal sellers will grow (or shrink) exponentially at this rate. Formally, we have $s_{fo}(t) = \lambda_{se} \rho(t) \cdot s_{fo}(t)$, where $\lambda_{se} > 0$ is some constant. This leads to the following dynamical equation

$$\dot{s}_{fo}(t) = -\dot{s}_{in}(t) = \lambda_{se} s_{fo}(t) s_{in}(t) \left( \tilde{U}_{se}^{fo} \left( \frac{b_{fo}(t)}{s_{fo}(t)} \right) - \tilde{U}_{se}^{in} \left( \frac{s_{in}(t)}{b_{in}(t)} \right) \right), \quad (24)$$

where $\lambda_{se} > 0$ is a constant. Again, this dynamical equation has both a discrete-time and a continuous-time interpretation. In either case, equation (10) is a necessary and sufficient condition for a population distribution $(s_{fo}, s_{in})$ to be a “nontrivial” fixed point of the dynamics. Here, “nontrivial” refers to the fact that the replicator dynamics always have “trivial” fixed points where $s_{fo} = 0$ or $s_{in} = 0$. However, unless these “pure population” equilibria arise from a solution to equation (10), they are generally unstable to small perturbations. Thus, a pure population of this type will be destabilized as soon as even one of the reproducing agents produces a “mutant” child of the opposite type. Thus, we can safely ignore these trivial equilibria, and focus only on the equilibria described by equation (10).

**Both Buyers and Sellers Switching between Markets**

Finally, we could suppose that the buyer/seller populations evolve according to replica-
tor/imitation dynamics. This yields dynamical equations:

\[
\dot{s}_{fo}(t) = -\dot{s}_{in}(t) = \lambda_{se} s_{fo}(t) s_{in}(t) \left( (1 - T_{ax}) U_{se} \left( \frac{b_{fo}(t)}{s_{fo}(t)} \right) - L_{se} - U_{se} \left( \frac{s_{in}(t)}{b_{in}(t)} \right) \right)
\]

and

\[
\dot{b}_{fo}(t) = -\dot{b}_{in}(t) = \lambda_{bu} b_{fo}(t) b_{in}(t) \left( U_{bu} \left( \frac{b_{fo}(t)}{s_{fo}(t)} \right) - L_{bu} - U_{bu} \left( \frac{s_{in}(t)}{b_{in}(t)} \right) \right),
\]

where \( \lambda_{se} > 0 \) and \( \lambda_{bu} > 0 \) are constants. Again, this dynamical equation has both a discrete-time and a continuous-time interpretation. In either case, equation (17) is a necessary and sufficient condition for a population distribution \((b_{fo}, b_{in}, s_{fo}, s_{in})\) to be a “nontrivial” fixed point of the dynamics (25). Here, “nontrivial” refers to the fact that the replicator dynamics always has “trivial” fixed points where either \( b_{fo} = 0 \) or \( b_{in} = 0 \) and either \( s_{fo} = 0 \) or \( s_{in} = 0 \).

Note that the vector field determined by (25) is obtained by multiplying the vector field defined by (16) by a scalar function which is positive everywhere in \((0, b) \times (0, 1)\). Thus, a stable fixed point for (16) is also a stable fixed point for (25).

### Appendix C: Alphabetical Index of Notation

**\( \alpha(q) \)** Benefit (to the formal buyers) of quality assurance (e.g. warranties, free repair service, etc.)

**\( b_{in} \)** Ratio of buyers in the informal market, relative to population of sellers in both markets.

**\( b_{fo} \)** Ratio of buyers in the formal market, relative to population of sellers in both markets.

**\( b = b_{in} + b_{fo} \)** Overall ratio of buyers to sellers in the whole economy.

**\( B_f := b_{fo}/s_{fo} \)** Ratio of buyers to sellers in formal market.

**\( B_i := b_{in}/s_{in} \)** Ratio of buyers to sellers in the informal market.

**\( c_{se}^{in} \)** Cost of production in the informal market.

**\( c_{se}^{fo} \)** Cost of production in the formal market. (Includes quality assurance, but not taxes or regulatory compliance.)

**\( \delta \)** Discount factor (in section 4).

**\( \eta \)** Bargaining strength of informal sellers (in section 3).

**\( g_{in} := v_{bu}^{in} - c_{se}^{in} \)** The gains from trade in the informal market.

**\( g_{fo} := v_{bu}^{fo} - c_{se}^{fo} \)** The gains from trade in the formal market.
\( L_{bu} \) Lump sum costs for informal buyers (e.g. inconvenience).

\( L_{bo} \) Lump sum costs for formal buyers (e.g. transportation and shoe leather costs).

\( L_{se} \) Lump sum costs for informal sellers (e.g. crime risk, bribery, protection money, shoe leather).

\( L_{so} \) Lump sum costs for formal sellers (e.g. regulatory compliance, license fees, rent).

\( L_{bu} \) “Net” lump sum costs for formal buyers.

\( L_{so} \) “Net” lump sum costs for formal sellers.

\( P_{bu}^i \) Match probability for informal buyers.

\( P_{bu}^f \) Match probability for formal buyers.

\( P_{se}^i \) Match probability for informal sellers.

\( P_{se}^f \) Match probability for formal sellers.

\( q \) Expenditure on quality assurance technology by formal sellers.

\( R_{se}^i = 1 - T_{se}^i \), the residual earnings rate for informal sellers.

\( R_{se}^f = 1 - T_{se}^f \), the residual earnings rate for formal sellers.

\( R_{se} = R_{se}^f / R_{se}^i \), the “net” residual earnings rate for formal sellers.

\( s^i \) Proportion of sellers in the informal market.

\( s^f \) Proportion of sellers in the formal market.

\( S_i := s^i / b^i \). Ratio of sellers to buyers in the informal market.

\( t \) Time (in dynamical interpretation of model).

\( T_{se}^i \) Expected costs of monetary crime for informal sellers.

\( T_{se}^f \) Taxes and unit regulatory costs for formal sellers.

\( T_{se} \) “Net” tax burden for formal sellers.

\( u_{bu}^i \) Utility of a purchase for informal buyers.

\( u_{bu}^f \) Utility of a purchase for formal buyers.

\( u_{se}^i \) Utility of a sale for informal sellers.
$u_{fo}^o$  Utility of a sale for formal sellers.

$U_{bu}^{in} = P_{bu}^{in} u_{bu}^{in}$, the expected utility of informal buyers.

$U_{bu}^{fo} = P_{bu}^{fo} u_{bu}^{fo}$, the expected utility of formal buyers.

$U_{so}^{in} = P_{so}^{in} u_{so}^{in}$, the expected utility of informal sellers.

$U_{so}^{fo} = P_{so}^{fo} u_{so}^{fo}$, the expected utility of formal sellers.

$v_{bu}^{in}$  Value of merchandise to informal buyer.

$v_{bu}^{fo}$  Value of merchandise to formal buyer.
Appendix D: Best response vector fields

Figure 5: The vector fields generated by the best response differential equations (16), for the four cases shown in Figure 1. (A) (A) The mixed-market equilibrium defined by the crossing of $S(0,0.5)$ and $B(-0.5)$ is a global attractor. (B) The mixed-market equilibrium defined by the crossing of $S(0,-0.4)$ and $B(0.4)$ is a global attractor. (C) The pure formal market equilibrium $(0,0)$ is a global attractor. (D) The pure informal market equilibrium $(b,1)$ is a global attractor.