Competition and the Hold-up Problem: a Setting with Non-Exclusive Contracts

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Abstract

This work studies how the introduction of competition to the side of the market offering trading contracts affects the equilibrium investment profile in a bilateral investment game. By using a common agency framework, where trading contracts are non-exclusive, I find that the equilibrium investment profile depends on the level of competition of the trading outcome. Full efficiency can only be implemented when the trading outcome is the most competitive. Moreover, a low level of competition is not always Pareto dominant for the parties offering the trading contracts, and larger welfare can be obtained with lower competitive equilibria.

Keywords: bilateral investment; common agency; competition; Pareto dominance; welfare.

JEL Classification Numbers: D44; L11.
1 Introduction

In many economic situations, parties undertake relation-specific investment to increase potential gains from their relationship. Consider for instance an insurer who researches possible contingencies to better suit the special needs of his client; or a seller that reduces the production cost of an intermediate good specific to a downstream producer. Specific investment increases the potential gains from trade, and the decision to invest depends on the extent in which the investing party appropriates those gains. Because in the trading stage investments costs are sunk, each party fears opportunistic behavior by their counterpart. Anticipating that part of the gains from trade coming from investment will be expropriated results in inefficient investment decisions, which have detrimental effects on resource allocation and economic welfare. For instance, Fisher Body, a manufacturer of body cars, refused to locate their body plants adjacent to General Motors assembly plans, a move that was necessary for production efficiency.

The existence of the “hold-up” problem is generally traced to incomplete contracts, that is, the inability of parties to write contracts depending on all relevant and publicly available information.\footnote{If specific investment is verifiable or enforceable ex-post, it is in the interest of the contractual parties to write compensation schemes linked to investment, Grossman & Hart (1986), Grout (1984), Hart & Moore (1988) and Williamson (1985).} To solve this problem, the economic literature has focused on two different approaches. The first approach, organization design, is closely related to the theory of the firm and searches for conditions to determine when transactions should be undertaken through a price mechanism - the market - or by fiat - the firm. It also establishes provisions for asset ownership and dictates that the residual right of control should be given to the party who is more prompt to suffer from ex-post opportunism, Hart (1995). The second approach is the design of long-term contracts. Its focus is on establishing contractual provisions such as default or option contracts, enforceable in case of disagreement, to relax potential conflicts of interests between the trading parties.

The main caveat from the previous solutions is the need of a sound and solid institutional system allowing for either a clear definition and allocation of property rights; or the existence of a third party, such as a neutral court of justice, able to enforce ex-ante contracts or impose rules to resolve disputes. Then, what happens when a sound and solid institutional system
does not exist? In this paper, I consider situations where investment contracts cannot be enforced and I explore how the introduction of competition to one side of the market gives incentives to undertake profitable specific investment.

I consider a model in which a single buyer trades with many sellers for the provision of an homogenous input. One of the sellers is aware of a technology which enables him to reduce the cost of input production. The buyer can also invest to improve her valuation for the homogenous input by adapting her production process. I characterize the equilibrium payoff of each part of the market and obtain the equilibrium investment profile. An application that fits this model is the provision of military or medical supplies to governments in states where economic institutions do not allow for the design or enforcement of ex-ante contracts. The model then proposes a normative analysis on how trade should take place in a market to provide incentives to both parts of the market who take profitable specific investment.

In the provision of military and medical supplies, it is normal to assume that neither part of the market has the whole bargaining power. Neither the government nor the sellers are able to appropriate all the potential gains from trade. Modeling the bargaining procedure in a common agency game with specific investment is a daunting task. I then take the methodology used in the existing literature and consider an analog of a first price auction in which, to compete for the buyer, each seller offers a menu of trading contracts; a trading contract consists of an amount of input provision and a transfer. The buyer then chooses the best trading contracts. This auction divides the trading surplus between the buyer and the sellers and the amount that each seller obtains is a measure of his contribution to that surplus. Indeed, by allowing the sellers to form coalitions to coordinate their out of equilibrium trading contracts, the payoff of each seller equals the loss of the trading surplus originated when the buyer excludes him from trade; this loss depends on the outside option available to the buyer. The larger the number of sellers coordinating their out of equilibrium trading contracts the larger is that outside option. Hence, the equilibrium outcome is more competitive, as the bargaining position for each seller is smaller, the larger the number of competing sellers coordinating their out of equilibrium trading contracts.

I obtain that the trading partners invest efficiently only when the trading outcome is the most competitive, or equivalently when the bargaining position of the sellers is minimized. In this case, investments do not affect the outside option available to the buyer after exclusion of
a seller and each seller appropriates his marginal contribution of the trading surplus. In any other situation where the equilibrium outcome is less competitive, investment is never efficient. The investing seller over-invests as his investment affects the outside option available to the buyer. Hence, larger bargaining positions for the sellers generate an asymmetric appropriation of the trading surplus creating investment inefficiencies. I also find a relationship between the equilibrium investment profile and the the number of sellers the buyer establishes trade with. With an infinite number of sellers the equilibrium investment profile tends to efficiency regardless of the equilibrium ex-post.

I further explore which equilibrium outcome or bargaining position the competing sellers prefers the most, and which leads to larger welfare. Because the equilibrium investment profile depends on the level of competition of the equilibrium outcome, the sellers not always prefer situations where competition is mild. Because a lower sellers’ bargaining position incentivizes the investment of the buyer, the sellers may obtain larger profits when the competition of the equilibrium outcome is large. Moreover, the results are influenced by the sensitivity that the investment of the investing seller has on the equilibrium trading allocation. Because relative productive efficiency changes with the investment of the seller, a larger seller’s investment translates into a reduction of the amount traded by the competing sellers. If this effect turns out to be small, all sellers prefer a less competitive equilibrium outcome granting them a more favorable bargaining position. Otherwise, different sellers prefer different bargaining positions. With regard to welfare, I obtain that it is not always maximized when the equilibrium outcome is the most competitive and this is due to the strategic complementarity of investment. Inefficiency of investment created to one side of the market can restore efficiency to the other side, leading to larger potential gains from trade. Surprisingly, I obtain that lower competitive outcomes may lead to higher levels of welfare. Therefore, a competition authority should be careful in analyzing an industry where ex-ante specific investments are important; promoting competitive outcomes may fail to maximize the welfare that can be generated in the market.

In the next section I discuss the related literature. In section 3 I introduce the formal set-up of the model and proceed to solve it backwards. Therefore, in section 4.1 I study the properties of the equilibrium allocation and characterize the equilibrium payoffs in section 4.2. I proceed to obtain the equilibrium investment profile in section 4.3. In section 5, I compare
equilibria, I begin with Pareto optimality and I continue by ranking them in terms of welfare. Finally, section 6 concludes. All proofs are in the appendix.

2 Literature

The present work builds on the literature on markets and contracts. In this literature instead of considering the impossibility of contracting over some states of nature or actions, there are limits on the number of parties that can be part of the same contract. In this paper, I use the most recent set-up, i.e., trading contracts are non-exclusive and a common agent can freely sign multiple bilateral trading contracts with different parties.\footnote{Earlier studies have centered the analysis on exclusive contracts. This is the spirit of Akerlof (1970), Rothschild & Stiglitz (1976) and Aghion & Bolton (1987) Biglasier & Mezzetti (1993, 2000).}

The first theoretical work to consider a general model of contracting between one agent and multiple principals is due to Segal (1999). In a general framework, he shows that with the absence of direct externalities, the equilibrium trading outcome is efficient.\footnote{There are no externalities when the principals’ payoffs depend only on their own trade with the agent.}

Efficiency is robust even in a bidding game, where multiple principals propose trading contracts to the common agent, and where inefficiencies may arise from the coexistence of offers made by different parties.\footnote{Bernheim & Whinston (1986) also consider a common agency game where a group of principals aim at providing incentives to a common agent. Hence, I have a special class of complete information common agency, where the agent’s choice is an N-dimensional vector specifying the amount of a good to be traded.}

While a unique and efficient trading outcome exists in the absent of direct externalities, it has been shown multiplicity in the equilibrium payoffs, Chiesa & Denicolò (2009).\footnote{The authors show that the set of equilibrium payoffs is a semi-open hyper-rectangle. Additionally, Martinort & Stole (2009) show multiplicity of equilibria in a public common agency game and use asymmetric information as a tool for equilibrium refinement.}

In a common agency framework, the equilibrium payoff that any principal obtains is determined by the “threat” of being replaced by his competitors, and this “threat” pins down to which type of “latent” or out of equilibrium trading contracts are submitted by the rest of the principals. Chiesa & Denicolò (2009) characterize the maximum payoff compatible for a non-cooperative notion of equilibrium, which is given by the “threat” of any principal to be unilaterally replaced by one of his competitors. In this paper, I characterize a subset of equilibrium payoffs by allowing the competing sellers to form a coalition to collectively replace any other seller. I then obtain a range of different equilibrium payoffs where the lowest one
coincides with the “truthful” equilibrium.\footnote{In a “truthful” equilibrium, the strategy is such that each party obtains its marginal contribution to the surplus.}

In a more recent paper, Chiesa & Denicolò (2012) undertake comparative statics of different equilibria and state that the Pareto dominant equilibrium for the sellers is the one where the rent of the buyer is minimized. In their framework, potential gains from trade are irrelevant on how these gains are redistributed and the sellers always prefers an equilibrium where the portion of the gains from trade is the most favorable to them. I challenge their finding by introducing a previous stage where parties can undertake specific investment before the trading stage. Moreover, by introducing an investment stage in the game, I am able to compare equilibria with regard to welfare. This analysis has not been carried out in the markets and contracts literature, where the different type of equilibria are only a way to distribute the gains from trade and welfare stays invariant. In my model, the redistribution of the gains from trade has implications on the investment decisions of the parties and on the final dimension of those gains.

The present paper is also closely related to the “hold-up” literature where an early formulation is due to Klein, Crawford & Alchian (1978) and Williamson (1979, 1983). In these papers the “hold-up” problem arises because parties are unable to bargain over specific investment due to its unverifyiability. In my model the “hold-up” problem comes from the lack of contract enforceability. The “hold-up” literature concludes that in the absence of ex-ante contracts, investments are likely to be inefficiently low under any possible bargaining game, Grossman & Hart (1986) and Hart & Moore (1990). This literature has then centered in ways of designing mechanisms to restore the efficient levels of investment, such as the allocation of property right or the design of ex-ante contracts as in Aghion, Dewatripont & Rey (1994), Chung (1991) and Edlin & Reichelstein (1996). In my model, ex-ante contracts cannot be enforced and this relates to the literature on competition and the “hold-up” problem as in Cole, Mailath & Postlewaite (2001a, 2001b); Mailath, Postlewaite & Samuelson (2013); Felli & Roberts (2012); Makowski (2004) and Samuelson (2013). All those models consider a matching mechanism where once investment has been undertaken, agents decide on the trading partner. Investment then works as a mechanism to increase the outside option, giving higher incentives to invest. Departing from this literature, I allow the offering part of the market
to compete by offering trading contracts to the monopolistic side. Trade in my model is non-exclusive, and I give a normative analysis on how trade should be organized in situations where both parts of the market undertake specific investment.

3 Model

I consider a bilateral investment game where a single buyer trades with many ex-ante identical sellers. The $N$ sellers are indexed by $i \in \{1, ..., N\}$ and produce an homogeneous input for the buyer.

The game consist of two stages that are played sequentially. In stage one, specific investment takes place. Here, only seller 1 invests in a cost-reducing technology, which allows him to reduce the cost of production. The amount of investment is a continuous variable $\sigma \geq 0$, with a convex cost $\psi(\sigma)$. The buyer undertakes also specific investment to enhance her valuation of the total amount traded. She takes a binary decision on whether or not to invest $b \in \{0, 1\}$, and incurs to a fixed costs of $K$.

I further assume that the investing parties do not have any budget constraint, and they are not financially restrained on the amount of investment that they can take.

In stage 2, each seller trades with the common buyer. Following Chiesa & Denicolò (2009), I consider a bidding game where trade is modeled as a first-price auction in which the sellers simultaneously submit a menu of trading contracts $M_i \subset \mathbb{R}^+$ for each $i \in N$, and the buyer chooses a single trading contract from each one of them. A typical trading contract is a pair $m_i = (x_i, T_i)$, where $x_i \geq 0$ is the quantity seller $i$ supplies and $T_i \geq 0$ is the corresponding total payment or transfer from the buyer towards seller $i$. Because trade is voluntary, I require that the null or zero contract is always offered in equilibrium, i.e., $m_i^0 = (0, 0)$.

As in Chiesa & Denicolò (2009), to guarantee the existence of an optimal choice for the buyer, I require the set $M_i$ to be a compact set $\mathcal{G}$. Hence, with the menu profile of trading contract $M = (M_1, M_2, ..., M_N) \in \mathcal{G}^N$, A strategy for the buyer is a function $\mathcal{M}(M) : \mathcal{G}^N \to (\mathbb{R}^+)^N$ such that $\mathcal{M}(M) \in \times_{i=1}^N M_i$ for all $M \in \mathcal{G}^N$. Later, when I obtain the equilibrium transfers, I elaborate more on the restrictions of the trading contracts that the sellers offer to

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7The buyer decides on whether to adapt his production process to the homogenous input provided by the sellers; obtaining a larger utility from consumption.
Finally, the model is one of complete information and the equilibrium concept is sub-game perfect Nash (SPNE). Even if investment is observable, it is not contractable and this is because investment cannot be enforced by a third party.\footnote{By not assuming any restriction on the trading offers that the sellers can offer to the buyer, Chiesa & Denicolo (2009) provide a complete characterization of the set of equilibrium payoffs.}

\section{3.1 Payoffs and trading surplus}

The payoffs of the buyer and the sellers are quasi-linear in transfers.\footnote{The model is one of private and delegated common agency. It is private since a seller cannot condition payments on the quantities traded by others, and it is delegated because the buyer is allowed to trade with a subset of sellers.} The buyer obtains

\begin{equation}
\Pi(M | b) = U(X | b) - \sum_{i=1}^{N} T_i - K \times b,
\end{equation}

where $X = \sum_{i=1}^{N} x_i$ is the total quantity traded; and seller’s 1 payoff is

\begin{equation}
\pi_1(M | \sigma) = \pi_1(M_1 | \sigma) = T_1 - C(x_1 | \sigma) - \psi(\sigma).
\end{equation}

The payoffs for the rest of the sellers do not directly depend on the investment profile and those are equal to

\begin{equation}
\pi_i(M | \sigma) = \pi_i(M_i | \sigma) = T_i - C(x_i | \sigma) - \psi(\sigma).
\end{equation}
\[ \pi_i(M_i) = \pi_i(M) = T_i - C(x_i), \quad \text{for all } i \neq 1. \] 

(3.3)

Finally, for a given investment profile \((b, \sigma)\) the maximum trading surplus is

\[ TS^*(b, \sigma) = \max_{x_1, \ldots, x_n} \left[ U(x_1 + \ldots + x_n | b) - C_1(x_1 | \sigma) - \sum_{i \neq 1} C_i(x_i) \right], \]

(3.4)

and \(x^* = (x_1^*, \ldots, x_N^*)\) is the vector of quantities that solves the problem. For later use, I denote by \(X^*_{-H} = \sum_{i \notin H} x_i^*\), for \(H \subset N\), the sum of the previous quantities without taking the quantities of the subset of sellers in \(H\). I finish by stating the assumptions of the utility and costs functions. Subscripts denote partial derivatives, and I also denote the utility of the buyer when she does not invest \(U(X | b = 0)\) by \(U(X)\).

1. \(U_x(\cdot) > 0, \ U_{xx}(\cdot) < 0, \ U(X | b = 1) > U(X)\) and \(U_x(X | b = 1) > U_x(X)\),
2. \(C_x(\cdot) > 0, \ C_{xx}(\cdot) > 0, \ C_\sigma(\cdot) < 0, \ C_{x\sigma}(\cdot) < 0, \ \psi_\sigma(\sigma) > 0 \) and \(\psi_{\sigma\sigma}(\sigma) > 0\),
3. \(\lim_{X \to 0} U_x(\cdot) = +\infty, \ \lim_{X \to \infty} U_x(\cdot) = 0, \ \lim_{x_1 \to 0} C_x(\cdot) = 0\) and \(\lim_{x_1 \to \infty} C_x(\cdot) = +\infty\).

4 Analysis

I solve the model backwards to obtain the sub-game perfect Nash equilibrium (SPNE). I begin with the equilibrium of the trading game played in stage 2. After describing the properties of the equilibrium trading allocation, I characterize a subset of the equilibrium transfers. Departing from the existing literature, I allow the sellers to form a coalition to coordinate their out of equilibrium trading contracts. Later, I solve stage 1 of the game and obtain the equilibrium investment profile. Finally, I rank equilibria with regard to Pareto dominance and welfare.

4.1 Equilibrium trading allocation

The equilibrium allocation in the trading game depends on the investment undertaken at stage 1. I then proceed to characterize the equilibrium allocation for a given vector of investment \((b, \sigma)\). Because the production cost of each seller depends only directly on the amount of input
he produces, his individual payoff is not directly affected by the trading contracts submitted by all other sellers. This is clear in equations (3.2) and (3.3) where the payoff of any seller \( i \) depends only on the strategy of the buyer undertaken to him \( M_i \) and not on the whole profile \( M \).\(^{11}\) Thus, given the trading contracts of the competing sellers, each seller effectively plays a bilateral trading game with the buyer where he has the whole bargaining power. Thus, when submitting a trading contract, each seller maximizes the potential gains from trade that can be generated between him and the buyer; hence, for any quantity traded with the other sellers \( X_{-i} \) I obtain

\[
\Pi(M \mid b) + \pi_i(M_i \mid \sigma) = U(X_{-i} + x_i^* \mid b) - \sum_{j \neq i} T_j - C(x_i^* \mid \cdot) \\
> U(X_{-i} + \hat{x}_i \mid b) - \sum_{j \neq i} T_j - C(\hat{x}_i \mid \cdot); \text{ for any } \hat{x}_i \geq 0, \forall i \in N.
\]

Seller \( i \) does not profit by deviating from the efficient trading amount \( x_i^* \) to any other amount \( \hat{x}_i \), and this holds true for every seller \( i \in N \).\(^{12}\) Consequently, the efficient allocation is a Nash equilibrium defined by the following system of equations:

\[
U_x(X^* \mid b) = C_x(x_i^* \mid \sigma) \quad \text{for } i = 1,
\]
\[
U_x(X^* \mid b) = C_x(x_i^*) \quad \text{for } i \neq 1,
\]

(4.1)

where, for a given investment profile, the marginal utility of consumption equals the marginal costs of production. How the equilibrium trading allocation changes with investment is shown in the following lemma, whose proof, presented in the appendix, page 39, comes directly from the previous system of equations.

**Lemma 1.** In the efficient trading allocation:

i) for a given investment of the buyer, an increase on the investment by seller 1 rises the amount of trade between the buyer and himself, but decreases the amount of trade with all

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\(^{11}\)There are no direct externalities, only contractual externalities between the sellers, which are because the buyer’s marginal willingness to pay for the good depends on the quantities traded with all the sellers. Inefficient equilibria arise if the buyer have to purchase a pre-set total quantity, Krishna & Tranaes (2002).

\(^{12}\)This result is due to “individual excludability” and “bilateral efficiency” which fully characterize the equilibrium of the game. For an exhaustive analysis see Bernheim & Whinston (1996) and Segal (1999).
other sellers. The total amount traded increases.

\[ \frac{dx^*_1}{d\sigma} > 0; \quad \frac{dx^*_j}{d\sigma} < 0 \text{ for all } j \neq 1 \text{ and } \frac{\partial}{\partial \sigma} X^* > 0. \]

ii) For a given investment of seller 1, the amount of trade of each seller increases with the investment of the buyer.

\[ x^*_i(1, \sigma) > x^*_i(0, \sigma) \quad \forall i \in N. \]

The higher the investment undertaken by seller 1, the more efficient he becomes with respect to the other sellers and the buyer substitutes trading from any other sellers to seller 1. This substitution effect is of second order and because the economy in aggregate is more efficient the total amount traded is higher. For a given investment of the seller, when the buyer invests, she trades a larger amount with every seller, as the relative efficiency of each seller stays the same.\(^{13}\)

Later, in the investment stage, the magnitude of change of the efficient allocation of the non-investing sellers due to an increase of investment of seller 1 is of the utmost importance. Hence, I introduce the following definition.

**Definition 1. (Allocative sensitivity)** I call \( \frac{dx^*_j}{d\sigma} \) for \( j \neq 1 \) the allocative sensitivity, which corresponds to the change on the efficient trading allocation of sellers \( j \neq 1 \) due to an increase of investment of seller 1.

In the appendix, page 39, it is shown that the magnitude of this allocative sensitivity depends on the primitives of the economy.\(^{14}\)

### 4.2 Equilibrium transfers

The literature on markets and contracts has established that the maximum payment or transfer that any seller obtains depends on the “threat” that the buyer decides to exclude him

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\(^{13}\)I am thankful to Martin Pollrich for his insights in proving the claim.

\(^{14}\)In the model, sellers produce completely homogeneous products. However, the degree of substitutability will have a strong effect on the sensitivity created in the equilibrium allocation. With perfect homogenous products, the buyer perfectly substitute products from sellers and the allocative sensitivity is big. In my model, the degree of substitutability depends on the primitives of the economy, i.e., on the convexity of the cost function. With heterogeneous products, the buyer will not reduce much the amount that she trades with the other sellers after an increase of the investment by seller 1. I am thankful to professor Sánchez-Pagués for this observation.
from trade, and this is directly related to the out of equilibrium or “latent” trading contracts that the competing sellers offer to the buyer.\textsuperscript{15} Chiesa & Denicolò (2009) have shown that in the absence of any restrictions on these out of equilibrium trading contracts, there is multiplicity in the equilibrium transfers. They have also characterized the maximum payoff compatible with a non-cooperative notion of equilibrium, which happens when there is only a single competing seller offering an out of equilibrium trading contract aimed at excluding any other seller. In my model, I use the same technique to obtain the equilibrium transfers, but departing from Chiesa & Denicolò (2009) I allow the sellers to form a coalition to coordinate their out of equilibrium trading offers.\textsuperscript{16} I then obtain a subset of the equilibrium payoffs, where the most competitive equilibrium outcome coincides with what the literature have called the “truthful” equilibrium, where each seller obtains his marginal contribution to the trading surplus.\textsuperscript{17}

To begin with the analysis, the following definition will be useful when assuming that a group of sellers are able to coordinate their offers.

**Definition 2. (Coordination)** A set of sellers $J \subset N$ coordinate their trading contracts if, for a given aggregate amount $X^*_H$ - where $J \subseteq H \subset N$ - and an investment profile $(b, \sigma)$, the gains from trade generated with the buyer and these sellers in $J$ are the largest and:

$$V_J \left( X^*_H \mid b, \sigma \right) = \max_{\{x_j\}_{j \in J}} \left[ U \left( X^*_H + \sum_{j \in J} x_j \mid b \right) - \sum_{j \in J} C_j(x_j \mid \cdot) \right],$$

(4.2)

where $\tilde{x}(J) = (\tilde{x}_j(\cdot \mid J), \ldots, \tilde{x}_{j'}(\cdot \mid J))$ for all $j, j' \in J$, are the quantities that solves the problem.

When any seller $i$ offers his equilibrium trading contract he takes into account how much the buyer can generate with the rest of the sellers. In other words, what is the outside option available to the buyer if she decides not to trade with him. This outside option depends on

\textsuperscript{15}The “latent” contracts are the trading contracts that are never accepted by the buyer but effectuate a constraint on the equilibrium transfer of the sellers.

\textsuperscript{16}Chiesa & Denicolò (2009) provide a partial characterization of the equilibrium strategies.

\textsuperscript{17}Truthful strategies are assumed in Bernheim & Whinston (1986), Bergemann & Välimäki (2003), Dixit, Grossman & Helpman (1997), Spence (1976) and Spulber (1979). A strategy is called to be “truthful” to a given action if it truly reflects the principal’s marginal preference for another action relative to the given action. In a private common agency, truthfulness means that each principal can ask payments that differ from his true valuations of the proposed trades only by a constant.
the out of equilibrium trading contracts offered by the competing sellers. With regard to
these out of equilibrium offers, I introduce the following assumption.

**Assumption 1.** A set of sellers $J_i \subset N$ for $i \notin J_i$ form a coalition to coordinate their out of
equilibrium trading contracts to exclude seller $i$.\(^{18}\)

Therefore, I impose that in the menu of trading contracts, the sellers belonging to the set
$J_i$ will coordinately offer out of equilibrium contracts with the properties stated in definition 2.
Indeed, what the buyer and the sellers in set $J_i$ obtain by coordinating their out of equilibrium
trading contracts is equal to

$$V_{J_i} \left( X_{*\{J_i,i\}}^{*} \mid b, \sigma \right) = \max_{\{x_j\}_{j \in J_i}} \left[ U \left( X_{*\{J_i,i\}}^{*} + \sum_{j \in J_i} x_j \mid x_i = 0, b \right) - \sum_{j \in J_i} C_j(x_j \mid \cdot) \right], \quad (4.3)$$

which coincides with the expression given in definition 2 by setting the set $H = \{J_i,i\}$ and
the trading quantity of seller $i$ to zero, $x_i = 0$. Expression (4.3) states the maximum trading
surplus that can be generated with the buyer and the sellers in $J_i$ if the buyer excludes seller
$i$ from trade. The set $J_i$ does not need to be equal for every seller $i$, but I further assume
that the cardinality of this set is the same for every seller $i$. Hence, I select equilibria where
the set $J_i$ is composed by 1, 2 and up to $N - 1$ sellers.\(^{19}\) The following lemma introduces the
result regarding the trading quantities submitted out of equilibrium, that will be useful for
the rest of the paper.

**Lemma 2.** For any investment profile $(b, \sigma)$ and a set of sellers in $J_i$, the aggregate trading
quantity offered out of equilibrium is smaller than the aggregate equilibrium trading quantity

$$X^{*}(b, \sigma) > X_{*\{J_i,i\}}^{*}(b, \sigma) + \sum_{j \in J_i} \tilde{x}_j(b, \sigma \mid J_i), \quad \text{for any } J_i \subset N,$$

but the individual out of equilibrium trading quantity for any seller $j \in J_i$ is larger than their
equilibrium trading quantity, and it is decreasing with the number of sellers in $J_i$.

$$\tilde{x}_j(b, \sigma \mid J_i') > \tilde{x}_j(b, \sigma \mid J_i) > x_j^{*}(b, \sigma); \quad \forall j \in J_i, J_i' \text{ and } J_i' \subset J_i.$$\(^{18}\)I assume that the set of sellers in $J_i$ reach a biding agreement to coordinate their out of equilibrium trading
contracts before investment is undertaken. Hence, I do not allow the equilibrium investment profile to be a
mechanism for equilibrium selection. I am grateful to Zhijun Chen for this observation.

\(^{19}\)Chiesa & Denicolo (2009) consider the case where the cardinality of the set is a singleton.
The formal proof is in the appendix, page 40, and here I give some intuition. Due to the convexity of the cost function, the total quantity traded when trading with seller $i$ always dominates the increase in the quantity traded with the set of sellers in $J_i$. It is immediate to see that the individual trading quantity that any seller $j \in J_i$ submits in his out of equilibrium trading contract is larger than his efficient quantity. Because the out of equilibrium trading contracts are aimed at excluding seller $i$, they have to offer a larger quantity of trade to the buyer as compensation for the trading quantity not traded due to the exclusion of seller $i$.

Following Chiesa & Denicolò (2009) with the assumption that a group of sellers in $J_i$ can form a coalition to coordinate their out of equilibrium trading contracts at no cost, the maximum equilibrium transfer that any seller $i$ obtains is equal to

$$T_i^e(J_i \mid b, \sigma) = V_{J_i} \left( X^* \mid b, \sigma \right) - V_{J_i} \left( X^* - \{J_i\} \mid b, \sigma \right), \ \forall i \in N. \quad (4.4)$$

The result crucially depends on the fact that there is no cost for the sellers in $J_i$ to form a coalition. The equilibrium transfers can then be obtained as in Chiesa & Denicolò (2009) where the out of equilibrium offers are such that the group of sellers in $J_i$ are indifferent between supplying their prescribed equilibrium trading contracts or proposing the buyer to collectively replace seller $i$ from trade.\footnote{I am thankful to Steffen Lippert for the observation of the zero cost for coalition formation.} Some notation and results from Chiesa & Denicolò (2009) consistent with my model can be found in page 37 of the appendix. To grasp a better intuition of the result, I algebraically modify expression (4.4) to obtain that its right hand side is equal to

$$L_i (J_i \mid b, \sigma) = \left( U \left( X^* \mid b \right) - \sum_{j \in J_i} C_j (x_j^* \mid \cdot) \right) - V_{J_i} \left( X^* - \{J_i, i\} \mid b, \sigma \right). \quad (4.5)$$

Expression (4.5) corresponds to the loss of exclusion of seller $i$. In other words, the trading gains that cannot be realized due to the exclusion from trade of seller $i$. Therefore, the loss of exclusion is computed by putting equal to zero the trading quantity of seller $i$, keeping constant the production of the sellers not in $J_i$, and choosing optimally the quantities of the sellers belonging to $J_i$. The convexity of the cost function, makes it straightforward to see that
the loss of exclusion $L_i(J_i)$ is weakly decreasing in the set $J_i : J_i \supset J'_i \implies L_i(J'_i) \geq L_i(J_i)$.$^{21}$

Hence, the larger the number of sellers coordinating their out of equilibrium trading contracts, the higher is the trading surplus that they can generate and the lower is the loss of exclusion.

Because $T_i^c(J_i \mid b, \sigma) = L_i(J_i \mid b, \sigma)$ for all $i \in N$, the equilibrium transfer for any seller $i$ cannot be greater than the loss of exclusion as the buyer will decide not to trade with him. This cannot be lower, as seller $i$ has a profitable deviation to increase it. Moreover, the equilibrium transfer is somehow related to the Groves-Clark mechanism. What seller $i$ obtains in equilibrium is linked to the externality that he creates to the sellers belonging to the set $J_i$. The larger the externality, in the sense that the set of sellers in $J_i$ change their amount of trade with the buyer due to the trading of seller $i$, the lower is the equilibrium transfer of the latter.$^{22}$

With the equilibrium transfers, the following proposition states the equilibrium payoffs in the trading game.

**Proposition 1.** i) For a given set of sellers in $J_i$ and an investment profile $(b, \sigma)$, the equilibrium payoff of the sellers is

$$\pi_1(b, \sigma \mid J_i) = TS^*(b, \sigma) - \tilde{T}S_{-1}(b \mid J_i) - \psi(\sigma); \quad \text{for } i = 1,$$

$$\pi_i(b, \sigma \mid J_i) = TS^*(b, \sigma) - \tilde{T}S_{-i}(b, \sigma \mid J_i); \quad \text{for } i \neq 1,$$

and the equilibrium payoff of the buyer is

$$\Pi(b, \sigma \mid J) = TS^*(b, \sigma) - \sum_i \left( TS^*(b, \sigma) - \tilde{T}S_{-i}(b, \sigma \mid J_i) \right) - K \times b,$$

where $\tilde{T}S_{-i}(b, \sigma \mid J_i)$ is the maximal trading surplus that can be generated without seller $i$ and when a subset of sellers in $J_i$ coordinate their out of equilibrium trading contracts.

ii) $\tilde{T}S_{-i}(b, \sigma \mid J_i) > \tilde{T}S_{-i}(b, \sigma \mid J'_i)$ for $J'_i \subset J_i$. Moreover, for $J_i \subset N \setminus \{i\}$ each seller obtains more than his marginal contribution to the trading surplus.

The formal proof is presented in the appendix, page 41. From proposition 1, if all sellers coordinate their out of equilibrium offers, i.e., $J_i = N \setminus \{i\} = \tilde{J}_i$, for all $i \in N$, each

$^{21}$In general the inequality will be strict if $J'_i$ is not equal to $J_i$.

$^{22}$The equilibrium transfers also represent the degree of indispensability of seller $i$. 

15
seller appropriates his marginal contribution to the surplus and the trading gains are evenly distributed to all players. In this case, the equilibrium outcome is very competitive; the bargaining position of the sellers is minimized and their portion of the trading surplus is the smallest. For a lower number of sellers coordinating their out of equilibrium trading contracts, i.e., $J_i \subset N \setminus \{i\}$, the distribution of the gains from trade favors the sellers in detriment of the buyer. In those cases, the equilibrium outcome is less competitive; the bargaining position of the sellers is larger and each one of them appropriates more than his marginal contribution to the surplus. I then proceed to state my notion of intensive competition.

**Definition 3. (Intensive competition)** An equilibrium outcome is more competitive the lower the portion of the gains from trade that the sellers appropriate. Hence, for a given number of active sellers $N$, and a given investment profile $(b, \sigma)$, a more competitive equilibrium is associated with a larger number of sellers in the set $J_i$. Therefore, the most competitive equilibrium is when $J_i = N \setminus \{i\} = \bar{J}_i$, for all $i \in N$ and the least competitive is when the set $J_i$ is a singleton $|J_i| = 1 = \bar{J}_i$ for all $i \in N$.

4.3 Investment profile

I begin by characterizing the efficient investment profile and I proceed with equilibrium. Efficiency serves as a benchmark and allows to see whether without ex-ante contracts full efficiency can be restored in equilibrium. I find that the decision to invest from both sides of the market depends on the competitiveness of the equilibrium outcome, as this determines the bargaining position of the sellers and eventually the gains that the investing parties are able to appropriate.

4.3.1 Efficient investment

The efficient vector of investment maximizes welfare; this equals the trading surplus minus the cost of investment. Both sides of the market decides to invest efficiently when they appropriate all the gains coming from their investment. Hence, the efficient investment is uniquely characterized by the solution of the following system of equations:

$$
\psi_\sigma(\sigma_E) = -C_\sigma \left( x_1^+(b, \sigma_E^b) \mid \sigma_E^b \right), \quad \forall b; \quad (4.9)
$$
where the upperscript on the investment of seller 1 represents the investment of the buyer. Accordingly, \( \sigma_1^E \) stands for the efficient investment of the seller when the buyer invests in equilibrium, i.e., \( b = 1 \).

Seller 1 sets the level of investment such that the marginal reduction of the production costs equals the marginal cost of investment. Similarly, the buyer invests if the fixed cost of investment \( K \) is lower than the increase in welfare arising from her investment, represented by the threshold \( \hat{K}_E \). A characteristic of the efficient investment profile - that also carries over in equilibrium - is that investments are strategic complements. Hence the more one of the parties invests, the higher are the incentives of the other party to increase investment. This result comes from a variant of super-modularity; lemma 1 shows that the investment of one party always increases the total amount of trade, and through this trade allocation, the value of investment from one party increases the marginal return of the other party’s investment.\(^{23}\)

4.3.2 Equilibrium investment profile

In equilibrium, because the investing party may not appropriate all the benefits coming from their own investment, the implementation of the efficient investment profile is generally not possible. I will show that full efficiency can only be implemented whenever the equilibrium outcome of the trading game is the most competitive, or equivalently when the bargaining position of the sellers is minimized. In the analysis that follows, I consider both the intensive and extensive degree of competition. The former, previously defined, takes into account how many sellers coordinate their out of equilibrium trading contracts, the latter, considers how the equilibrium investment profile is affected by the number of active sellers in the industry.

\(^{23}\)Lemma 1 shows that the amount traded with each seller increases if the buyer is investing, this implies that for a given level of investment from seller 1, \( x^*_1(1, \sigma) > x^*_1(0, \sigma) \) and together with assumption \( C_{\sigma}(\cdot) < 0 \) the right hand side of (4.9) increases with buyer’s investment.

\[
\partial(rhs) = -C_{\sigma}(x^*_1(1, \sigma) | \sigma) + C_{\sigma}(x^*_1(0, \sigma) | \sigma) = -\int_{x^*_1(0, \sigma)}^{x^*_1(1, \sigma)} C_{\sigma}(\tau) d\tau > 0,
\]

A similar argument can be used to see that the investment threshold of the buyer increases with the investment of the seller. If the investment of the buyer was continuous, i.e., \( \bar{b} = 0 \), there would be investment complementarity if the function \( TS^*(b, \sigma) \) is super-modular in investments, i.e., \( TS^*_E(b, \sigma) > 0 \). For an exhaustive analysis see Donald Topkis (1978).
4.3.3 Intensive competition

The equilibrium investment decisions are best-response actions. The following definition states an equilibrium in the investing game.

**Definition 4.** The vector \((b_e^f, \sigma_e^f)\) constitutes an equilibrium, if and only if:

\[
\begin{align*}
    b_e^f & \in \arg\max_{b \in \{0, 1\}} \Pi (b, \sigma_e^f | J), \\
    \sigma_e^f_{J_1} & \in \arg\max_{\sigma \geq 0} \pi_1 (b_e^f, \sigma | J_1).
\end{align*}
\]

Because the equilibrium payoff depends on the number of sellers belonging to the set \(J\), there is a direct link between the level of competition in the trading outcome and the equilibrium investment profile. The next proposition, proven in the appendix page 42, states that the efficient investment profile can be implemented in equilibrium.

**Proposition 2.** The efficient investment profile is implementable if and only if the outcome of the trading game is the most competitive, i.e., \(J_i = N \setminus \{i\} = \bar{J}_i\) for all \(i \in N\).

The investment decisions depend on how each party appropriates the gains coming from investment. When the outcome of the trading game is the most competitive, the gains from trade are evenly distributed among all market participants, and each seller obtains his marginal contribution of the trading surplus. This is because the investment of seller 1 does not have any effect on the outside option available to the buyer, or equivalently on the gains from trade that the sellers are able to generate with their out of equilibrium trading contracts. Hence, seller 1 exclusively appropriates the increase of the trading surplus coming from his own investment. Because, under some values of the investment cost, the buyer takes the efficient level of investment, there exists full efficiency. This is never the case when the outcome of the trading game is less competitive. With an increase on the bargaining position of the sellers, each one of them obtains more than their marginal contribution of the trading surplus, which distorts the incentives to invest efficiently.

From the previous discussion, I can easily characterize the investment profile when the equilibrium outcome of the trading game is the most competitive. This is introduced in the following corollary whose proof is relegated to the appendix, page 43.
Corollary 1. When in the most competitive equilibrium, the investment of the buyer is not efficient, the equilibrium investment profile is characterized by underinvestment.

The previous two results state that the investment decision of seller 1, in the most competitive equilibrium, is constrained efficient. For a given investment of the buyer, seller 1 always takes the efficient investment decision. However, when the buyer fails to invest efficiently, the equilibrium investment profile is characterized by the “hold-up” problem, and both seller 1 and the buyer underinvest. Downward distortion of investment arises because of strategic complementarity; a lower investment of the buyer creates lower potential gains from trade, and this makes seller 1 to decrease his level of investment.

I proceed to study investment when the equilibrium outcome is less competitive and the sellers have a more favorable bargaining position, that is, when the set of sellers coordinating their out of equilibrium trading contracts is \( J_i \subset N \setminus \{i\} \). The result is stated in the following proposition.

Proposition 3. When the equilibrium trading outcome is not the most competitive, \( J_i \subset N \setminus \{i\} \) for all \( i \in N \), then, for a given investment of the buyer, the magnitude of seller 1 over-investment depends on the level of ex-post competition and the degree of the allocative sensitivity \( dx^* m / d\sigma \). The magnitude of over-investment is

\[
\gamma(J_1) = - \sum_{m \notin J_1, 1} \left( \int_{X^*}^{X^*(J_1, 1)} + \sum_{j \in J_1} \tilde{x}_j(J_1) \right) U_{xx}(\tau) d\tau \frac{dx^*_m}{d\sigma},
\]

and it decreases with the level of intensive competition, i.e., \( \gamma(J'_1) > \gamma(J_1) \) for \( J'_1 \subset J_1 \).

The formal proof is in the appendix, page 44. Contrary to the case where the outcome of the trading game is the most competitive, here the investment of seller 1 is distorted upwards. With a less competitive outcome, seller 1 does not only appropriate all the direct gains coming from his investment, but also part of the payoffs from his competing sellers. Seller’s 1 loss of exclusion depends on his investment through the allocation of the out of equilibrium offers that remains unchanged for the sellers not coordinating their offers. In other words, the investment of the seller reduces the trading allocation of the competing sellers who do not coordinate their out of equilibrium trading contracts, and this puts a constraint on the gains from trade that can be generated out of equilibrium. Hence, the larger the allocation sensitivity \( dx^*_m / d\sigma \)
for \( m \notin \{ J_1, 1 \} \), the bigger is the loss of exclusion of seller 1 and the larger is his equilibrium transfer. Moreover, for a fixed investment of the buyer, the magnitude of over-investment \( \gamma(J_1) \) decreases with the number of sellers belonging to the set \( J_1 \). The lower is the level of competition - which implies a smaller \( J_1 \) - the distortion of investment is larger.

With regard to the equilibrium investment profile, I introduce the following corollary; this states that with low levels of competition in the trading game, i.e., \( J_i \subset N \setminus \{ i \} \), inefficiencies may arise to both sides of the market. The formal proof is the appendix, page 45.

**Corollary 2.** When the buyer takes the efficient investment decision, seller 1 over-invests.

1) If the investment decision of the buyer is not efficient, the inefficiency created is two-sided:
   
   - A) the buyers underinvest, and
   - B) seller 1 over-invests or underinvest depending on the magnitude of the allocative sensitivity.

   Over-investment always appears in equilibrium if the allocative sensitivity is sufficiently large, i.e.,
   
   \[
   - \frac{d\tau^*}{d\sigma} > \frac{\int_{\tau^*}^{\tau^*_1(0,\sigma_0^0)} C_{2\sigma}(\tau) \, d\tau}{(N \setminus \{ 1 \} - J) \times \int_{X^*(0,\sigma_0^0) + \sum_{j \in J} \tau_j(0,\sigma_0^0) J} \, U_{xx}(\tau) \, d\tau} = \lambda(J).
   \]

   The results so far state how the equilibrium investment profile depends on the competition of the trading outcome. However, to compare equilibrium investment profiles, I need to be more explicit on the investment decision of the buyer. The buyer decides to invest if the gains obtained from her investment, represented by the threshold

   \[
   \hat{K}(J) \equiv TS^*(1, \sigma^1(J)) - TS^*(0, \sigma^0(J)) - \sum_{i \in N} (T_i^1(J) - T_i^0(J)),
   \]

   are larger than the fixed cost of investment \( K \).

   The following proposition, proven in page 46 of the appendix, states how the investment threshold changes with respect to the level of competition in the trading game.

**Proposition 4.** The change on the investment threshold \( \hat{K}(J) \) with respect to the level of competition in the trading game depends on the degree of the allocative sensitivity:

1) when the allocative sensitivity is small, the investment threshold of the buyer is monotonically increasing with the level of competition in the trading game, i.e., \( \hat{K}(J) > \hat{K}(J') \) for \( J' \subset J \).
b) With a large enough allocative sensitivity, the investment threshold of the buyer is non monotone with the level of ex-post competition.

With a fixed investment of seller 1, the investment threshold of the buyer increases with the level of competition. Higher competition entails that a larger portion of the trading surplus goes to the buyer, who has more incentives to invest. However, in equilibrium, the investment threshold of the buyer also depends on the investment undertaken by seller 1, and due to investment complementarity, this is positively affected by lower levels of competition. A larger investment of seller 1 reduces the equilibrium transfers of the competing sellers, and this benefits the buyer. Whenever the allocative sensitivity is small, from proposition 3, the magnitude of $\gamma(J_1)$ is small and the investment of seller 1 is quite stable with respect to the level of competition in the trading game. As a result, the buyer is always better-off with larger levels of competition. Conversely, when the allocative sensitivity is important, seller’s 1 investment is very sensitive to the level of competition of the trading game and larger investments from seller 1 are obtained with outcomes that are less competitive. This investment effect counterbalances the previous competition effect, and the investment threshold of the buyer fails to be monotone with competition. Consequently, larger incentives to invest may arise in low competitive equilibria, i.e., $\hat{K}(J) < \hat{K}(J')$ for $J' \subset J$.

To give more clarity of the results, I graphically represent the equilibrium investment profile depending on the degree of competition of the trading outcome. Points further away from the origin of the horizontal axis represent a higher competitive equilibrium. On the upper part of figure 2, pictures a) and b) represent the equilibrium investment of seller 1, and this crucially depends on the level of ex-post competition and the degree of the allocative sensitivity. When the allocative sensitivity is small, picture a), the slope of the curve is much flatter than with a large allocative sensitivity, picture b). There are also discontinuous jumps on seller’s 1 investment decisions and those are due to the investment of the buyer. Thus, in picture c), the investment threshold of the buyer is monotone as stated in proposition 4, and, a less competitive equilibrium, which entails a less favorable portion of the trading surplus for the buyer, gives her less incentives to invest. The point where the fixed cost of investment, represented by the dashed red line, is above the investment threshold, the buyer switches his

\[\text{Figure 2: Equilibrium Investment Profile}\]

The figure is aimed at giving a simple illustration of the results and the lines represented do not stand for computed equilibrium.
Figure 2: Equilibrium investment profile of seller 1 and the buyer depending on the level of ex-post competition. The fixed investment cost of the buyer is represented by the red line in pictures c) and d). The left pictures represent a situation with a moderate allocative sensitivity, and the pictures on the right a situation with large allocative sensitivity.

investment decision from investment - in more competitive equilibria - to non investment - in less competitive equilibria. This generates the discreet jump downwards on the investment of the seller represented in picture a).

With a large allocative sensitivity, the investment threshold fails to be monotone, and this is because the constraint created to the transfers of the non-investing sellers, coming from seller’s 1 investment, dominates the more unfavorable portion of the trading surplus of a less competitive equilibria. Here, lower competition makes the buyer undertake a positive level of investment that does not come about with higher levels of competition. This is represented in picture d), where for low levels of competition the buyer decides to invest.

So far I have established how the competitiveness of the trading outcome affects the investment decisions of both sides of the market. It is left to study how the equilibrium investment profile is affected by the extensive degree of competition, in other words, the
number of active sellers in the industry.

4.3.4 Extensive competition

“Ceteris paribus”, the larger the number of sellers, the lower is the loss of exclusion of a given seller. The trading amount of the excluded seller can be easily substituted by trading more with the rest, and the higher the number of sellers, the easier it becomes. The outside option of the buyer after exclusion of a seller is bigger with an increase on the number of sellers, and this is equivalent to a reduction of the bargaining position of the sellers. Hence, the equilibrium transfer of each seller is a decreasing function of the number of active sellers, and this has an effect on the equilibrium investment profile. This result stated in the following proposition is proven in the appendix page 48.

**Proposition 5.** Regardless of the competition of the trading outcome, full efficiency is implemented when the number of sellers tends to infinity.

As the number of sellers increases, each seller appropriates less from the trading surplus, as his loss of exclusion is a decreasing function with the number of active sellers. When the number of sellers is infinite, each seller is only able to appropriate his marginal contribution of the trading surplus regardless of the equilibrium in the trading outcome. The buyer also takes the efficient investment decision with infinite sellers; in the limit she is able to appropriate the whole gains coming from her investment. Therefore, the buyer invests efficiently no matter her fixed costs of investment. The evolution of unilateral investment decisions with the number of sellers is illustrated in figure 3.

The picture illustrates that with one seller, there is a situation of a bilateral monopoly. Because the seller has the whole bargaining power, the buyer is completely “held-up” and she never invests. Conversely, the investment decision of the seller is efficient. With more than one seller, the investment threshold of the buyer is positive. This is because the sellers start to compete for the trading contracts and the buyer is able to appropriate part of the benefits coming from her investment. With low levels of competition, seller 1 over-invests, as he gets more than his marginal contribution to the surplus. However, since competition increases with a larger number of sellers, in the limit and with an infinite number of them, each seller obtains only his marginal contribution of the trading surplus; the buyer obtains
all the benefits coming from her investment.

With the link between investment and competition, it is only left to undertake equilibria comparison. I start by introducing the concept of Pareto optimality and I indicate which equilibrium is preferred by the sellers. Later, I depart from the analysis of surplus distribution, and I study which equilibrium performs best in terms of welfare.

5 Comparison of equilibria

5.1 Pareto optimality

I examine which equilibrium gives higher payoffs to the sellers. With a given investment profile, the analysis is simple, and this is because the sellers always prefer a situation where the equilibrium outcome is less competitive. When the trading surplus remains unchanged, and it does not depend on the competitiveness of the equilibrium outcome, all sellers prefer a situation where the distribution of the surplus is more favorable to them.\footnote{This is the case in Chiesa & Denicolo (2009, 2012). They state that the minimum rent equilibrium - the least competitive equilibrium - is Pareto dominant. This result comes from the fact that as long as parties do not invest the potential trading surplus stays the same. All the sellers are identical and an equilibrium of the trading game represents only a split of the surplus.} However, since in my model the equilibrium investment profile depends on the level of ex-post competition, low equilibrium outcomes may not always be preferred. For seller 1, the trade-off is whether

Figure 3: Unilateral investment decisions as a function of number of active sellers. On the left, the investment of seller 1 and on the right, the threshold below which the buyer decides to invest. The thick solid line stands for the efficient investment profile, the solid line represents a situation where competition is the most severe and the dashed line corresponds to the lowest level of competition.
a more favorable distribution of the trading surplus has an effect on the investing decision of the buyer. For the other sellers, there is, in addition, the investment decision of seller 1 and how this affects their equilibrium trading allocation. The result is presented in the following proposition.

**Proposition 6.** Whenever the allocative sensitivity is “small” so that $\hat{K}(J) > \hat{K}(J')$ for $J' \subset J$:

i) the least competitive equilibrium is Pareto dominant for the sellers if the investment decision of the buyer is equilibrium invariant,

ii) otherwise, Pareto dominance is attained with an intermediate level of competition.

Whenever the allocative sensitivity is “big” so that the investment threshold of the buyer is non monotone, the least competitive equilibrium is never Pareto dominant for the sellers,

iii) while it is always preferred for seller 1,

iv) the rest of the sellers are better-off with a larger competition of the trading outcome.

The formal proof is relegated to the appendix, page 49. In figure 4 and 5, I give a graphical interpretation of the results of proposition 6. Points in the horizontal axis farther from the origin stand for equilibrium outcomes that are more competitive. Figure 4 represents the situation where the allocative sensitivity is small, and figure 5 stands for a large allocative sensitivity.

![Figure 4](image_url)

**Figure 4:** Payoffs of the sellers as a function of the level of competition in the trading outcome when the allocative sensitivity is small. The black line represents the payoff of seller 1 and the dashed line stands for the non-investing sellers. The picture on the left stands for a situation where the equilibrium investment of the buyer is equilibrium invariant.

Because in equilibrium the investment of the buyer is affected by the investment of seller
1 and vice-versa, the buyer and seller 1 are always better-off the higher the investment in equilibrium. However, this is not always the case for the rest of the sellers where the investment of the buyer and seller 1 have opposite effects on their payoffs.

Seller 1 obtains a higher payoff in less competitive equilibria given that the buyer’s investment is equilibrium invariant. The rest of the sellers also prefer a less competitive equilibria provided that the seller’s 1 investment is relatively stable in all possible equilibria. An increase in the level of seller’s 1 investment generates a reduction in the equilibrium trading allocation for the rest of the sellers, reducing their equilibrium transfers. This negative effect dominates a more favorable portion of the surplus when the allocative sensitivity created by seller’s 1 investment is large.

Therefore, Pareto dominance of a less competitive equilibrium is not robust when the investment by both parts of the market is introduced. Under some situations, it may be beneficial to agree to an even distribution of the potential gains from trade among all parties rather than an asymmetric distribution; asymmetries may induce poor investment decisions by the unfavorable side of the market.

Figure 5: Payoffs of the sellers as a function of the level of competition in the trading outcome when the allocative sensitivity is large. The black line depicts the payoff of seller 1 and the dashed line stands for the non-investing sellers. The picture on the left stands for a situation where the equilibrium investment of the buyer is equilibrium invariant.
5.2 Welfare

This section departs from distributional issues and ranks equilibria according to welfare. Welfare is equal to the trading surplus minus the costs of investment,

\[ W^*(b, \sigma) = TS^*(b, \sigma) - K \times b - \psi(\sigma). \]

The analysis so far shows that ex-ante inefficiencies are more prompt to emerge whenever the trading outcome is less competitive, see proposition 2 and corollary 2. Hence, the most competitive equilibrium, in general, performs better in terms of welfare. However, investments decisions in the model are strategic complements. Therefore, under some parameters, I find that decreasing the level of ex-post competition may bring larger welfare whenever the allocative sensitivity created to the non-investing sellers is sufficiently big. Hence, the inefficiencies in investment, arising with seller 1, may work as a mechanism to restore the efficient investment of the buyer. As long as buyer’s investment has a big impact on the trading surplus, this is welfare enhancing. The following proposition states the result.

**Proposition 7.** When in the most competitive equilibrium, the investment decision of the buyer is not efficient and the allocative sensitivity is sufficiently big such that the buyer’s investment threshold is not monotone with the level of ex-post competition, welfare is maximized with an intermediate level of competition. Otherwise, the largest welfare is obtained with the highest level of competition.

The formal proof of the proposition is in the appendix, page 51. The graphical interpretation of the result is represented in figure 6, where a point in the horizontal axis that is further away from the origin stands for an equilibrium outcome that is more competitive. Welfare is monotonically increasing with the level of ex-post competition in situations where the investment decision of the buyer is equilibrium invariant. Welfare presents jumps whenever the investment decision of the buyer depends on the competitiveness of the equilibrium outcome. In this case, welfare is the largest with an intermediate level of competition only if the decision of the buyer switches from non-investing to investing when the equilibrium outcome is less competitive. In the situation when the buyer is taking the efficient investment decision in the most competitive equilibrium any reduction on the level of competition ex-post translates
Figure 6: Welfare as a function of the level of competition in the trading outcome. The figure on the right stands for the situation where the investment of the buyer, in the highest level of competition, is efficient; the figure on the left is when the contrary occurs. Jumps in the curves stand for a switch in the investing decision of the buyer and a higher slope of the curves represent larger levels of the allocative sensitivity.

6 Conclusion

In this paper, I show that introducing competition to the side of the market offering the trading contracts, may solve the “hold-up” problem without the introduction of ex-ante contracts. Organizing trade in a certain manner may circumvent situations were economic institutions do not allow for designing or enforcing ex-ante contracts. The “invisible” hand works in an efficient way to solve possible economic failures. Yet, the scope where market failures are eliminated is quite restricted. In the model, full ex-ante efficiency can only be achieved when the outcome of the trading game is the most competitive. In this case, all active sellers coordinate their out of equilibrium trading contracts, and the loss arising when a seller is excluded from trade is minimized. Here, each seller appropriates his marginal contribution to the trading surplus which gives the incentives to invest efficiently. In any other equilibrium of our trading game, full efficiency does not materialize. As a result, if the government should organize the procurement of a given input by using different suppliers, he must ensure that the competition among them is the largest.

The model also shows that, by introducing investment to both sides of the market, the
equilibrium played in the trading game is not only a way to redistribute rents between the sellers and the buyer, but it has also an effect on the size of the potential gains that can be attained. Previous analysis have stated that an equilibrium where the competition is maximal is not necessarily very attractive for the part of the market offering the trading contracts. This is because the offering part can “tacitly” coordinate to reduce competition in order to obtain a more favorable portion of the potential gains from trade. However, this paper demonstrates that an equilibrium with higher competition generally displays a more efficient investment profile resulting in larger welfare. However, if the efficient trading allocation is very sensitive to the investment of seller 1, lower levels of ex-post competition might perform better than higher competition. This is possible in the model because investment is undertaken by both sides of the market, and decisions to invest are strategic complements. With a large allocative sensitivity, seller 1 invests more and the loss of exclusion of the remaining sellers is smaller. Hence, the buyer decides to invest in a lower competitive equilibrium bringing about a larger surplus.

A question that deserves attention is what equilibrium is more likely to arise. This issue has already been addressed in the literature but there does not exist a clear answer. Yet, in the present model this is a question of great importance due to its affects on welfare. Despite the fact that there is no way to be sure of the equilibrium played in the trading game, an external player might induce some set of equilibria to be played. For instance, in markets that has recently been liberalized, to maximize welfare, a competition authority has to ensure that real competition exists in the market. In this model, inducing a particular type of equilibrium has to do with the number of sellers coordinating their out of equilibrium trading contracts. Hence, a third party might be able to induce an equilibrium or another by imposing restrictions on the number of contracts submitted, or on putting obstacles to coordination. Nevertheless, I am aware that more work about equilibrium refinement must be done; in the present work I have selected equilibria by imposing the assumption that a number of sellers coordinate their out of equilibrium trading contracts.

The results of the paper go trough if I relax some of the assumptions. In the model, I consider the case that only seller 1 knows the new technology, and can undertake invest-

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26Some of the works addressing equilibrium selection are Bernheim & Whinston (1986), Martimort & Stole (2009) and Klemperer & Meyer (1989).
ment to reduce his production cost. A natural extension is to consider the case where all sellers have knowledge of a technology to become more efficient. However, even if unilateral investment decisions are easy to obtain and coincide with the ones obtained in this paper, the characterization of the equilibrium investment profile seems daunting. This is so because the investment decisions of the buyer between the sellers are strategic complements, while the ones among sellers are strategic substitutes. However, I conjecture that the strategic substitutability among sellers’ investment might be of second order, and an increase in the investment of the buyer gives to all sellers higher incentives to invest.

Another extension is to consider a setting without a monopolistic buyer. In this case, non-exclusivity also comes from the fact that a seller can sign multiple trading contracts with different buyers. In such a case, a buyer differentiates and creates an indirect externality to the other buyers if she decides to invest. I believe that the equilibrium menus offered are complicated to obtain and I suspect that the competitive advantage that the buyer gets, with respect to the rest, may induce him to over-invest.

References


A Appendix

Lemma 3. The total gains from trade are larger with a higher investment of seller 1, that is, \( TS^*(b, \sigma) > TS^*(b, \sigma) \) for \( \sigma > \sigma \) and any \( b \).

Proof. I consider the case where \( b = 0 \) but the case where \( b = 1 \) is analogous.

\[
TS^*(0, \sigma) = U(X^*(0, \sigma)) - C(x^*_1(0, \sigma) \mid \sigma) - \sum_{i \neq 1} C_i(x^*_i(0, \sigma))
\]

\[
< U(X^*(0, \sigma)) - C(x^*_1(0, \sigma) \mid \sigma_i) - \sum_{i \neq 1} C_i(x^*_i(0, \sigma)) + U(X^*(0, \sigma_i)) - U(X^*(0, \sigma_i))
\]

\[
< U(X^*(0, \sigma)) - U(X^*(0, \sigma_i)) + TS^*(0, \sigma_i)
\]

\[
\implies TS^*(0, \sigma_i) - TS^*(0, \sigma) > U(X^*(0, \sigma_i)) - U(X^*(0, \sigma)) = \int_{X^*(0, \sigma)}^{X^*(0, \sigma_i)} U_x(\tau) d\tau > 0,
\]

where the first strict inequality comes from the lower cost of production due to larger investment and the second from efficiency. The strict inequality in the last line is due to lemma 1, where I showed that \( X^*(0, \sigma_i) > X^*(0, \sigma) \) for any \( \sigma_i > \sigma \).

Lemma 4. The total gains from trade are bigger when the buyer invests, that is, \( TS^*(1, \sigma^1) > TS^*(0, \sigma^0) \).

Proof. This only states that the potential gains from trade are larger with larger amounts of investment.

\[
TS^*(1, \sigma^1) = U(X^*(1, \sigma^1) \mid b = 1) - C(x^*_1(1, \sigma^1) \mid \sigma^1) - \sum_{i \neq 1} C_i(x^*_i(1, \sigma^1))
\]

\[
= U(X^*(1, \sigma^1) \mid b = 1) - U(X^*(1, \sigma^1)) + U(X^*(1, \sigma^1)) - C(x^*_1(1, \sigma^1) \mid \sigma^1) - \sum_{i \neq 1} C_i(x^*_i(1, \sigma^1))
\]

\[
\geq U(X^*(1, \sigma^1) \mid b = 1) - U(X^*(1, \sigma^1)) + TS^*(0, \sigma^0)
\]

\[
\implies TS^*(1, \sigma^1) - TS^*(0, \sigma^0) \geq U(X^*(1, \sigma^1) \mid b = 1) - U(X^*(1, \sigma^1)) > 0.
\]

The first inequality comes from lemma 3 and the last strict inequality comes by the assumption that \( U(X^* \mid b = 1) - U(X^*) > 0 \).

Lemma 5. The increase on the total gains from trade by any non-investing seller are higher when the buyer is investing and the level of competition in the trading game is the largest, i.e., \( \bar{J} \):

\[
TS^*(1, \sigma^1) - TS_{-i}(1, \sigma^1 \mid \bar{J}_1) \geq TS^*(0, \sigma^0) - TS_{-i}(0, \sigma^0 \mid \bar{J}_1) \text{ for } i \neq 1.
\]

Proof. Here I state that the contribution that a seller have on the trading surplus is larger if the buyer invests. I will make explicit use of lemma 4. Observe that the previous expression is equivalent to
Proof. I know that the lower bound of the expression on the left is \( \bar{D} = U(X^*(1, \sigma^1) \mid b=1) - U(X^*(1, \sigma^1)) \). I proceed by obtaining the upper bound of the difference \( \bar{T}S_{-i}(1, \sigma^1 \mid \bar{J}_1) - \bar{T}S_{-i}(0, \sigma^0 \mid \bar{J}_1) \).

\[
\bar{T}S_{-i}(1, \sigma^1 \mid \bar{J}_1) = U \left( \sum_{j \neq i} \tilde{x}_j(1, \sigma^1 \mid \bar{J}) \mid b = 1 \right) - C \left( \tilde{x}_1(1, \sigma^1 \mid \bar{J}) \mid \sigma^1 \right) - \sum_{j \neq i, 1} C_j \left( \tilde{x}_j(1, \sigma^1 \mid \bar{J}) \right)
\]

\[
\leq U \left( \sum_{j \neq i} \tilde{x}_j(1, \sigma^1 \mid \bar{J}) \mid b = 1 \right) - C \left( \tilde{x}_1(1, \sigma^0 \mid \bar{J}) \mid \sigma^0 \right) - \sum_{j \neq i, 1} C \left( \tilde{x}_j(1, \sigma^0 \mid \bar{J}) \right)
\]

\[
= U \left( \sum_{j \neq i} \tilde{x}_j(1, \sigma^1 \mid \bar{J}) \mid b = 1 \right) - U \left( \sum_{j \neq i} \tilde{x}_j(1, \sigma^1 \mid \bar{J}) \right) + U \left( \sum_{j \neq i} \tilde{x}_j(1, \sigma^1 \mid \bar{J}) \right)
\]

\[
- C \left( \tilde{x}_1(1, \sigma^0 \mid \bar{J}) \mid \sigma^0 \right) - \sum_{j \neq i, 1} C_j \left( \tilde{x}_j(0, \sigma^0 \mid \bar{J}) \right)
\]

\[
\leq U \left( \sum_{j \neq i} \tilde{x}_j(1, \sigma^1 \mid \bar{J}) \mid b = 1 \right) - U \left( \sum_{j \neq i} \tilde{x}_j(1, \sigma^1 \mid \bar{J}) \right) + \bar{T}S_{-i}(0, \sigma^0 \mid \bar{J}_1)
\]

\[
\Rightarrow \bar{T}S_{-i}(1, \sigma^1 \mid \bar{J}_1) - \bar{T}S_{-i}(0, \sigma^0 \mid \bar{J}_1) \leq U \left( \sum_{j \neq i} \tilde{x}_j(1, \sigma^1 \mid \bar{J}) \mid b = 1 \right) - U \left( \sum_{j \neq i} \tilde{x}_j(1, \sigma^1 \mid \bar{J}) \right) = \bar{D}.
\]

Where the inequalities comes from efficiency. I proceed to show that the difference between the lower and the upper bound is positive \( \bar{D} - \overline{\bar{D}} > 0 \) because

\[
\bar{D} - \overline{\bar{D}} = U(X^*(1, \sigma^1) \mid b=1) - U(X^*(1, \sigma^1)) - \left[ U \left( \sum_{j \neq i} \tilde{x}_j(1, \sigma^1 \mid \bar{J}) \mid b = 1 \right) - U \left( \sum_{j \neq i} \tilde{x}_j(1, \sigma^1 \mid \bar{J}) \right) \right]
\]

\[
= U(X^*(1, \sigma^1) \mid b=1) - U \left( \sum_{j \neq i} \tilde{x}_j(1, \sigma^1 \mid \bar{J}) \mid b = 1 \right) - \left[ U(X^*(1, \sigma^1)) - U \left( \sum_{j \neq i} \tilde{x}_j(1, \sigma^1 \mid \bar{J}) \right) \right]
\]

\[
= \int_{\sum_{j \neq i, \tilde{x}_j(1, \sigma^1 \mid \bar{J})}^X} \left( U_x(\tau \mid b=1) - U_x(\tau) \right) d\tau > 0,
\]

which is positive by lemma 2 and by the assumption that \( U_x(\tau \mid b=1) > U_x(\tau) \). \( \square \)

Lemma 6. The increase on welfare given by seller 1 is higher when the buyer is investing, i.e.,

\[
TS^*(1, \sigma^1) - \bar{T}S_{-i}(1 | \bar{J}_1) - \psi(\sigma^1) \geq TS^*(0, \sigma^0) - \bar{T}S_{-i}(0 | \bar{J}_1) - \psi(\sigma^0).
\]

Proof. I am going to proceed by contradiction. Take the contrary and assume that

\[
TS^*(1, \sigma^1) - \bar{T}S_{-i}(1 | \bar{J}_1) - \psi(\sigma^1) < TS^*(0, \sigma^0) - \bar{T}S_{-i}(0) - \psi(\sigma^0 | \bar{J}_1).
\]
This implies that the investing seller is worst-off when the buyer is investing and hence he has less incentives to invest. This would imply that $\sigma^1 < \sigma^0$, but this contradicts the fact that investments are strategic complements.

**Lemma 7.** For any $J_i \subset N \setminus \{i\}$ and $J_i$, and with a small allocative sensitivity, i.e. the level of investment by seller 1 is similar, we have that

$$T S_{-i}(1, \sigma_E | J_i) - T S_{-i}(1, \sigma_J | J_i) > T S_{-i}(0, \sigma_E | J_i) - T S_{-i}(0, \sigma_J | J_i).$$

**Proof.** By using the same procedure as in lemma 5 I obtain:

$$T S_{-i}(1, \sigma_E | J_i) - T S_{-i}(1, \sigma_J | J_i) > T S_{-i}(0, \sigma_E | J_i) - T S_{-i}(0, \sigma_J | J_i)$$

$$\geq U \left( \sum_{j \neq 1} \hat{x}_j(J) | 1 \right) - U \left( X_{(J,1)}^* + \sum_{j \in J} \hat{x}_j(J) | 1 \right) - U \left( \sum_{j \neq i} \hat{x}_j(J) \right) - \left( X_{(J,1)}^* + \sum_{j \in J} \hat{x}_j(J) \right)$$

$$= \int_{X_{(J,1)}^* + \sum_{j \in J} \hat{x}_j(J)} \left( U_x(\tau | b = 1) - U_x(\tau) \right) d\tau > 0,$$

and this is positive by lemma 2 and by assumption $U_x(X | b = 1) > U_x(X)$. □

**Lemma 8.** When the buyer invests in $J'$ but not in $J$ and $J' \subset J$, the non-investing seller is always better in a more competitive equilibrium. For any $J' \subset J$ it has to be that

$$T S^*(0, \sigma_J^0) - T S_{-i}(0, \sigma_J^0 | J) > T S^*(1, \sigma_J^1) - T S_{-i}(1, \sigma_J^1 | J'), \quad \forall i \neq 1.$$

**Proof.** I proceed by contradiction, consider the contrary

$$T S^*(0, \sigma_J^0) - T S_{-i}(0, \sigma_J^0 | J) < T S^*(1, \sigma_J^1) - T S_{-i}(1, \sigma_J^1 | J'),$$

but then it has to be the case than the investment threshold for $J'$ is lower than when $J$, i.e., $\hat{K}(J') < \hat{K}(J)$. However, this implies that if the buyer decides to invest in $J'$ she also has to invest in $J$, and I reach a contradiction. □

**Lemma 9.** For a given investment profile $(b, \sigma)$ the amount that each seller trades with the buyer decreases with the number of active sellers, but the aggregate level of trade is higher.

$$x_i^*(N + 1) < x_i^*(N) \quad \forall i \in N \quad \text{and} \quad X^*(N + 1) > X^*(N).$$

**Proof.** The results comes directly from the concavity of the utility function and the convexity of the
cost function. In order to ease notation, I do not consider investment. For an number of \( N + 1 \) active sellers, the amount traded in equilibrium needs to satisfy

\[
U_x \left( \sum_{i=1}^{N+1} x_i^*(N+1) \right) = C_x \left( x_i^*(N+1) \right).
\]

I prove the claim by contradiction, assume that \( x_i^*(N+1) \geq x_i^*(N) \) \( \forall i \in N \), and since \( N + 1 > N \), I have that \( \sum_{i=1}^{N+1} x_i^*(N+1) > \sum_{i=1}^{N} x_i^*(N) \) and by the concavity of the utility function \( U(\cdot) \) and optimality, it has to be the case that

\[
C_x \left( x_i^*(N+1) \right) = U_x \left( \sum_{i=1}^{N+1} x_i^*(N+1) \right) < U_x \left( \sum_{i=1}^{N} x_i^*(N) \right) = C_x \left( x_i^*(N) \right) \ \forall i \in N,
\]

but the convexity of \( C_x(\cdot) \) implies that \( x^*(N+1) < x^*(N) \), which leads to a contradiction. The previous also implies that \( X^*(N+1) > X^*(N) \).

**Lemma 10.** The function \( V_J \left( X^*_{\{J,i\}} \right) \) is well defined, strictly increasing and strictly concave in \( X^*_{\{J,i\}} \). The maximizer \( \tilde{x}_j \left( X^*_{\{J,i\}} \right) \) for \( j \in J_i \) is decreasing in \( X^*_{\{J,i\}} \).

**Proof.** This is the general case of Chiesa & Denicolò (2009) for any set \( J_i \). That the function \( V \left( X^*_{\{J,i\}} \right) \) is well defined follows from the Inada conditions. By the envelop theorem I obtain \( V_x \left( X^*_{\{J,i\}} \right) > 0 \) and \( V_{xx} \left( X^*_{\{J,i\}} \right) < 0 \), which implies that the function is strictly increasing and strictly concave. By the implicit function theorem, I find that:

\[
\frac{\partial \tilde{x}_j \left( X^*_{\{J,i\}} \right)}{\partial X^*_{\{J,i\}}} = \frac{U_{xx}(\cdot)}{C_x(\cdot) - U_{xx}(\cdot)} < 0.
\]

**B Appendix**

**Results of Chiesa & Denicolò (2009)** Following the notation of Chiesa & Denicolò (2009) a Nash equilibrium in the trading game is a list of strategies \( \langle \tilde{M}(M), \tilde{M}_1, \tilde{M}_2, ..., \tilde{M}_N \rangle \) such that:

\[
\mathcal{M}(M) \in \arg\max_{\mathcal{M} \in \times_{i=1}^N M_i} \Pi(\mathcal{M}) \ \forall \ M \in \Gamma^N
\]

and

\[
\tilde{M}_i \in \arg\max_{\tilde{M}_i \in \Gamma} \pi_i(\tilde{M}(\tilde{M}_{-i}, M_i)) \ \forall \ i \in N
\]
where \((\hat{M}_{-i}, M_i) \equiv (\hat{M}_1, ..., \hat{M}_{i-1}, M_i, \hat{M}_{i+1}, ..., \hat{M}_N)\).

The following result and propositions can be found in Chiesa & Denicolò (2009).

**Result CD (2009):** The maximum equilibrium transfer of seller \(i\) when there is only a single competing seller submitting an out of equilibrium trading contract aiming to exclude him from trade is
\[ T_i^e = V_{\nu_i} \left( X^*_{-\{\nu_i\}} \right) - V_{\nu_i} \left( X^*_{-\{\nu_i, i\}} \right). \]

The strategies to get this payoff in my model is the same as in Chiesa & Denicolò (2009), and I only need to substitute the seller \(\nu_i\) by a set of sellers \(J_i\). The out of equilibrium offers are designed in the same way as in Chiesa & Denicolò (2009) such that the group of sellers belonging to the set \(J_i\) are just indifferent between offering their out of equilibrium trading contract and their equilibrium contract. I need to assume that there exist no cost in forming a coalition of sellers who coordinate their out of equilibrium contracts. Observe that the trading quantities that are offered in those out of equilibrium trading contracts are given in equation (4.3) in the main text.

What matters to construct these equilibria is the cardinality of the set \(J_i\). As in Chiesa & Denicolò (2009) I assume that the most efficient seller submits out of equilibrium contract. Hence, in any equilibria where a number of sellers \(J\) coordinate their out of equilibrium trading contracts to exclude a seller \(i \neq 1\), seller 1 will form a coalition with a number of sellers \(J - 1\). Because all other sellers are equal, the identity of the remaining sellers does not matter. To exclude seller 1 any number of sellers \(J\) coordinating their out-of-equilibrium offers will do.

**Proposition 1 CD (2009):** A vector of payoffs \((\Pi, \pi_1, \pi_2, ..., \pi_N)\) is a vector of equilibrium payoffs if and only if it satisfies \(\Pi + \pi_1 + \pi_2 + ... + \pi_N = TS^*\) and \(0 < \pi_i \leq \bar{\pi}_i \forall i \in N\).

Indeed selecting equilibria by allowing a group of sellers to form a coalition to coordinate their out of equilibrium contracts, I obtain a subset of equilibrium payoffs that belong to the range stated in the proposition. In my model, the sellers’ equilibrium payoffs belong to the range \(\pi_i^{TE} \leq \pi_i \leq \bar{\pi}_i\) and satisfy \(\Pi + \pi_1 + \pi_2 + ... + \pi_N = TS^*(b, \sigma)\). The lowest possible payoff equals the “truthful” equilibrium, and this occurs when all the sellers form a coalition to coordinate their out of equilibrium contracts to exclude any other seller from trade. In this equilibrium, every seller obtains his marginal contribution to the surplus.

**Proposition 3 CD (2009):** An equilibrium outcome is supported by a menu of trading contracts with minimum cardinality \(|\hat{M}_i| = 2\) for \(N - 2\) sellers and \(|\hat{M}_i| = 3\) for the remaining 2 sellers.
Equilibrium supply schedules must contain some contracts that will never be accepted. An equilibrium then cannot be supported by take-it-or-leave-it offers (TIOLI). This is because the revelation principle does not hold true in a common agency game. Therefore, Chiesa & Denicolò (2009) provide some insights into the structure of the equilibrium supply schedules or menus of trading contracts. Individual excludability means that the buyer must earn her equilibrium payoffs if she excludes any seller $i$. But if she excluded seller $i$ while continuing to accept the efficient contracts of the remaining of the sellers she would earn less than her equilibrium rent. This implies that there must be an additional contract of any other seller $j \neq i$ that the buyer would accept in place of seller $j$’s efficient contract if the buyer refused to trade with seller $i$. Hence, in Chiesa & Denicolò (2009), the threat of exclusion is undertaken by one of the sellers. In their model it is assumed that seller 1 is more efficient than the rest and he will offer an out of equilibrium contract aimed at replacing any other seller. There is another seller, that will also offer an out of equilibrium contract to threaten the exclusion of seller 1. Hence, when submitting the equilibrium transfers any seller $i \neq 1$ fears to be excluded by seller 1 and seller 1 fears to be excluded by any other seller, say seller 2.

In my model, the threat of exclusion is undertaken by a coalition of sellers. Consider for instance an equilibrium where two sellers enter into a coalition to submit out of equilibrium contracts to replace any seller. Then, the threat of exclusion of any seller $i \neq 1, 2$ is undertaken by the coalition of the out of equilibrium offers of seller 1, the most efficient, and any other seller, say seller 2. The quantities in their out of equilibrium contracts must be given by equation (4.3). As in Chiesa & Denicolò (2009) I assume that there is another seller, say seller 3, offering an out of equilibrium contract in my model making a coalition with seller 1 in order to replace seller 2, and a coalition with seller 2 in order to replace 1. Then any seller $i \neq 1, 2$ when submitting the equilibrium trading contract fears to be excluded by the coalition of seller 1 and 2. Seller 1 fears to be excluded by the coalition of 2 and 3, and seller 2 fears to be excluded by the coalition of 1 and 3. Notice that the the identity of the sellers forming the coalition to exclude any seller $i \neq 1, 2$ coincide, and they are different for the exclusion of seller 1 and 2. What matters is the cardinality of the set, in this case the cardinality is 2. The same procedure applies for a set with cardinality 3 till $N - 1$ sellers. Hence, the minimum cardinality of trading contracts when a number of sellers $J$ form a coalition is $\lvert \hat{M}_i \rvert = 3$ for $(J + 1)$ sellers and $\lvert \hat{M}_i \rvert = 2$ for $(N - (J + 1))$ sellers.

Proof of lemma 1: I start by showing how the investment of seller 1 affects the equilibrium allocation. I consider the case where the buyer decides not to invest, i.e., $b = 0$ but the proof is analogous for $b = 1$. Differentiating the first-order conditions given in (4.1) for $x_j^*$ and $j \neq i$ with
respect to $\sigma$ I obtain

$$U_{xx}(X^*) \times \sum_{h=1}^{N} \frac{dx_h^*}{d\sigma} = C_{xx}(x_j^*) \times \frac{dx_j^*}{d\sigma}. \quad (B.1)$$

Because the left hand side is independent of $j$ I find that all $dx_j^*/d\sigma$ have the same sign. Now suppose also that $dx_1^*/d\sigma$ has that same sign. Then also the sum has that same sign and since $U_{xx}(\cdot) < 0$ and $C_{xx}(\cdot) > 0$ this leads to a contradiction. Now suppose $dx_1^*/d\sigma < 0$. The other signs therefore have to be positive. By (B.1) I find that $\sum_{h=1}^{N} dx_h^*/d\sigma < 0$. But the first-order condition for $x_1^*$, differentiated with respect to $\sigma$ is

$$U_{xx}(X^*) \times \sum_{h=1}^{N} \frac{dx_h^*}{d\sigma} = C_{xx}(x_1^* | \sigma) \times \frac{dx_1^*}{d\sigma} + C_{x\sigma}(x_1^* | \sigma), \quad (B.2)$$

which would then have a positive left hand side and a negative right hand side due to $C_{x\sigma}(\cdot) < 0$ - a contradiction.

Thus I have shown the first and the second part of point i) of the lemma. Again by (B.1) the last claim follows from $\partial X^*/\partial \sigma = \sum_{h=1}^{N} dx_h^*/d\sigma$ and the level of the allocative sensitivity is implicitly characterized in expression (B.1). I proceed by analyzing the effect that the investment of the buyer has on the equilibrium allocation. Again, I am going to make use of the conditions for the equilibrium allocation represented in equation (4.1), and for a fixed investment of seller 1 I obtain

$$C_x(x_1^* | \sigma) = U_x(X^* | b = 1) > U_x(X^*) = C_x(x_1^*), \quad \text{for } 1,$$

$$C_x(x_j^*) = U_x(X^* | b = 1) > U_x(X^*) = C_x(x_j^*), \quad \text{for } j \neq 1.$$

The strict inequality is by assumption and by the convexity of the cost function I obtain the result.

**Proof of lemma 2:** I have to show that $X^*(b, \sigma) > X_{-\{J_i\},i}^*(b, \sigma) + \sum_{j \in J_i} \hat{x}_j(b, \sigma | J_i)$. I am going to consider the case where there buyer is not investing, i.e., $b = 0$ but the proof is analogous for the case when the buyer invests $b = 1$. Also, consider any set of sellers $J_i \subset N$. With the same investment profile, I know that $\sum_{h \neq J_i} x_h^* = X_{-\{J_i\},i}^*$, and the expression above is equivalent to $\sum_{j \in J_i} x_j^* + x_i^* > \sum_{j \in J_i} \hat{x}_j(J_i)$. Therefore and from the Inada conditions I have that $x_i^* > 0$ if $\sum_{j \in J_i} (x_j^* - \hat{x}_j(J_i)) > 0$ I am done. Observe that for a given investment profile, if the above is true, it has to be true for any $j \in J_i$, hence $x_j^* > \hat{x}_j(J_i)$. If the contrary occurs, $x_j^* < \hat{x}_j(J_i)$, then from the equilibrium allocation I have

$$U_x \left( X_{-\{J_i\},i}^* + \sum_{j \in J_i} \hat{x}_j(J_i) \right) = C_x(\hat{x}_j(J_i)) > C_x(x_j^*) = U_x(X^*),$$

40
and by concavity of $U(\cdot)$ I prove the claim. The previous also implies that for any $j \in J_1$ I have $\tilde{x}_j(J_i) > x^*_j$. Using the same procedure I can easily prove that for any $J_i' \subseteq J_i$ I obtain

$$X^*_{\{J_i,1\}} + \sum_{j \in J_i} \tilde{x}_j(J_i) \geq X^*_{\{J_i',1\}} + \sum_{j \in J_i'} \tilde{x}_j(J_i'),$$

and by using the same argument as before, I get $\tilde{x}_j(J_i') \geq \tilde{x}_j(J_i)$.

**Proof of proposition 1:** The equilibrium transfer of seller 1 depends on the number of sellers belonging to the set $J_1$ and for a given investment profile this is equal to

$$T_1(J_1 | b, \sigma) = U(X^* | b) - \max_{(x_j) \in J_1} \left[ U \left( X^*_{\{J_1,1\}} + \sum_{j \in J_1} x_j | \tilde{x}_1 = 0, b \right) - \sum_{j \in J_1} C_j(x_j) \right] + \sum_{j \in J_1} C_j(x_j^*).$$

Operating further I obtain

$$T_1(J_1 | b, \sigma) = U(X^* | b) - \sum_{j \in J_1} C_j(x_j^*) - \left[ U \left( X^*_{\{J_1,1\}} + \sum_{j \in J_1} \tilde{x}_j(J_1) | \tilde{x}_1 = 0, b \right) - \sum_{j \in J} C_j(\tilde{x}_j(J_1)) \right]
+ \left[ \sum_{j \in J_1, 1} \left( C_j(x_j^*) - C_j(x_{j'}^*) \right) \right] + [C(x_1^* | \sigma) - C(x_1^* | \sigma)]
= TS^*(b, \sigma) - \left[ U \left( X^*_{\{J_1,1\}} + \sum_{j \in J_1} \tilde{x}_j(J_1) | \tilde{x}_1 = 0, b \right) - \sum_{j \in J_1} C_j(\tilde{x}_j(J_1)) - \sum_{j \notin J_1, 1} C_j(x_j^*) \right]
+ C(x_1^* | \sigma)
= TS^*(b, \sigma) - TS_{-1}(b | J_1) + C(x_1^* | \sigma).

Introducing this equilibrium transfer to the payoffs of seller 1 in expression (3.2) I get the equilibrium payoffs stated in the proposition. The payoff of the buyer is

$$\Pi(b, \sigma | J) = U(X^* | b) - \sum_i T^*_i(J_i | b, \sigma) - K \times b
= U(X^* | b) - \left[ \sum_i TS^*(b, \sigma) - TS_{-1}(b | J_i) + C(x_i^* | \cdot) \right] - K \times b
= U(X^* | b) - \sum_i C_i(x_i^* | \cdot) - \left[ \sum_i TS^*(b, \sigma) - TS_{-1}(b | J_i) \right] - K \times b
= TS^*(b, \sigma) - \sum_i \left( TS^*(b, \sigma) - TS_{-1}(b, \sigma | J_i) \right) - K \times b.$$
I proceed to show point (ii), i.e., \( T_j - \pi_1(j) > T_j - \pi_i(j) \) for \( j_i \subset j \). I take \( b = 0 \) and the payoffs of seller 1

\[
\tilde{T}_j - \pi_1(j) = U \left( X^* - x(j), \sum_j x_j(J) \right) - \sum_j C_j(x_j(J)) - \sum_j C_j(x_j) \\
= U \left( X^* - x(j), \sum_j x_j(J) \right) - \sum_j C_j(x_j(J)) - \sum_j C_j(x_j) + \tilde{T}_j - \pi_1(j)
\]

\[
\Rightarrow \tilde{T}_j - \pi_1(j) - \tilde{T}_j - \pi_1(j) \geq U \left( \sum_j x_j(J) \right) - U \left( X^* - x(j), \sum_j x_j(J) \right)
\]

The first inequality comes from efficiency and the last strict inequality comes from lemma 2. This result can be applied to any seller \( i \in N \). Because each seller obtains his marginal contribution whenever \( J_i = N \setminus \{i\} \), then it immediate to see that for any other \( J_i \subset J_i \) any seller gets more than his marginal contribution to the trading surplus.

**Proof of proposition 2:** To show existence of efficiency in the equilibrium investment profile, I pay attention to seller’s 1 investment. I later show that there always exists a region of the fixed cost of investment of the buyer where she always takes the efficient investment.

I first show the “if” part of the proposition. From proposition 1, I obtain that the payoff of seller 1 in the most competitive equilibrium is equal to

\[
\pi_1(b, \sigma | \tilde{J}) = T_j - \pi_1(j) - \psi(\tau).
\]

The term \( T_j - \pi_1(j) \) does not depend on the amount invested \( \sigma \). Therefore using \( T_j - \pi_1(j) \) given in the main text, and by the envelope-theorem, the first-order condition for the seller 1 is given by

\[
\psi(\sigma) = -C_{x_j}(x_j, \sigma), \quad \forall b;
\]

which is the same expression obtained in (4.9). Because seller 1 receives the marginal contribution of the trading surplus, he becomes the residual claimant and invests efficiently.
To show the “only if” part, I take any $J_1 \subset N \setminus \{1\}$. Now the equilibrium payoff of seller 1 is

$$\pi_1(b, \sigma \mid J_1) = TS^*(b, \sigma) - T\bar{S}_{-1}(b, \sigma \mid J_1),$$

and calculating the first order condition and applying the envelope theorem I obtain that the equilibrium investment profile is

$$\psi_\sigma(\sigma) = -C_\sigma(x^*_1(b, \sigma^b) \mid \sigma^b) - \frac{\partial (T\bar{S}_{-1}(b, \sigma \mid J_1))}{\partial \sigma},$$

where the extra term depends on the investment of the seller 1 from the allocation that remains unchanged $X^*_{-(J_1, 1)}(b, \sigma)$. As a result, $\frac{\partial (T\bar{S}_{-1}(b, \sigma \mid J_1))}{\partial \sigma} \neq 0$ and this creates a distortion of the investment of the seller. Hence, the efficient investment profile is only implementable when the trading outcome is the most competitive.

**Proof of corollary 1:** The investment decision of seller 1 is as in proposition 2 and the investment of the buyer is given by:

$$K = \begin{cases} 
TS^*(1, \sigma^1_E) - TS^*(0, \sigma^0_E) - \kappa(\bar{J}) = \hat{K}(\bar{J}) & \text{then } b = 1 \\
\bar{K}(\bar{J}) & \text{then } b = 0,
\end{cases}$$

where the term $\kappa(\bar{J})$ is the difference in the payoff of the sellers when the buyer decides to invest and it is equal to

$$\kappa(\bar{J}) \equiv \pi_1(1, \sigma^1_E \mid \bar{J}_1) - \pi_1(0, \sigma^0_E \mid \bar{J}_1) + \sum_{i \neq 1} \left[ \pi_i(1, \sigma^1_E \mid \bar{J}_i) - \pi_i(0, \sigma^0_E \mid \bar{J}_i) \right]$$

$$= TS^*(1, \sigma^1) - T\bar{S}_{-1}(1 \mid \bar{J}_1) - TS^*(0, \sigma^0) + T\bar{S}_{-1}(0 \mid \bar{J}_1)$$

$$+ \sum_{i \neq 1} \left[ TS^*(1, \sigma^1) - T\bar{S}_{-i}(1 \mid \bar{J}_i) - TS^*(0, \sigma^0) + T\bar{S}_{-i}(0 \mid \bar{J}_i) \right].$$

The magnitude $\kappa(\bar{J})$ represents how much the sellers benefit from the investment of the buyer and those are the gains that cannot be appropriated by the latter. By making an explicit use of the lemmas in appendix A I show that the appropriation of the gains by the sellers is bigger than the cost of investment $\kappa(\bar{J}) > \psi(\sigma^1_E) - \psi(\sigma^0_E)$. I split $\kappa(\bar{J})$ into two parts

$$A = \sum_{i \neq 1} \left[ TS^*(1, \sigma^1) - T\bar{S}_{-i}(1 \mid \bar{J}_i) - TS^*(0, \sigma^0) + T\bar{S}_{-i}(0 \mid \bar{J}_i) \right]$$

43
and
\[ B = TS^*(1, \sigma^1) - T\bar{S}_{-1}(1 | J_1) - TS^*(0, \sigma^0) + T\bar{S}_{-1}(0 | J_1). \]

In lemma 5, I show that \( A > 0 \) and in lemma 6 I show that \( B > \psi(\sigma_E^1) - \psi(\sigma_E^0). \)

Hence, the investment threshold below which the buyer invests is lower compared to efficiency \( \hat{K}(\bar{J}) < \hat{K}_E \). Thus, because the buyer cannot appropriate all the gains coming from her investment, she underinvests whenever the fix cost of investment lies between \( K \in (\hat{K}(\bar{J}), \hat{K}_E) \). Finally, since investments are strategic complements implies that seller 1 also underinvests in equilibrium.

**Proof of proposition 3:** From proposition 2 I know that the seller’s 1 investment fails to be efficient whenever \( J_1 \subset N \setminus \{1\} \). Here, I show that there exist over-investment and I characterize the magnitude. Without loss of generality I consider that the investment of the buyer to be \( b = 0 \). I take the first order condition of the equilibrium payoffs from seller 1 with respect to investment, and by applying the envelope condition I obtain
\[
\psi_\sigma(\sigma) = -C_x(x_1^*(b, \sigma) | \sigma) - \sum_{m \neq J_1, 1} \left( U_x \left( X^*_{-1} + \sum_{j \in J_1} \tilde{x}_j(J_1 | b) - C_x(x_j^*) \right) \right) \times \frac{dx_m^*}{d\sigma},
\]

where the transformation in the second line is due to the fact that, at the equilibrium allocation, marginal benefit equals marginal cost, i.e. \( U_x(x^*) = C_x(x^*_j), \forall j \in N \). Comparing this condition with efficiency (4.9), I see that the difference is the additional term is
\[
\gamma(J_1) \equiv - \sum_{m \neq J_1, 1} \left( U_x \left( X^*_{-1} + \sum_{j \in J_1} \tilde{x}_j(J_1 | b) - U_x(x^*) \right) \right) \times \frac{dx_m^*}{d\sigma},
\]

and by applying the fundamental theorem of calculus I obtain
\[
\gamma(J_1) \equiv - \sum_{m \neq J_1, 1} \left( \int_{X^*}^{X^*_{-1}} U_{xx}(\tau)d\tau \right) \times \frac{dx_m^*}{d\sigma} > 0,
\]

and the whole expression is positive. By lemma 2 and the concavity of the utility function \( U(\cdot) \), I know that the part in brackets is positive. By lemma 1 I know that the amount traded with the sellers that are not investing is decreasing with the amount invested by seller 1. Therefore, this term is strictly positive which means that the seller 1 over-invests and its magnitude depends on the allocative sensitivity that the investment of seller 1 creates to the rest of the sellers.
To show that the degree of over-investment decreases with the number of sellers in $J_1$, I use a continuous approximation and I show that $\partial \gamma(J_1)/\partial J_1 < 0$. Hence, by applying the Leibniz rule of differentiation to the previous expression I obtain

$$
\frac{\partial \gamma(J_1)}{\partial J_1} = \left( \int_{X^*} \frac{X^* - \sum_{j \in J_1} \tilde{x}_j(J_1)}{U_x(t) \sigma} \right) \times \frac{dx_m}{\sigma} 
- U_{xx} \left( \frac{X^* - \sum_{j \in J_1} \tilde{x}_j(J_1)}{\sigma} \right) \times \frac{\partial \gamma(J_1)}{\partial J_1} \left( \frac{X^* - \sum_{j \in J_1} \tilde{x}_j(J_1)}{\sigma} \right) \times \frac{dx_m}{\sigma} < 0,
$$

(B.4)

and the sign is due to lemma 2.

**Proof of corollary 2:** The first point is shown in proposition 3. To show point A) I take the investment decision of the buyer, where for any $J \subset N \setminus \{i\}$ this is:

$$
K \begin{cases} 
\leq TS^*(1, \sigma_1^i) - TS^*(0, \sigma_0^i) - \kappa(J) \equiv \tilde{K}(J) & \text{then } b = 1 \\
\geq \tilde{K}(J) & \text{then } b = 0,
\end{cases}
$$

(B.5)

where the extra term $\kappa(J)$ is the difference in the payoff of the sellers when the buyer invests. Again this represents how much the sellers benefit from the investment of the buyer and those benefits can not be appropriate by the latter.

$$
\kappa(J) \equiv \pi_1 \left( 1, \sigma_1^i \mid J_1 \right) - \pi_1 \left( 0, \sigma_0^i \mid J_1 \right) + \sum_{i \neq 1} \left[ \pi_i \left( 1, \sigma_1^i \mid J_1 \right) - \pi_i \left( 0, \sigma_0^i \mid J_i \right) \right] 
= TS^*(1, \sigma^1) - \tilde{T}S_{-1}(1, \sigma^1 \mid J_1) - TS^*(0, \sigma^0) + \tilde{T}S_{-1}(0, \sigma^0 \mid J_1) 
+ \sum_{i \neq 1} \left[ TS^*(1, \sigma^1) - \tilde{T}S_{-1}(1, \sigma^1 \mid J_i) - TS^*(0, \sigma^0) + \tilde{T}S_{-1}(0, \sigma^0 \mid J_i) \right].
$$

The buyer underinvests, under some fixed cost of investment, if the threshold of investment is below efficiency, that is,

$$
\tilde{K}(J) \leq \tilde{K}_E \iff TS^*(1, \sigma_1^i) - TS^*(0, \sigma_0^i) - \kappa(J) \leq TS^*(1, \sigma_1^E) - TS^*(0, \sigma_0^E) - \left( \psi \left( \sigma_1^E \right) - \psi \left( \sigma_0^E \right) \right) 
\iff \psi \left( \sigma_1^E \right) - \psi \left( \sigma_0^E \right) \leq TS^*(1, \sigma_1^E) - \tilde{T}S_{-1}(1, \sigma_1^E \mid J_1) - \left( TS^*(0, \sigma_0^E) - \tilde{T}S_{-1}(0, \sigma_0^E \mid J_1) \right) 
+ \sum_{i \neq 1} \left[ TS^*(1, \sigma_1^E) - \tilde{T}S_{-1}(1, \sigma_1^E \mid J_i) - \left( TS^*(0, \sigma_0^E) - \tilde{T}S_{-1}(0, \sigma_0^E \mid J_i) \right) \right].
$$

By using the same procedure as in lemma 5 I can show that the last part in brackets is positive.
Therefore, I only need to verify that
\[
TS^*(1, \sigma_E^1) - \tilde{TS}_{-1}(1, \sigma_J^1 | J_1) - \left( TS^*(0, \sigma_E^0) - \tilde{TS}_{-1}(0, \sigma_J^0 | J_1) \right) \geq \psi (\sigma_E^1) - \psi (\sigma_E^0).
\]

Here I apply lemma 7 that states \(\tilde{TS}_{-1}(1, \sigma_J | J) > \tilde{TS}_{-1}(1 | \tilde{J}_1) - \tilde{TS}_{-1}(0 | \tilde{J}_1) + \tilde{TS}_{-1}(0, \sigma_J^0 | J_1)\), and by introducing this the the previous expression I have that
\[
TS^*(1, \sigma_E^1) - \tilde{TS}_{-1}(1, \sigma_J^1 | J_1) - \left( TS^*(0, \sigma_E^0) - \tilde{TS}_{-1}(0, \sigma_J^0 | J_1) \right) > TS^*(1, \sigma_E^1)
- \left[ \tilde{TS}_{-1}(1 | \tilde{J}_1) - \tilde{TS}_{-1}(0 | \tilde{J}_1) + \tilde{TS}_{-1}(0, \sigma_J^0 | J_1) \right] - \left( TS^*(0, \sigma_E^0) - \tilde{TS}_{-1}(0, \sigma_J^0 | J_1) \right)
= TS^*(1, \sigma_E^1) - \tilde{TS}_{-1}(1 | \tilde{J}_1) - \left( TS^*(0, \sigma_E^0) - \tilde{TS}_{-1}(0 | \tilde{J}_1) \right) > \psi (\sigma_E^1) - \psi (\sigma_E^0),
\]
where the last inequality comes by lemma 6. Therefore, the investing threshold in equilibrium is lower than the efficiency level regardless of the investment decision of the buyer.

To show point B) I need to compare the right hand side of the expression determining the investment of the seller 1 in equilibrium (B.3) evaluated at \(b = 0\), with the right hand side of expression determining the efficient investment (4.9) evaluated at \(b = 1\).

\[
Rhs^J(b = 0) = -C_\sigma (x^*_1(0, \sigma_J^0) | \sigma) - \sum_{m \neq J_1, 1} \left( \int_{X^*}^{X^*_1(J_1, 1)} + \sum_{j \in J_1} \bar{\delta}_j(J_1) \right) UXx(\tau)d\tau \frac{dx^*_m}{d\sigma},
\]

\[
Rhs^E(b = 1) = -C_\sigma (x^*_1(1, \sigma_E^1) | \sigma),
\]
and I will have that the efficient investment is higher if
\[
Rhs^E(b = 1) > Rhs^J(b = 0)
\]
\[
\implies -C_\sigma (x^*_1(1, \sigma^1) | \sigma) > -C_\sigma (x^*_1(0, \sigma^0) | \sigma) - \sum_{m \neq J_1, 1} \left( \int_{X^*}^{X^*_1(J_1, 1)} + \sum_{j \in J_1} \bar{\delta}_j(J_1) \right) UXx(\tau)d\tau \frac{dx^*_m}{d\sigma}
\]
\[
\implies \int_{x^*_1(1, \sigma^1)} C_{xx}(\tau)d\tau > - \sum_{m \neq J_1, 1} \left( \int_{X^*}^{X^*_1(J_1, 1)} + \sum_{j \in J_1} \bar{\delta}_j(J_1) \right) UXx(\tau)d\tau \frac{dx^*_m}{d\sigma}
\]
\[
\implies - \frac{dx^*_m}{d\sigma} > \frac{\int_{x^*_1(0, \sigma_E^0)} C_{xx}(\tau)d\tau}{(N \setminus \{1\} - J_1) \times \int_{X^*(0, \sigma_J^0)}^{X^*_1(J_1, m)} + \sum_{j \in J_1} \bar{\delta}_j(0, \sigma_J^0 | J_1) UXx(\tau)d\tau} = \lambda(J_1).
\]
otherwise, the contrary occurs. Therefore, if the allocative sensitivity is large, seller 1 invests more than the efficiency level regardless of the investment decision of the buyer.
**Proof of proposition 4:** For a given subset of sellers in $J$, the investment threshold of the buyer is given by

$$
\hat{K}(J) = TS^*(1, \sigma^1(J)) - TS^*(0, \sigma^0(J)) - \sum_{i \in N} (T_i^1(J) - T_i^0(J)).
$$

I redefine this threshold as

$$\hat{K}(J) = \mathcal{N}(J) - \mathcal{I}(J),$$

where $\mathcal{N}(J) = TS^*(1, \sigma^1(J)) - TS^*(0, \sigma^0(J))$ and $\mathcal{I}(J) = \sum_{i \in N} (T_i^1(J) - T_i^0(J))$.

When the allocative sensitivity is very small, I have shown in proposition 3 that the distortion of investment is little and for any $J' \subset J$ I have that $\gamma(J) \approx 0$. Hence, for any $J' \subset J$, I obtain that $\sigma_{J'} \approx \sigma_J$. Then it is immediate to get that $\hat{K}(J') < \hat{K}(J)$ because $\mathcal{N}(J') \approx \mathcal{N}(J)$ and $\mathcal{I}(J') > \mathcal{I}(J)$, which comes directly from lemma 7, but considering the investment of seller 1 fixed, in appendix A. Therefore, the higher the level of competition in the trading game, the more incentives for the buyer to invest. With a similar investment of seller 1, the trading surplus remains constant over the level of competition ex-post and the buyer is better-off when she can appropriate a larger proportion of those gains.

When, the allocative sensitivity is significant, I obtain that the seller’s 1 investment changes with competition and for $J' \subset J$ I obtain that $\sigma_{J'} > \sigma_J$. Then, I also obtain that $\mathcal{N}(J') > \mathcal{N}(J)$ because

$$
\frac{\partial \mathcal{N}(J')}{\partial \sigma} \approx \frac{\partial (TS^*(1, \sigma^1) - TS^*(0, \sigma^0))}{\partial \sigma} \times (\gamma(J') - \gamma(J)) = \int_{x_1^*(1, \sigma^1)}^{x_1^*(0, \sigma^0)} C_{x\sigma}(\tau) d\tau \times [\gamma(J') - \gamma(J)] > 0.
$$

Moreover, the part $\mathcal{I}(J)$ is now affected by the change of investment and I obtain that $\partial \mathcal{I}(J)/\partial \sigma < 0$ because

$$
\frac{\partial \mathcal{I}(J')}{\partial \sigma} \approx \sum_i \left( \int_{x_i^*(1, \sigma^1)}^{\tilde{\xi}_i(1, \sigma^1|J')} C_{x\sigma}(\tau) d\tau - \int_{x_i^*(0, \sigma^0)}^{\tilde{\xi}_i(0, \sigma^0|J')} C_{x\sigma}(\tau) d\tau \right) \times [\gamma(J') - \gamma(J)]
$$

$$
= \sum_i \left( \int_{x_i^*(1, \sigma^1|J') + \tilde{\xi}_i(0, \sigma^0|J')}^{x_i^*(0, \sigma^0|J') + \tilde{\xi}_i(1, \sigma^1|J')} C_{x\sigma}(\tau) d\tau \right) \times [\gamma(J') - \gamma(J)] < 0
$$

Hence, I obtain that

$$
\hat{K}(J') - \hat{K}(J) = -\left( \mathcal{I}(J') - \mathcal{I}(J) \right)
$$

$$
+ \left[ \int_{x_1^*(1, \sigma^1)}^{x_1^*(0, \sigma^0)} C_{x\sigma}(\tau) d\tau + \sum_i \left( \int_{\tilde{\xi}_i(1, \sigma^1|J') + \tilde{\xi}_i(0, \sigma^0|J')}^{\tilde{\xi}_i(0, \sigma^0|J') + x_i^*(0, \sigma^0)} C_{x\sigma}(\tau) d\tau \right) \right] \times [\gamma(J') - \gamma(J)]
$$

Thus, with a big enough allocative sensitivity $[\gamma(J') - \gamma(J)]$ is big enough such that $\hat{K}(J') > \hat{K}(J)$.
Proof of proposition 5: I proceed by construction and I study a situation where the number of active sellers tends to infinity. I consider first the investment decision of seller 1 and later I study the investment threshold of the buyer. Regarding the investment of seller 1, it is without loss of generality to take the case where the outcome in the trading game is the least competitive, that is, whenever the set of sellers in \( J_1 \) is a singleton \(|J_1| = 1 = J_{1,1}\).

I have shown in proposition 3 that the investment distortion of the seller in the least competitive equilibrium is given by

\[
\gamma(N \mid J_{1,1}) \equiv - \sum_{m \neq J_{1,1}} \left( \int_{X^*(N)}^{X^*_1(N)} \left( \frac{\partial^2 U}{\partial x \partial x} \right) \left( \tau \right) d\tau \right) \times \frac{dx_m}{d\sigma}.
\]

(B.6)

The magnitude of this object depends on the number of active sellers \( N \) as represented by the bounds of the integral, and the difference is equal to \( x^*_1(N) + x^*_1(N) - \tilde{x}_{J_{1,1}}(N \mid J_{1,1}) > 0 \). I now show that this difference tends to zero when the number of active sellers is arbitrarily large and hence the investment distortion is also zero.

By lemma 9 in appendix A, I have shown that as the number of sellers increase, the per seller amount of trade decreases. In the limit, from the concavity of the utility function, together with the Inada condition, I obtain that for any investment \( b \) the individual trading amount tends to zero, that is, \( \lim_{N \to \infty} x^*_1(N) \approx 0 \). With the equilibrium trading allocation given in expression (4.1) I get

\[
\lim_{N \to \infty} \left[ X^*(N) \right] \approx \infty \Rightarrow \lim_{N \to \infty} \left[ U_x(X^* \mid b) \right] \approx 0 \Rightarrow \lim_{N \to \infty} \left[ \frac{\partial U}{\partial x} \right] \approx 0
\]

With regard to how the amount \( \tilde{x}_{J_{1,1}}(N \mid J_{1,1}) \) evolves with the number of sellers, I know that this object is the solution of the value function \( V_{J_{1,1}}(X^*_{\cdot(J_{1,1})}) \), whose properties are stated in lemma 10 in appendix A. From lemma 9, the amount \( X^*_{\cdot(J_{1,1})}(N) \) increases, and the change on \( \tilde{x}_{J_{1,1}}(N \mid J_{1,1}) \) equals

\[
\frac{d\tilde{x}_{J_{1}}(N \mid J_{1,1})}{dX^*_{\cdot(J_{1,1})}} = \frac{U_{xx}(\cdot)}{C_{xx}(\cdot)} - \frac{U_{xx}(\cdot)}{U_{xx}(\cdot)}.
\]

By totally differentiating the first order condition of the equilibrium trading allocation, the change on the efficient amount is also decreasing and equals

\[
\frac{dx^*_{J_{1}}(N)}{dX^*_{\cdot(J_{1,1})}(N)} = \frac{U_{xx}(\cdot)}{C_{xx}(\cdot)} - \frac{U_{xx}(\cdot)}{U_{xx}(\cdot)}.
\]

But the decrease on the first amount dominates

\[
\frac{U_{xx}(\cdot)}{C_{xx}(\cdot)} - \frac{U_{xx}(\cdot)}{U_{xx}(\cdot)} < \frac{U_{xx}(\cdot) - U_{xx}(\cdot)}{C_{xx}(\cdot)} \Rightarrow C_{xx}(\cdot) < C_{xx}(\cdot) - U_{xx}(\cdot) \Rightarrow U_{xx}(\cdot) < 0.
\]
and whenever the number of sellers tend to infinity I obtain that $\lim_{N \to \infty} \left[ \tilde{x}_{J_1}(N \mid J_1) \approx x_{J_1}(N) \right]$. Therefore, when the number of sellers tend to infinity the difference between the upper and the lower integrand of (B.6) tends to zero, and the distortion of investment is also zero. This happens for any set of sellers belonging to $J_1 \subset N$.

I now show that the investment thresholds of the buyer converges to efficiency when the number of active sellers tend to infinity. For any $J \subset N$ the investment threshold of the buyer is

$$\hat{K}(J) \equiv TS^*(1, \sigma^1) - TS^*(0, \sigma^0) - \kappa(J).$$

Because seller’s 1 investment tends to efficiency with an infinite number of sellers, the first part of the investment threshold tends to efficiency as represented in (4.9)

$$\lim_{N \to \infty} \left[ TS^*(1, \sigma^1) - TS^*(0, \sigma^0) \right] \approx TS^* (1, \sigma^1_E) - TS^* (0, \sigma^0_E).$$

Moreover, the appropriation of the gains from trade, coming from an investment of the buyer, by the non investing sellers tends to zero, because

$$TS^* (1, \sigma^1_E) - TS^* (0, \sigma^0_E) \approx \lim_{N \to \infty} \left[ T\tilde{S}_{-i} (1, \sigma^1_E \mid J_i) - T\tilde{S}_{-i} (0, \sigma^0_E \mid J_i) \right]$$

$$\implies TS^* (1, \sigma^1_E) - T\tilde{S}_{-i} (1, \sigma^1_E \mid J_i) - \left( TS^* (0, \sigma^0_E) - T\tilde{S}_{-i} (0, \sigma^0_E \mid J_i) \right) \approx 0$$

and this result comes from the fact that $\lim_{N \to \infty} [x^*_i (N) \approx 0]$. By the same argument, the appropriation of the trading surplus by seller 1 has to be very close to the difference of investment costs $\psi (\sigma^1_E) - \psi (\sigma^0_E)$. Hence, I finally obtain that the investment threshold of the buyer also tends to efficiency $\lim_{N \to \infty} \left[ \hat{K}(J) \right] \approx \hat{K}_E$, for any $J \subset N$.

**Proof of proposition 6:** I begin by considering the case where the allocative sensitivity is small. In this case, I have established in proposition 4 that the investment threshold of the buyer is monotonically increasing with competition. The lower portion of the surplus appropriated by the buyer with lower competition dominates the higher investment of seller 1, i.e., $\hat{K}(J') < \hat{K}(J)$ for any $J' \subset J$. I first consider point i) when the investment of the buyer is equilibrium invariant. Here, to show that seller 1 is better-off with lower levels of competition I only need to verify that his investment increases with lower levels of competition and this is the case since I know that $\gamma(J') > \gamma(J)$, for any $J' \subset J$. 

49
For the non-investing sellers, it is easy to see that for any $J' \subset J$, I obtain

$$TS^*(b, \sigma^b_J) - \tilde{T}S_{-i}(b, \sigma^b_J, | J') > TS^*(b, \sigma^b_{J'}) - \tilde{T}S_{-i}(b, \sigma^b_J | J)$$

$$\implies \tilde{T}S_{-i}(b, \sigma^b_J | J) - \tilde{T}S_{-i}(b, \sigma^b_J, | J') > TS^*(b, \sigma^b_{J'}) - TS^*(b, \sigma^b_J) \approx 0$$

$$\implies \tilde{T}S_{-i}(b, \sigma^b_J | J) - \tilde{T}S_{-i}(b, \sigma^b_J | J') > 0.$$

The right hand side of the second line is close to zero because with a small allocative sensitivity, the investment of the seller is similar regardless to the equilibrium ex-post $\sigma^b_J \approx \sigma^b_{J'}$. The third line is positive by point ii) in proposition 1.

When the investment of the buyer depends on the equilibrium played ex-post and with a small allocative sensitivity, from proposition 4 I know that the investment threshold monotonically increases with competition, and hence there must exist a $J' \subset N$ where the buyer invests whenever $J' \subset J$ and does not invest otherwise. Due to complementarity of investment and proposition 3, I know also that $\sigma^b_{J'} > \sigma^b_J$. Furthermore, because the allocative sensitivity is small, I obtain that the variation $(\gamma(J) - \gamma(J'))$ is small so $\sigma_{J'} > \sigma^b_{J'}$ for any $J' \subset J$. This implies that the largest payoff of seller 1 is achieved with an intermediate level of competition.

With regard to the non-investing sellers, I get the same result. Hence, for any $J' \subset J$ I obtain $TS^*(1, \sigma^b_{J'}) - \tilde{T}S_{-i}(1, \sigma^b_J, | J') > TS^*(1, \sigma^b_J) - \tilde{T}S_{-i}(1, \sigma^b_J | J)$. Thus, I only need to show that for $J \subset J'$ I get

$$TS^*(1, \sigma^b_J) - \tilde{T}S_{-i}(1, \sigma^b_J, | J') > TS^*(0, \sigma^b_J) - \tilde{T}S_{-i}(0, \sigma^b_J | J).$$

(B.7)

By lemma 5 and proposition 1 I know that

$$TS^*(1, \sigma^b_J) - \tilde{T}S_{-i}(1, \sigma^b_J | J) > TS^*(0, \sigma^b_J) - \tilde{T}S_{-i}(0, \sigma^b_J | J) \quad \forall J \subset N,$n

$$TS^*(0, \sigma^b_J) - \tilde{T}S_{-i}(0, \sigma^b_J | J) > TS^*(0, \sigma^b_J) - \tilde{T}S_{-i}(0, \sigma^b_J, | J').$$

By summing up both expressions and letting $J = J'$ in the first inequality then

$$TS^*(1, \sigma^b_J) - \tilde{T}S_{-i}(1, \sigma^b_J, | J') > -TS^*(0, \sigma^b_J) + \tilde{T}S_{-i}(0, \sigma^b_J | J)$$

$$+ 2 \left[ TS^*(0, \sigma^b_J) - \tilde{T}S_{-i}(0, \sigma^b_J, | J') \right],$$

(B.8)

and by summing up expression (B.7) and (B.8) I obtain

$$2 \left[ TS^*(1, \sigma^b_J) - \tilde{T}S_{-i}(1, \sigma^b_J, | J') \right] > 2 \left[ TS^*(0, \sigma^b_J) - \tilde{T}S_{-i}(0, \sigma^b_J, | J') \right]$$

where the last inequality holds by lemma 5.

When the allocative sensitivity is big, by proposition 4 I know that there exist a $J' \subset J$ such
that $\hat{K}(J') > \hat{K}(J)$. Point iii) is easy to obtain. Because of investment complementarity, seller 1 always obtains larger payoffs the less competitive the equilibrium outcome is, he is not only able to appropriate a larger partition of the trading surplus, but the investment of the buyer goes in his favor. Contrarily, the non-investing sellers obtain the largest payoffs when the outcome in the trading game is the most competitive. When, there exist a $J' \subset J$, where the buyer decides to invest in any $J'' \subset J'$, I obtain

$$TS^*(0, \sigma^0_J) - \tilde{T}S_{-i}(0, \sigma^0_J | J) > TS^*(1, \sigma^1_J) - \tilde{T}S_{-i}(1, \sigma^1_J | J''),$$

which comes from lemma 8 in appendix A. Moreover, when the investment of the buyer is equilibrium invariant, as long as the allocative sensitivity is big enough I will also have that

$$TS^*(b, \sigma^b_J) - \tilde{T}S_{-i}(b, \sigma^b_J | J) > TS^*(b, \sigma^b_{J'}) - \tilde{T}S_{-i}(b, \sigma^b_{J'} | J'); \quad \text{for any } J' \subset J.$$

And the non-investing sellers are better with a lower partition of the surplus as this also implies a smaller equilibrium investment of seller 1.

**Proof of proposition 7:** I first consider the case when the allocative sensitivity is small such that $\hat{K}(J) > \hat{K}(J')$ for any $J' \subset J$. This entails that the investment decision of the buyer is only efficient in an equilibrium with a set of sellers in $J'$ if it is also in $J$. Then, because the investment decision of seller 1 is inefficient in any $J' \subset J$ as stated in proposition 3, I obtain that the highest level of welfare is obtained when competition is the most severe, that is, when $J_i = N \setminus \{i\}$.

With a big allocative sensitivity, I know that the investment threshold of the buyer is not monotonically increasing with the level of competition ex-post, and there exist a $J' \subset J$ such that $\hat{K}(J') > \hat{K}(J)$. In this case, I show that welfare is maximized for $J'$.

For the case when $J \subset J'$, it is immediate and the argument is the same as in the first paragraph. For any $J \supset J'$, I only need to show that $W^J(\cdot) > W^{J'}(\cdot)$. Then, I define the difference in welfare

$$D(\cdot) = W^{J'}(1, \sigma^1_J) - W^J(0, \sigma^0_E) = TS^*(1, \sigma^1_J) - \hat{K}(J') - \psi(\sigma^1_J) - TS^*(0, \sigma^0_E) + \psi(\sigma^0_E).$$

Because I only want to know if there exists a situation where a less competitive equilibrium does better in terms of welfare, I take the lowest possible value of the fixed investment costs, $K = \hat{K}(J) = TS^*(1, \sigma^1_E) - TS^*(0, \sigma^0_E) - \kappa(\bar{J})$. By introducing this in the previous expression, I obtain that the lower
bound of the difference is given by

\[ D(\cdot) = TS^*(1, \sigma^1_{1J}) - TS^*(1, \sigma^1_{1E}) + TS^*(0, \sigma^0_{1J}) + \kappa(\bar{J}) - \psi(\sigma^1_{1J}) - TS^*(0, \sigma^0_{1E}) + \psi(\sigma^0_{1E}) \]

\[ = TS^*(1, \sigma^1_{1J}) - TS^*(1, \sigma^1_{1E}) + \kappa(\bar{J}) - (\psi(\sigma^1_{1J}) - \psi(\sigma^0_{1E})) \]

\[ > TS^*(1, \sigma^1_{1J}) - TS^*(1, \sigma^1_{1E}) + \psi(\sigma^1_{1J}) - \psi(\sigma^0_{1E}) - (\psi(\sigma^1_{1J}) - \psi(\sigma^0_{1E})) \]

\[ = TS^*(1, \sigma^1_{1J}) - TS^*(1, \sigma^1_{1E}) - (\psi(\sigma^1_{1J}) - \psi(\sigma^1_{1E})) \),

where the first inequality comes from the proof of corollary 1 in the appendix. Hence, I obtain that the difference is positive whenever the increase in the trading surplus coming from a higher investment of seller 1 is bigger than the investment cost, i.e. \( TS^*(1, \sigma^1_{1J}) - TS^*(1, \sigma^1_{1E}) > \psi(\sigma^1_{1J}) - \psi(\sigma^1_{1E}) \). Therefore, I additionally require that the effect of the investment of seller 1 in the trading surplus is sufficiently big.