Leverage and speculation

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November 2014

Abstract

I suggest a continuous-time model of speculation in which size plays a role. An agent controls the drift of a geometric Brownian process and may engage in secret speculation to enhance it; doing so exposes the process to large Poisson losses. The optimal contract uses size as an additional instrument, so there is downsizing on the equilibrium path to preserve incentive compatibility. I characterise the value function and present important properties of the optimal contract. This work is particularly pertinent to leverage regulation.

Keywords: JEL Classification: D82, D86, G28, L43.

1 Introduction

The Global Financial Crisis has rekindled a long-standing debate on both leverage and speculation in financial institutions. Leverage is typically seen as risky in its own right because it is associated with financial fragility. A firm that is too highly levered is less able to withstand adverse shocks such as credit losses. Because no firm operate in isolation, this makes for a less resilient economy. Speculation is also, perhaps rightly so, mistrusted on its own; some suggest it is necessarily wasteful and should be banned (sources?).

An important dimension of the recent financial crisis is the conjunction of both these characteristics: very large, highly levered, financial firms engaged in speculation and failed. This differs from failure due solely to speculation, such as that of Long-Term Capital Management in 1998, which did not precipitate such a major crisis. It also differs from the failure other very large firms like Worldcom in 2002 (the largest then), Enron in 2001 or Chrysler and General Motors in 2009.

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In this paper I present a model in which the opportunity to speculate and firm size interact. It takes the form of a dynamic contracting model under moral hazard. The agent controls the profitability of a firm in a geometric Brownian process, which captures firm size. That agent must be given incentives to not divert funds (equivalently, to exert effort); she can also engage in speculative activities that improve profitability but expose the firm to a Poisson process of very large losses. I assume that the firm is essential: it cannot be shut-down, which fits many large financial institutions. Hence the threat of liquidation (and simultaneous termination of the agent) has no bite and a special resolution mechanism is required. This fits, for example, firm like Global Systemically Important Banks (there are 29 worldwide as of writing) and Global Systemically Important Insurers (there are 9).

I characterize incentive compatibility and show that firm size emerges as a necessary control to satisfy it. Thus controlling firm size is a contractual instrument that is required by incentive compatibility. The incentive compatible, optimal contract includes downsizing on the equilibrium path in order to satisfy incentive compatibility, as well as a stopping time (termination and restart) that is driven by a minimum scale of operation. This downsizing option makes for particularly rich dynamics. If downsizing has to be so severe that the firm becomes inefficiently small, the principal prefers replacing the agent with a new one and restart the firm at the efficient scale. The contract also features cash payments at the reflective boundary (beyond which increasing the continuation becomes too costly). However where that boundary exactly lies depends on the severity of the speculation problem. If it is mild, the dominant problem is that of cash diversion. Then, up to incentive compatibility, the problem resembles that of He (2008) and the reflecting boundary is pinned by the cash diversion problem. Cash payments worsen the no-speculation constraint, so if speculation is the dominant problem cash payment must be further delayed. That is, the principal must increase the agent’s continuation value to preserve incentive compatibility. The value function reflects these characteristics.

There is a connection between cash flow diversion and speculation through the incentive contract, although these activities are independent. The more efficient is diversion the more attractive is the instantaneous return on speculation, and so the more difficult it is to deter. Deterring speculation thus requires a higher continuation value; so more efficient cash flow diversion induces a higher social cost (because of the option to speculate).
The model maps naturally into standard regulatory instruments such as leverage ratios. However here leverage must be limited not to render the firm resilient to potential losses (such as credit losses) but to deter speculation. The reason is that limiting leverage increases the stake of the insiders (their continuation value) and so enforces incentive compatibility.\textsuperscript{1}

The papers closest to this work are those of He (2008), Biais, Mariotti, Rochet and Villeneuve (2010, now BMRV), Biais and Casamatta (1999), as well as Rochet and Roger (2014) and DeMarzo, Livdan and Tchistyi. He (2008) extends the work of DeMarzo and Sannikov (2006) to a model with a geometric motion. Incentive compatibility is characterised as in DeMarzo and Sannikov (2006) and so is independent of scale. When principal and agents have the same discount factor the contract prescribes an absorbing state for the continuation value: incentives are free in that the agent’s state is large enough. BMRV study the problem of large Poisson risks (accidents or losses), the probability of which is controlled through the agent’s effort. Incentive compatibility dictates that the firm be downsized following an accident as a punishment to the agent. Here I also find that downsizing is a necessary instrument, however to deter speculation (not for spur effort). More precisely, the downsizing decision must be taken before the agent exerts her actions – so it is not a punishment. Biais and Casamatta (1999), like Rochet and Roger (2014) and DeMarzo et al (2013) (independently) study the double moral hazard problem of diversion (or effort) and speculation. Biais and Casamatta use a static model and consider a restricted set of instruments (equity, debt and options) and show that the optimal contract always include debt and equity. Debt turns the agent into the residual claimant and so is good to spur effort. Equity is necessary to deter speculation. Sometimes options are required to add to that (i.e. when the speculation problem is severe). Rochet and Roger (2014) and DeMarzo et al (2013) develop continuous-time versions however with arithmetic Brownian motions. Incentive compatibility requires the continuation value to remain sufficiently high, but because of the ABM it is size-independent. Rochet and Roger prescribe a deterministic termination when the continuation value is too low because liquidation of the project is ruled out. DeMarzo et al suggest a stochastic termination rule to avoid inefficient termination (which is allowed). Here termination is an instrument too but only because the firm must operate at some efficient scale $X_0$ (and cannot be stopped); in the limit ($X_0 \to 0$) there is no

\textsuperscript{1}This is not to say that leverage ratios are not useful to also buffer credit losses – but there are none in equilibrium in this model.
This paper also connect to the literature on leverage and risk-taking, especially in financial institutions. In a series of papers, Anat Admati (and at times her coauthors) makes the argument for less leverage on grounds of less fragility for individual firms, less subsidies from society and greater systemic resilience. I add a simple but salient point: with smaller (less levered) institutions, there is also less speculation. The optimal contract can be implemented with a minimal equity requirement imposed on the firm together with a leverage ratio (these jointly completely control size). The purpose of which is to ensure the agent has enough at stake not to engage in excessive risk-taking. VanHoose (2007) provides a survey that suggests a persistent lack of consensus as to the role and benefit of capital requirements. Furlong and Keeley (1989) establish that asset risk decreases when the capitalization of a bank increases. Milne (2002) observes that a bank’s portfolio choice depends on its capitalization. This model accords well with both, and minimum capital requirements induce the institution to choose the less risky path. The reason is that breaching the capital requirement triggers downsizing. Morrison and White (2005) propose a model of adverse selection and moral hazard in which capital requirements are also used to solve the moral hazard problem and to screen out bad banks (or bankers). Last, this work connects to a more recent literature on interventions and bailouts. Zentefis (2014) shows the nature of the rescue matters: if the institution is burdened by excessively large repayments ex post (as a debtor, for example) it has incentives to default. In this model there is no default but early intervention takes the form of downsizing. Mariathasan, Merrouche and Werger (2014) show empirically that the provision of implicit guarantee enhances risk-taking. Instead here the resolution mechanism is explicit and the only guarantee is that of intervention (to preserve incentive compatibility).

2 Model

Consider a firm (the agent) providing a service that must be continued in all circumstances. Operating cash flows follow the geometric process

\[ dX_t = \mu_t(a)X_t dt + \sigma X_t dZ_t - LdN_t(a) \]  

(2.1)

where \( \mu(a) > 0 \ \forall a \) is interpreted as the profitability, or yield, of the firm and \( X_t \) its size. For a bank one can think of its balance sheet. \( Z \equiv \{Z_t, \mathcal{F}_t; 0 < t < \infty \} \) is a Brownian motion associated
with a filtration \( \mathcal{F}_t \) on a probability space \((\Omega, \mathcal{F}, \mathcal{P})\). For technical reasons I let \( Z \) be a bounded process, where the bound \( \kappa \) may be arbitrarily large. This may be interpreted as finite returns, which is not shocking to economists. The term \( L \) is a large loss that occurs with instantaneous probability \( dN_t \) (a Poisson process). The action \( a \in \{0, 1\} \) of the agent controls both the drift \( \mu(a) \) and the probability \( dN_t(a) \); that is, \( \mu(1) = \mu \) and \( dN_t(1) = 0 \), while \( \mu(0) = \mu + \Delta \mu \) and \( dN_t(0) = \lambda > 0 \). Action \( a = 0 \) may be interpreted as speculation, or asset shifting; it increases the drift but introduces exposure to large losses \( L \). These losses are sufficient to wipe out the firm – more on this later. I suppose that the initial value of \( X_t \) is \( X_0 > 0 \) and corresponds to some kind of efficient scale. All agents are risk-neutral; the principal discounts future payments at rate \( r > 0 \) and the agent is (weakly) more impatient: \( \gamma \geq r \). If an agent is terminated she must be replaced at cost \( \gamma \); these may be search costs, restructuring costs and the like.

To be clear, there are two sources of frictions. First, in the spirit of DeMarzo and Sannikov (2006) the operating cash flow at any moment \( t \) can be diverted by the agent: a dollar diverted brings her \( \eta \leq 1 \) dollars. Second, the agent can secretly engage in excessively risky ("speculative") activities that generate the additional cash-flow \( \Delta \mu \) per unit of time but expose the firm to catastrophic losses \( L \). For example, the firm sells (but does not buy) CDS (like AIG or Morgan Stanley during the Global Financial Crisis) or issues options. Or it speculates on electricity contracts (like Enron in the late 1990’s).

The principal seeks to maximises the ex ante value of the firm. The first best is given by

\[
\mathbb{E}_t \left[ \int_t^{\infty} e^{-r(s-t)} X_s ds \right] = \frac{1}{r - \mu} X_t
\]

I postpone the discussion of the contract until after all necessary elements have been introduced because satisfying incentive compatibility imposes some structure on the contract. A contract \( \Xi \) is incentive compatible if it is designed in such a way that the agent never finds it optimal to divert cash, nor to engage in speculative activities. For reasons that will be obvious I suppose that \( \eta \Delta \mu / \lambda \leq 1 \).

## 3 The Contract

From the work of Spear and Srivastava (1987) who introduced the recursive approach to contracting, any contract can be characterized by the stochastic process \( W \) describing the continuation payoff
of the agent when the contract Ξ is executed. Given the contract Ξ, at any moment \( t \) and for an arbitrary stopping time \( \tau \), the agent’s continuation utility at date \( t \) takes the form

\[
W_t(\Xi) = \mathbb{E}_t \left[ \int_t^\tau e^{-r(s-t)} dI_s + e^{-r(\tau-t)} W_\tau \mid \mathcal{F}_t \right]. \tag{3.1}
\]

Using the martingale representation theorem as in Sannikov (2008), we know there exists a RCLL process \( \beta/\sigma \) that delivers an equivalent representation. Then the dynamics of \( W \) write

\[
dW_t = rw_t dt + \frac{\beta_t}{\sigma} (dX_t - \mu(a) X_t) - P_t (dN_t - \mathbb{E}[dN_t]), \tag{3.2}
\]

where \( \beta_t/\sigma \) represents the sensitivity of the agent’s continuation payoffs to cash flows, and \( P_t \) is the penalty incurred in case of a large loss. The derivations of this equation can be found in DeMarzo and Sannikov (2006) and in He (2008) for the term \( \beta_t/\sigma \). Incentive compatible contracts can be directly characterized by simple conditions on these sensitivity parameters \( \beta_t/\sigma \) and \( P_t \).

### 3.1 Incentive compatibility

Recall that the process \( I \) of payments to the agent must satisfy the limited liability constraint \( dI_t \geq 0 \). From the definition of \( W_t \) this implies

\[
W_t \geq 0,
\]

which has implications for incentive compatibility.

**Proposition 1** There is no cash diversion as long as

\[
\beta_t \geq \eta \sigma \equiv \beta \tag{3.3}
\]

and there is no speculation if and only if

\[
P_t \geq \frac{\beta_t}{\sigma} \Delta \mu X_t \tag{3.4}
\]

Combining these two conditions gives with the limited liability constraint gives

\[
W_t \geq \eta \frac{\Delta \mu}{\lambda} X_t \tag{3.5}
\]
The first part of the proposition can be found in DeMarzo and Sannikov (2006) and in He (2008). By sharing the random part of the cash-flow with the agent the principal can deter diversion because the agent would steal from her own pocket. I remark that this constraint is independent of the size $X_t$ of the firm.

The second part is novel. Engaging in speculation generates an additional gain $\Delta \mu X_t$, of which the agent appropriates a fraction $\eta \leq \beta_t / \sigma$. It may trigger a penalty that must be sufficiently large to deter it: $\lambda P_t \geq \Delta \mu X_t$. The incentives are strongest when $P_t \equiv W_t$: the agent must be wiped out after a large adverse event. Any further penalty would violate limited liability, thus $W_t \geq W_m$ so as to preserve incentive compatibility.

**Remark 1** Condition (3.5) is noteworthy: it requires that the continuation value has to grow linearly with the size of the firm $X_t$. This condition arises naturally here and may contribute to explain the increase in executive compensation at large firms (see Landier, 2008).

**Remark 2** From Conditions (3.4) and (3.6) we see it is immaterial that losses not be a function of the size of the firm. To deter them one needs a large enough penalty $P_t$ regardless of their structure.

### 3.2 Elements of the contract

Consider some value function $V(X, W)$ of the two (sufficient) state variables. Thanks to the linearity of the process $dX$ in $X$ we know that the value function $V(X, W)$ is homogenous of degree 1 in $X$ – see He (2008) or BMRV. So we can let

$$\frac{W_t}{X_t} = y_t$$

with this $V(X, W) = X_t v(y_t)$. Redefining the continuation value in this fashion does not affect Condition (3.3) since it is independent of $X_t$. It turns the second incentive constraint (3.5) into

$$y_t \geq \eta \frac{\Delta \mu}{\lambda} \equiv y_m,$$

which states that the size-adjusted continuation value must exceed a constant threshold. This may be thought of as the share of the agent in the firm. Using (3.6), (3.5) reads

$$W_t \geq y_m X_t$$
which introduces the scale $X_t$ as a control. This resembles BMRV, where the agent’s continuation value is linear in the size to deter her from consuming her private benefit. Here this condition is necessary to prevent speculation in this model – the benefit of which is linear in $X_t$. In BMRV it is a punishment that follows a large loss; there the rent is linear in the size.

When $X_t \leq X_0$ the operation is not efficient and as soon as
\[
V(X_t, W_t) = V(X_0, W_0) - \gamma
\] (3.7)
the agent must be terminated and replaced, where $\gamma \geq 0$ is a transaction cost. This condition asserts that the principal replaces the agent as soon as he is better off paying $\gamma$ to restart the contract at the minimum efficient scale, with the corresponding continuation value. In summary,

Proposition 2 To satisfy incentive compatibility, a contract $\Xi$ must

- include a non-decreasing cash flow process $I = \{I_t\}_{t \geq 0}$;
- include a stopping time $\tau \geq 0$;
- include a sensitivity $\beta_t$ characterised by (3.3);
- satisfy the no-speculation constraint (3.5), which requires:
  1. a left-continuous downsizing process $X^d = \{X^d_t\}_{t \geq 0}$ and
  2. the termination condition (3.7).

The point of Proposition 2 is that it is not obvious a priori that the scale $X_t$ must be used as a control. It comes about because of the incentive problem characterised by (3.5) and summarized by (3.6): at any moment $t$ the continuation must exceed a fraction of the size $X_t$. Downsizing is the natural alternative to termination, which is the instrument of choice with a fixed-size process like an arithmetic Brownian motion – see Rochet and Roger (2014). Termination becomes necessary here out of efficiency consideration.

For completeness the process $N$ of large losses generate a filtration $\mathcal{F}^N_t, t \geq 0$. In anticipation, under the contract $\Xi$ characterised by Proposition 2, the principal decides on the size $X_t$ that is, the process $X$ is $\mathcal{F}^N$-predictable, and so is the decision to restructure. The stopping time $\tau$ is also a $\mathcal{F}^N$-predictable stopping time. The process $I$ is $\mathcal{F}^N$-adapted instead because it follows the realisation of $dN$.  

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For now I conjecture, and will verify later, that the optimal contract is such that

- \( \beta_t \equiv \beta \) (minimum cash flow sensitivity that prevents cash diversion);
- \( P_t/X_t \equiv y_m \) (the agent loses everything in case of a catastrophe);

The principal allows for the smallest fraction \( \beta_t \) of the volatile component of the cash flow \( dX_t - E[dX_t] = X_t \sigma dZ_t \) to be left to the agent. Imposing a higher (size-adjusted) penalty than \( y_m \) may only trigger earlier downsizing without altering the shareholder’s incentives. Then along the optimal path \( W_t \) is subject to the dynamics

\[
dW_t = rW_t dt - dI_t + \beta X_t dZ_t.
\]

4 The value function and the optimal contract

The task now is to construct the dynamics of the value function \( V(X, W) \) of the principal. This is not a simple exercise because (i) the agent is subject to downsizing at any point in time to preserve incentive compatibility – Condition (3.5) – and (ii) if downsizing leads it to become too small, it may be terminated – Condition (3.7).

One first needs to establish the downsizing rule, which follows from the need to preserve incentive incentive compatibility; that is, it follows from Condition (3.5). Let \( X_{t+} \) denote the size of the firm at time \( t + dt \) and suppose \( W_t < y_m X_t \) for some \( t \geq 0 \). To satisfy incentive compatibility, the new size \( X_{t+} \) must satisfy

\[
W_t = y_m X_{t+}
\]

and for \( X_{t+} < X_0 \), the principal terminates the contract as soon as

\[
V(W_t, X_{t+}) \leq V(X_0, W_0) - \gamma
\]

where \( W_0 = y_m X_0 \).

Next I must ascribe probabilities to these events. The fact that \( W_t \) is a martingale has an additional virtue in that I can call on the Martingale Central Limit Theorem: a martingale (with bounded increments)converges in probabilities to a normal distribution with mean 0 and variance
Because of the limited liability constraint \((W_t \geq 0)\) I can only consider the left-truncated (at zero) standard normal. Therefore, for arbitrary \(t, t^+\),

\[
\Pr(W_{t^+} < W_t) = \Phi^T(W_t),
\]

where \(\Phi^T(\cdot)\) is the truncated \((0, 1)\) normal distribution with density computed as

\[
\phi^T(\xi) = \frac{\phi(\xi)}{\Phi(0)} = 2\phi(\xi).
\]

Hence I can define

\[
\Pr(W_{t^+} < W_t | W_t \geq 0, X_{t^+} \geq X_0) \text{ and } \Pr(W_{t^+} < W_t | W_t \geq 0, X_{t^+} < X_0)
\]

as the probabilities of downsizing and terminating, respectively. Then the value function \(V(X, W)\) must satisfy the Hamilton-Jacobi-Bellman equation

\[
rV = \max_{dI_t, X_t} X_t dt - dI_t + V_X dX_t + V_W dW_t + \frac{1}{2}X^2\sigma^2 [V_{XX} + 2\eta V_{WX} + \eta^2 V_{XX}] \tag{4.1}
\]

\[
- \Phi(W_t | X_t \geq X_0) [V(X, W_t) - V(X_{t^+}, W_t)]
\]

\[
- \Phi(W_t | X_t \geq X_0) [V(X, W_t) - V(X_0, W_0 + \gamma)]
\]

over the controls \((dI_t, X_t)\). To make sense of this equation one substitutes the definitions (2.1) and (3.2) of the dynamics of the cash flow process and the agent’s continuation value, subject to incentive compatibility (3.3), (3.5). Then making use of of the definition of \(y\) and the homogeneity property, the equation can be rewritten in terms of \(y\) and \(v(y)\). Finally, over the range where there is no cash payment \((dI_t = 0)\), one has

**Lemma 1** Let \(yv(y) = V(X, W)\) with \(y_t = W_t / X_t \, \forall t \geq 0\). The HJB of the function \(V(X, W)\) is equivalent to

\[
(r - \mu)v(y) = 1 + (\gamma - \mu)gv'(y) + \frac{1}{2} \sigma^2(\gamma - y)v''(y) \tag{4.2}
\]

\[
- \Phi(y_m | 1 \geq x_0) [v(y_m) - v(y_{t^+})]
\]

\[
- \Phi(y_m | 1 < x_0) [v(y_m) - v(y_0)]
\]

where \(x_0 = X_0 / X_{t^+}\) as a convention.

\(^2\)The Martingale CLT requires that the increments of the Brownian process be bounded; \(I_t\) is necessarily bounded and \(Z_t\) is also bounded by assumption.
To complete the description of the problem I add boundary conditions and claim

**Proposition 3** The function \( v(\cdot) \) is the unique solution of the differential equation (4.2) subject to

\[
v'(\bar{y}) = -1 \quad \text{for some } \bar{y} \quad (4.3)
\]

\[
v''(\bar{y}) = 0 \quad (4.4)
\]

and the termination condition (3.7).

This results paves the way to the description of some properties of the optimal contract. First there is no payment before \( y_t \) reaches \( \bar{y} \), which acts as a reflective barrier. From that point on providing incentives by increasing the continuation value \( y \) becomes too costly and the principal is better off disbursing cash. This is in line with BMRV, DeMarzo and Sannikov (2006), He (2008) and others. Second, there is downsizing until the scale of operations becomes inefficient. At that point the firm is not liquidated but the agent is terminated; it is necessary to preserve incentive compatibility and it a form of preemptive termination.

Third, when principal and agent have the same discount factor, He (2008) shows that the continuation value \( y_t \) reaches an absorbing state at \( y_t = \eta \). Then the agent never diverts funds; incentive provision in the form of an increasing continuation utility \( y_t \) becomes redundant (so there are no incentive costs). Indeed,

\[
dy_t = ry_t dt - i_t + \beta dZ_t
\]

\[
= ry_t dt + \eta \sigma dZ_t, \quad (i_t = 0)
\]

\[
= y_t(\sigma dt + \sigma dZ_t)
\]

so

\[
\frac{dy_t}{y_t} = \sigma dt + \sigma dZ_t
\]

is independent of the diversion problem.

Whether it holds here depends on the risk-taking problem too. From the dynamics of \( W \) given by (3.2), disbursing cash decreases the continuation value, which according to the incentive constraint (3.5) must remain non-decreasing at all times. I now turn to this problem. Condition (3.6)

\[^3\text{After good enough a history the scaled continuation value of the agent reaches an absorbing state; from that time on the principal can disburse cash to the agent at no cost (in terms of incentives).}\]
illustrates that providing incentives to deter diversion may not be sufficient to also deter speculation. Deterring excessive risk-taking requires the continuation value to grow at least at a constant rate $\eta \Delta \mu / \lambda$. So whether the continuation value reaches high-enough a state to solve both the diversion problem and the speculation problem depends on the severity of the risk-taking problem, i.e. whether

$$\Delta \mu \leq (>\lambda$$
5 Implementation

6 Discussion

6.1 Leverage and scale

6.2 Monitoring

6.3 Transmission channels

7 Conclusion
A Technical background

The main text spares the reader some technicalities. Underlying the choice of whether to speculate is an action: \( a \in \{0, 1\} \) that alters the drift \( \mu(a)dt \) and introduces the Poisson process \( dN_t \) of losses \( L \). That action generates a probability distribution over the paths of both \( \mu(a) \) and \( N \); so (implicitly) all expectations are taken with respect to that distribution.

To write Proposition 1 we need the dynamics of the agent’s continuation value \( W_t \). When she has a history of reports \( \tilde{x} = x \) up to time \( t \) and does not speculate at \( t \), the agent’s value reads

\[
\Psi_t = \int_0^t e^{-\gamma s} dI_s(x) + e^{-\gamma t} W_t(x)
\]

and is clearly a martingale. Hence there exists a process \( \beta_t(x) \) such that

\[
d\Psi_t = e^{-\gamma t} \frac{\beta_t(x)}{\sigma} [dX_t - (\mu(a) X_t) dt] \]

Differentiate the first expression and re-arrange these two expressions to obtain

\[
dW_t = \gamma W_t - dI_t + \beta_t X_t dZ_t.
\]

If instead the agent speculates and is subject to the penalty \( P_t \),

\[
\Gamma_t = \int_0^t e^{-\gamma s} dI_s(x) + \eta \int_0^t e^{-\gamma s} \Delta \mu X_s ds + e^{-\gamma t} W_t(x) - P_t dN_t
\]

and is also a martingale with respect to the filtration \( \mathcal{F}_t \) given the action \( a = 0 \). The auxiliary process becomes

\[
d\hat{\Psi}_t = e^{-\gamma t} \frac{\beta_t(x)}{\sigma} [dX_t - (\mu + \Delta \mu) X_t dt] = e^{-\gamma t} \beta_t(x) X_t dZ_t
\]

Differentiate \( \hat{\Psi}_t \):

\[
d\hat{\Psi}_t = e^{-\gamma t} dI_t(x) + \eta e^{-\gamma t} \Delta \mu X_t - re^{-\gamma t} W_t(x) dt + e^{-\gamma t} dW_t(x) - P_t dN_t,
\]

so when she speculates the agent’s continuation utility follows the dynamics

\[
d\hat{W}_t = \gamma W_t + \eta \Delta \mu X_t dt - dI_t + \beta_t X_t dZ_t - P_t dN_t
\]

B Proofs

Proof of Proposition 1: Condition (3.3) mirrors DeMarzo and Sannikov (2006) or He (2008) and follows from the derivations above. Note that because \( d\Psi_t = d\hat{\Psi}_t \), the sensitivity \( \beta_t \) is the
same regardless of whether the agent speculates. To deter her from engaging in (excessively) speculative activities, one needs the penalty to be large enough so that

\[ dW_t \geq d\hat{W}_t \]

that is

\[ P_t \geq \eta \frac{\Delta \mu}{\Delta\lambda} X_t \]

Combining with (3.3) binding one has \( P_t \geq \eta \frac{\Delta \mu}{\Delta\lambda} X_t \equiv y_m X_t \), which is feasible only when \( W_t \geq W_m \) by limited liability.

**Proof of Proposition 2:**

**Proof of Lemma 1:** with

\[
H_0(0) = 1 = H'_1(0) \\
H_1(0) = 0 = H'_0(0).
\]

By the Cauchy-Lipschitz theorem the functions \( H_0 \) and \( H_1 \) are uniquely defined.

**Proof of Proposition 3:**


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