Hunting the Unobservables for Optimal Social Security: A General Equilibrium Approach

Frank N. Caliendo and Emin Gahramanov
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Abstract

We study the optimal size of a pay-as-you-go social security program for an economy composed of both permanent-income and hand-to-mouth consumers. While previous work on this topic is framed within a two-period partial equilibrium setup, we study this issue in a life-cycle general equilibrium model. Because this type of welfare analysis depends critically on unobservable preference parameters, we methodically consider all parameterizations of the unobservables that are both feasible and reasonable—all parameterizations that can mimic key features of macro data (feasible) while still being consistent with micro evidence and convention (reasonable). The model predicts that the optimal tax rate is between 6 percent and 15 percent of wage income.

JEL Classification: E62, E21, H55, D50

Key Words: Optimal Social Security, Unobservable Preference Parameters, General Equilibrium Calibration, Permanent Income, Hand to Mouth.

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1 Introduction

Social security is the biggest government program in the U.S. and it can serve several functions: (i) redistribution of income from the elderly wealthy to the elderly poor, (ii) insurance against dying sooner than expected or becoming disabled, (iii) insurance against living longer than expected (outliving one’s savings), and finally, (iv) it may help people save for retirement. We will follow a literature which focuses on the last function and we will study the optimal provision of social security when some households are myopic (e.g., Feldstein 1985 and Docquier 2002).

In an economy with a mix of permanent-income and hand-to-mouth consumers, the optimal social security tax rate is strictly positive if period utility satisfies the usual conditions. A utilitarian social planner would never favor a zero tax rate since this would leave infinite marginal utility during the retirement years of hand-to-mouth consumers. Feldstein (1985) and Docquier (2002) study the optimal tax rate for such a mixed economy using a two-period partial equilibrium model with logarithmic preferences. However, the optimal tax rate depends critically on unobservable preference parameters, and social security also distorts capital accumulation and hence factor prices.¹ We study a general equilibrium life-cycle model with continuous-time overlapping generations in the spirit of Bullard and Feigenbaum (2007). Our model is populated with hand-to-mouth and permanent-income consumers, and we calibrate the unobservable parameters (the discount rate, the elasticity of intertemporal substitution, and the share of hand-to-mouth consumers in the economy) to some of the salient features of the U.S. data.

We use the following terminology to characterize the results of our calibration experiment. The feasible parameter space contains all combinations of the unobservable parameters.

¹Feldstein (1985) appears to be the first to rigorously study the optimal level of social security benefits for a mixed economy of permanent-income and hand-to-mouth consumers, and Docquier (2002) shows how some of the results are sensitive to the definition of social welfare. Neither author focuses on the general equilibrium calibration issues that are at the center of our paper. İmrohoroğlu et al. (2003) provide an authoritative general equilibrium discussion of the role of social security in improving the welfare of time-inconsistent hyperbolic agents, but this is also not the focus of our paper.
parameters that can replicate key targets in the aggregate U.S. data. The *reasonable* parameter space contains parameter values that are plausible and consistent with other outside evidence from microeconomic experiments and with convention in the macroeconomics literature.

Our goal is to compute optimal tax rates for the entire feasible-reasonable intersection. This process leads to the prediction that the optimal tax rate is between 6 percent and 15 percent of wage income. The current rate of 10.6 falls at the midpoint of this range.

2 Bullard-Feigenbaum Economy with Hand-to-Mouth Consumers

Our general equilibrium model is a variant of Bullard and Feigenbaum’s (2007) macroeconomic model. Bullard and Feigenbaum explore the role of leisure in the period utility function as a solution to the consumption hump puzzle, given that all consumers have perfect foresight. Alternatively, the present setup focuses on the welfare effects of social security with an additional assumption that some people do not have perfect foresight, while abstracting from the labor-leisure choice. We also include population and productivity growth since the internal rate of return in a pay-as-you-go program is critically dependent on growth (Feldstein 1985), while Bullard and Feigenbaum assume a stationary population.

Calendar time is continuous and is indexed by \( t \). All people enter the workforce at birth and work for \( T \) years and pass away after \( \bar{T} \) years. At each instant a new cohort is born and an old cohort dies. The size of each successive cohort grows at rate \( n \) (and hence the total population grows at rate \( n \)), so that the size of a cohort born at time \( \tau \) is \( N(\tau) = N(t)e^{n(\tau-t)} \), where \( N(t) \) is the size of the cohort born at time \( t \). The number of workers, therefore, at any time \( t \) is \( \int_{t-\bar{T}}^{t} N(t)e^{n(\tau-t)}d\tau \), and the number of retirees is \( \int_{t-\bar{T}}^{t-T} N(t)e^{n(\tau-t)}d\tau \). The ratio of workers to retirees, \( R \), is time-independent: \( R = \frac{(1 - e^{-nT})}{(e^{-nT} - e^{-n\bar{T}})} \).
All individuals from cohort $\tau$ are born and start work at time $\tau$, retire at $\tau + T$, and then exit the model at $\tau + T$. The workers take the economy-wide wage $w(t)$ as given, and $w(t)$ grows at a constant rate $x$ according to the stable competitive equilibrium that will be explained later. Every worker alive at time $t$ earns $w(t)$, implying that younger cohorts have higher levels of lifetime wealth than older cohorts. The agent’s financial asset account, $k(t, \tau)$, grows at the endogenously-determined risk-free real rate of return, $r$. Although factor prices are determined inside the model, the agent takes them as given. Following Bullard and Feigenbaum (2007), agents purchase claims on capital by lending directly to firms, and they also enter into debt contracts with agents from other cohorts; the rate of return is the same in both markets. Agents can also sell their claims on the capital of firms to agents from other cohorts. For example, a retiree who wishes to consume more than his income (from claims on capital plus social security benefits), can liquidate his claims and consume the proceeds. Likewise, a young agent who wants to spend more than he earns can borrow from middle-age savers. Hence, aggregate household saving (total demand for capital less demand for borrowed funds) flows to firms and is converted to capital, and firms then compensate households according to the productivity of that capital, less the amount needed to cover depreciation. Because firms use part of their revenues to repair/replace depreciated capital, the market value of the agent’s claims on capital always equals the value of the initial loan to the firm.

Because we do not have wage heterogeneity or mortality risk in the model, we abstract from the redistributive role and the insurance role of social security. Instead, our intent is to follow the literature outlined above and focus on the role of social security in helping to provide retirement income to those who do not save on their own.

The social security tax rate is $\theta$ and workers bear the full burden since labor is supplied inelastically (Feldstein 1985). Pay-as-you-go social security benefits per retiree at time $t$ are $b(t) = \theta w(t)R$. Note that $w(t)$ and $b(t)$ depend only on calendar time, and not on the birth date of the agent. The environment is free from risk and inflation.
Following Feldstein (1985), we divide consumers into two types. Let $\lambda$ be the share that live hand-to-mouth, and $1 - \lambda$ is the share that follow the permanent-income rule. For the former type, consumption during the working period is $c^h(t, \tau) = (1 - \theta)w(t)$ and consumption during retirement is $c^h(t, \tau) = b(t)$. The “h” superscript stands for “hand-to-mouth.”

A given permanent-income consumer solves the following control problem

$$\max \int_{\tau}^{\tau+T} e^{-\rho(t-\tau)}u(c(t, \tau))dt$$

subject to:

$$\frac{dk(t, \tau)}{dt} = rk(t, \tau) + (1 - \theta)w(t) - c(t, \tau) \text{ for } t \in [\tau, \tau + T]$$

(2)

$$\frac{dk(t, \tau)}{dt} = rk(t, \tau) + b(t) - c(t, \tau) \text{ for } t \in [\tau + T, \tau + \bar{T}] $$

(3)

$$k(\tau, \tau) = 0$$

(4)

$$k(\tau + \bar{T}, \tau) = 0$$

(5)

$$b(t) = \theta w(t)R$$

(6)

$$R = (1 - e^{-n\tau})/(e^{-nT} - e^{-n\bar{T}})$$

(7)

The period utility function is of the CRRA variety: $u(c(t, \tau)) = c(t, \tau)^{1-\sigma}/(1-\sigma)$, and the discount rate is $\rho$.

The Maximum Principle gives

$$c^p(t, \tau) = \Omega w(\tau)e^{\rho(t-\tau)}$$

(8)
where \( g \equiv \frac{r-p}{\sigma} \) and \( \Omega \) is invariant to birth and calendar dates

\[
\Omega \equiv \left\{ \frac{(1 - \theta) [e^{(x-r)T} - 1] + \theta R \left[ e^{(x-r)T} - e^{(x-r)\tau} \right]}{x - r} \right\} \frac{e^{x\tau} (g - r)}{e^{g\tau} - e^{r\tau}}
\]

and the “p” superscript stands for “permanent income.”

It follows that

\[
k(t, \tau) = \frac{(1 - \theta) w(\tau)}{x - r} \left[ e^{x(t-\tau)} - e^{r(t-\tau)} \right] + \frac{\Omega w(\tau)}{g - r} \left[ e^{r(t-\tau)} - e^{g(t-\tau)} \right]
\]

for \( t \in [\tau, \tau + T] \), and

\[
k(t, \tau) = \frac{\theta w(\tau) R}{x - r} \left[ e^{x(t-\tau)} - e^{r(\tau + t - T)} \right] - \frac{\Omega w(\tau)}{g - r} \left[ e^{g(t-\tau)} - e^{g(\tau + t - T)} \right]
\]

for \( t \in [\tau + T, \tau + T] \).

We close the model by endogenizing factor prices and we limit our attention to steady-state equilibria. We assume constant returns with competitive factor pricing. Total income is

\[ Y(t) = K(t)^\alpha [A(t)L(t)]^{1-\alpha} \]

where \( K(t) = (1 - \lambda) \int_{t-T}^{t} N(t)e^{\lambda(t-t)}k(t, \tau)d\tau \) is the total stock of capital at time \( t \), \( A(t) \) is the stock of labor-augmenting technology (where \( \dot{A}(t) = xA(t) \)), and \( L(t) = \int_{t-T}^{t} N(t)e^{\lambda(t-t)}d\tau \) is the total number of workers (where \( \dot{L}(t) = nL(t) \)). Capital depreciates at rate \( \delta \).

**Definition (Stable Competitive Equilibrium).** A stable competitive equilibrium is characterized by:

(i) \( r(t) = \partial Y(t)/\partial K(t) - \delta = \alpha Y(t)/K(t) - \delta, \)

(ii) \( w(t) = \partial Y(t)/\partial L(t) = (1 - \alpha)Y(t)/L(t), \)

(iii) \( Y(t) = [r(t) + \delta]K(t) + w(t)L(t), \)

(iv) \( \dot{r}(t) = 0 \) and \( \dot{w}(t) = xw(t), \)

(v) The savers follow a permanent-income rule \( c^p(t, \tau) = \Omega w(\tau)e^{g(t-\tau)}. \)
We will briefly illustrate the existence of such an equilibrium.

**Proposition (Existence).** This economy has a stable competitive equilibrium.

**Proof.** Parts (i)-(iii) combine the assumption of competitive factor markets with Euler’s theorem and are therefore self evident. Note that if \( Y(t) = N(t) = L(t) \) grows at rate \( x \), then \( \dot{r}(t) = 0 \) and \( \dot{w}(t) = xw(t) \). Thus, to prove existence it is sufficient to show that \( \dot{Y}(t)/Y(t) = \dot{K}(t)/K(t) = x + n \) when \( \dot{r}(t) = 0 \), \( \dot{w}(t) = xw(t) \), and a fraction \( 1 - \lambda \) of consumers obey \( e^p(t, \tau) = \Omega w(\tau)e^{g(t-\tau)} \). Let’s begin by constructing aggregate capital using (9) and (10) (which impose the assumptions \( \dot{r}(t) = 0 \), \( \dot{w}(t) = xw(t) \), and \( e^p(t, \tau) = \Omega w(\tau)e^{g(t-\tau)} \))

\[
K(t) = (1 - \lambda) \int_{t-T}^{t} N(t)e^{n(\tau-t)} \left\{ \frac{(1 - \theta)w(\tau)}{x - r} \left[ e^{x(t-\tau)} - e^{r(t-\tau)} \right] + \frac{\Omega w(\tau)}{g - r} \left[ e^{r(t-\tau)} - e^{g(t-\tau)} \right] \right\} d\tau \\
+ (1 - \lambda) \int_{t-T}^{t} N(t)e^{n(\tau-t)} \left\{ \frac{w(\tau)R}{x - r} \left[ e^{x(t-\tau)} - e^{xT + r(t-\tau) - T} \right] - \frac{\Omega w(\tau)}{g - r} \left[ e^{g(t-\tau)} - e^{gT + r(t-\tau) - T} \right] \right\} d\tau \\
= z_1(t)q_1 + z_1(t)q_2 + z_2(t)q_3 + z_2(t)q_4 + z_3(t)q_5 + z_3(t)q_6 + z_2(t)q_7 + z_2(t)q_8
\]

where the components of this polynomial are defined by

\[
\begin{align*}
z_1(t) & \equiv \frac{N(t)(1 - \theta)(1 - \lambda)w(t)}{x - r} \\
q_1 & \equiv \frac{1 - e^{-nT}}{n} \\
q_4 & \equiv \frac{1 - e^{(g - x - n)T}}{g - x - n} \\
q_7 & \equiv \frac{e^{-x(\tau + T)(x - n)T}}{x + n - r} \\
q_2 & \equiv \frac{N(t)(1 - \lambda)w(t)\Omega}{g - r} \\
q_5 & \equiv \frac{e^{-nT - n\tau}}{n} \\
q_8 & \equiv \frac{e^{(g - x - n)T - e^{(g - x - n)T}}}{g - x - n} \\
z_3(t) & \equiv \frac{N(t)\theta(1 - \lambda)w(t)R}{x - r} \\
q_3 & \equiv \frac{1 - e^{(r - x - n)T}}{x + n - r} \\
q_6 & \equiv \frac{e^{-(x + n)T + T + (T - \tau)e^{(r - x - n)T}}}{r - x - n}
\end{align*}
\]
Note that \( \dot{z}_1(t) = (x + n)z_1(t), \dot{z}_2(t) = (x + n)z_2(t), \dot{z}_3(t) = (x + n)z_3(t), \) hence it must be the case that \( \dot{K}(t)/K(t) = x + n. \) Also note that \( Y(t) \) can be expressed in growth terms: \( \dot{Y}(t)/Y(t) = \alpha \dot{K}(t)/K(t) + (1 - \alpha)(x + n) = x + n. \) This proves existence. Q.E.D.

3 Calibration of the Model and Computation of the Optimal Tax Rate

Our first goal is to calibrate the model to some of the salient features (or targets) from U.S. macroeconomic data, using available evidence to select values for observable parameters and leaving free the unobservable preference parameters in order to match the targets. After calibrating the model, we will compute the optimal social security tax rate in general equilibrium.

Individuals work for 40 years and live for another 15 years after retirement before passing away and exiting the model (i.e., \( T = 40 \) and \( \bar{T} = 55 \)), corresponding to an agent who starts work at 25, retires at 65, and dies at 80 (which is the average date of death in the U.S. over the past few decades, see Bullard and Feigenbaum 2007).

Because the individual supplies labor inelastically, he bears the full burden of the social security tax, so we will set \( \theta = 10.6\% \) to reflect the full OASI rate in the U.S. at present. We will normalize demographics and technology by \( N(0) = 1 \) and \( A(0) = 1. \) Bullard and Feigenbaum (2007) and Feigenbaum (2007) use a value of 1.56 percent for real wage growth in their macrocalibrations, and we will do likewise (\( x = 1.56\% \)). The rate of population growth is set to \( n = 1\% \) to reflect recent trends in the U.S. This value also conveniently produces a worker-to-retiree ratio of 3.5—close to the average value in the U.S. over the period 1980-2005. Following convention we set \( \alpha = 35\%; \) and, loosely following Bullard and Feigenbaum (2007) and Feigenbaum (2007), we use \( \delta = 8\% \) for the rate of capital depreciation. We will set calendar time to \( t = 0. \)

The remaining unobservable parameters are \( \sigma, \rho, \) and \( \lambda. \) We choose these free parame-
ters in effort to match the following three targets from the U.S. macro data: (i) a safe real rate of return equal to 3.5 percent (following the recent macroeconomic work in life-cycle consumption, see Gourinchas and Parker 2002, Bullard and Feigenbaum 2007, Feigenbaum 2007), (ii) a capital-output ratio equal to 3.0 (Gourinchas and Parker 2002, Bullard and Feigenbaum 2007, Feigenbaum 2007), and (iii) a discrete drop in aggregate life-cycle consumption at the date of retirement, which is often reported to be in the range of 10 to 20 percent (e.g., see Bernheim et al. 2001, and Ameriks et al. 2007 and the many references therein).

The Feasible Space \((F)\)

Let \(f \in F\) be any triplet \(\{\sigma, \rho, \lambda\}\) that can replicate (or nearly replicate) these targets. The first question that we ask is whether \(F \neq \emptyset\). We first note that the discrete drop can be pinned down with exactness simply by adjusting \(\lambda\) since the size of the drop in aggregate consumption is \(\frac{\lambda(1 - \theta - \theta R)}{1 - \theta}\). For example, \(\lambda = 17.2\%\) delivers a drop equal to 10 percent (which is the lower end of the target range), regardless of the values for \(\sigma\) and \(\rho\). Likewise, a value of \(\lambda = 25.8\%\) generates a drop equal to 15 percent (the mid-point of the target range) and a value of \(\lambda = 34.4\%\) generates a drop equal to 20 percent (the upper end of the target range). Hence, feasible values of \(\lambda\) necessarily lie between 17.2 percent and 34.4 percent.

To get a feel for the values of \(\sigma\) and \(\rho\) that belong to the feasible space, we run the following procedure. \textit{Step 1}: set \(\lambda = 17.2\%\) and then search for \((\sigma, \rho)\) ordered pairs that come close to hitting the target rate of return and capital output ratio (holding all other parameters besides \(\sigma\), \(\rho\), and \(\lambda\) at baseline values). By “close” we mean that the model produces a rate of return between 3.0 and 4.0 percent and a capital-output ratio between 2.9 and 3.1; we allow for some slack since the targets are not exactly defined in the first place.\(^2\) \textit{Step 2}: repeat step 1, but with a value of \(\lambda = 25.8\%\). \textit{Step 3}: repeat step 1 again,\(^2\)

\(^2\) Since we hold capital’s share and the depreciation rate constant across all the calibrations, the restriction \(K(t)/Y(t) \leq 3.1\) will bind before the restriction \(r(t) \geq 3.0\%\). Likewise, the restriction \(r(t) \leq 4.0\%\) will bind before the restriction \(K(t)/Y(t) \geq 2.9\).
but with a value of $\lambda = 34.4\%$.

The results from following steps 1 through 3 are shown in Table 1. Each panel represents a two-dimensional slice of the three-dimensional feasible space (cut along the $\lambda$-dimension). The feasible space is larger than presented in these tables but we run out of room trying to show the entire space; however, the rest of the unshown portion of the feasible space is not reasonable anyway and it is therefore of no interest to us.\(^3\)

*The Reasonable Space (S)*

The next question is whether we can rule out any portions of the feasible space on the grounds that the parameter values are not plausible or are inconsistent in some way with other outside evidence. Macroeconomists often consider values of $\sigma$ ranging from 1 to 10, and sometimes as low as about 0.5 (e.g., Gourinchas and Parker 2002; Feigenbaum 2007), so reasonable values for $\sigma$ are taken to be between 0.5 and 10. Although a negative discount rate is occasionally used in macro work, we will follow tradition and exclude negative values from the reasonable space (including negative values will lead to higher optimal tax rates). Finally, a large body of evidence documents that the share of non-savers in the economy is close to the range identified above as the feasible space (see Huang and Caliendo 2007 for a survey), so we are not able to rule out any portion of the feasible space along this dimension. Table 2 shows a 2-dimensional slice of some of the reasonable space, for $\lambda \in [17.2\%; 34.4\%]$.

*The Intersection of the Feasible and Reasonable Spaces*

We now focus our attention on the intersection of these two spaces, and we compute the optimal tax rate for each possible triplet from this intersection. Since all cohorts have an identical mix of consumer types (i.e., $\lambda$ is constant across cohorts), we can focus on the welfare of a weighted average of the two types of consumers, born at some date $\tau$ along

\(^3\)Because we have discretized the parameter space, the coarseness of the grid may have some effect on the results, but we suspect this issue is very minor.
the steady-state equilibrium growth path

\[
U(\tau) \equiv \lambda \int_\tau^{\tau+\bar{T}} e^{-\rho(t-\tau)} \frac{c^h(t, \tau)^{1-\sigma}}{1-\sigma} \, dt + (1-\lambda) \int_\tau^{\tau+\bar{T}} e^{-\rho(t-\tau)} \frac{c^p(t, \tau)^{1-\sigma}}{1-\sigma} \, dt \\
= \lambda \int_\tau^{\tau+\bar{T}} e^{-\rho(t-\tau)} \frac{[(1-\theta)w(t)]^{1-\sigma}}{1-\sigma} \, dt + \lambda \int_\tau^{\tau+\bar{T}} e^{-\rho(t-\tau)} \frac{b(t)^{1-\sigma}}{1-\sigma} \, dt \\
+ (1-\lambda) \int_\tau^{\tau+\bar{T}} e^{-\rho(t-\tau)} \frac{[\Omega w(\tau) e^{\theta(t-\tau)}]^{1-\sigma}}{1-\sigma} \, dt \\
= \chi_1(\tau)\chi_2 + \chi_3(\tau)\chi_4 + \chi_5(\tau)\chi_6
\]

where

\[
\chi_1(\tau) \equiv \frac{(1-\theta)w(\tau)^{1-\sigma}}{1-\sigma}, \quad \chi_2 \equiv \frac{(\lambda w(\tau) \tau - 1)}{1-\sigma}, \quad \chi_3(\tau) \equiv \frac{\lambda[\theta w(\tau) \tau]^{1-\sigma}}{1-\sigma}, \\
\chi_4 \equiv \frac{\lambda[\theta w(\tau) \tau - 1]}{1-\sigma}, \quad \chi_5(\tau) \equiv \frac{(\lambda[\theta w(\tau) \tau]^{1-\sigma})}{1-\sigma}, \quad \chi_6 \equiv \frac{(\lambda[\theta w(\tau) \tau - 1]}{1-\sigma}
\]

We define \( \theta^* \equiv \arg \max \{U(\tau)\} \). Notice \( w(\tau) \) can be factored out and ignored, so \( \theta^* \) does not depend on the birth date.

In Table 3 we compute the optimal tax rate for the intersection of the feasible and reasonable spaces in general equilibrium. Some of the parameterizations lead to a tax rate that is lower than the current rate and other parameterization lead to the opposite result. The lowest optimal tax rate from this intersection is 6 percent and the highest is 15 percent, while the average of the optimal tax rates (10.9 percent) is surprisingly close to the current rate in the U.S. (10.6 percent).
4 Conclusion

We have studied the optimal social security tax rate in a general equilibrium life-cycle economy with hand-to-mouth and permanent-income consumers, paying particular attention to all possible combinations of unobservable parameters that are feasible and reasonable. Hunting through this space, we find that the optimal tax rate may be close to the current rate in the U.S. This study can and should be expanded to include additional features of the life-cycle experience, while maintaining the general equilibrium structure which imposes discipline on the choice of preference parameters and forces the modeler to consider the full space of unobservables that can match key data targets.
5 References


Table 1. The Feasible Space, $F$. ($f \in F$)

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Top panel $\lambda = 17.2\%$, middle panel $\lambda = 25.8\%$, bottom panel $\lambda = 34.4\%$. $F$ is the space that can reproduce the designated targets in U.S. data.
Table 2. The Reasonable Space, $S$. ($s \in S$)

<table>
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Note: This is a portion of the space $S$ that is most often used in macroeconomic work and validated by microevidence.
Table 3. The Optimal Social Security Tax Rate for the Intersection of the Feasible ($F$) and Reasonable ($S$) Spaces.

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Top panel $\lambda = 17.2\%$, middle panel $\lambda = 25.8\%$, bottom panel $\lambda = 34.4\%$. 