SIMULTANEOUS VOLATILITY TRANSMISSION
AND SPILLOVER EFFECTS

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Abstract
Simultaneous Volatility models are developed and shown to be separate from Multivariate GARCH estimators. An example is provided that allows for simultaneous and uni-directional volatility and volume of trade effects. These effects are tested using intra-day data from the Australian cash index and index futures markets. Overnight volatility spillover effects from the S&P500 index futures markets are tested using alternative estimates of this U.S. market volatility. The simultaneous volatility model proves to be robust to alternative specifications of returns equations and to mis-specification of the direction of volatility causality.
1 INTRODUCTION

Some empirical applications in the finance and econometrics literatures have attempted to quantify volatility transmission and spillover effects by employing variants of multivariate volatility structures. However, these structures typically impose exogeneity restrictions or causality restrictions in the estimation process. Given that information transmission is virtually instantaneous it is more logical to employ structural systems which can incorporate simultaneous endogenous effects in the formalization. As well, when contemporaneous volatility and volume effects are synchronously observed at high frequency then these effects dominate lagged time series effects in estimated structural systems. The effective bias in parameter estimates obtained from GARCH models when accumulated volume of trade within the daily observation interval is included as an exogenous variable is demonstrated in Board et al (2001). However, modelling the intra-day contemporaneous volume of trade effects within a structural systems framework was first reported in Gannon (1994). We extend this latter approach and provide a formal econometric foundation for these latter models.

This leads to one key focus in this paper i.e., quantifying intra-day trading patterns and information effects. By quantifying contemporaneous and unidirectional volatility transmissions effects we are better able to deal with market microstructure effects. Structural estimation of intra-day volatilities will be important in quantifying (i) spillover effects between markets (information spillovers), (ii) volatility transmissions between underlying assets and derivatives, (iii) exogenous market specific political and economic announcement effects, and (iv) endogenous volume of trade effects. These effects cannot be adequately captured by employing data observed from particular time points from day to day.

Parameter estimates obtained from well specified systems then supply information with important economic content. These structural approaches are not difficult to employ providing necessary assumptions required for these estimators are not violated. When structural volatility estimates are invariant to alternative specifications of the mean equation then the volatilities can be regarded in intra-day data as transformations of returns. It is important to note that if the original innovations are I(0) each of the volatility measures are...
I(0) and if volume of trade is also I(0) endogenous volatilities and volumes can then be regarded as a system of structural equations. It follows then that the identification and estimation of such a system can proceed as for a system of structural equations in the mean.

This structural formalization permits a wider class of models than in the conditional volatility literature, for example, different transformations such as squared returns, the absolute value of returns and logarithmic transformations of squared returns. Different sets of endogenous and exogenous variables such as volume measures and microstructure factors can enter the system. This structural approach permits sensible theoretical restrictions to be placed on the structural volatility system, restrictions that can be used for identification. The reduced forms can be employed to obtain recursive step ahead forecasts for endogenous volatilities and volumes. The final forms can be employed to simulate shocks to time paths of included volatilities and variables. Attention is restricted to specification and testing these systems within this paper.

The foregoing arguments suggest compelling theoretical reasons for specifying a simultaneous set of volatility equations but there are further considerations. First, how do these structures differ from the class of time varying conditional variance structures? These time series structures include the class of GARCH structures initiated by Bollerslev (1986) and extended to a multivariate form in Engle and Kroner (1995), the EGARCH structure of Nelson (1991), variants of the absolute value volatility structure initiated by Schwert (1989), and extended to a bi-variate system by Bessembinder and Seguin (1992 and 1993), and a stochastic volatility structure initiated by Taylor (1986) and extended to the multivariate case in a sequence of papers following Harvey et al (1994). In section two the relationship between multivariate GARCH structures and multivariate time series structures for volatility processes is considered. It is shown that neither of these two types of structure allow endogenous volatility or endogenous volume effects to enter these systems. It follows that econometric identification of parameter estimates cannot be obtained within these systems estimators. Then these systems cannot be considered within the class of simultaneous volatility structures.

The second consideration is concerned with separating and selecting between competing structural volatility systems. The concept of structural systems that are not observationally equivalent is considered in section four. Structures are presented where the classical matrix rank and order conditions for identification are satisfied. The concept of 'local' identification is then introduced so that these systems can be compared and a dominant
system selected. Theoretically this selection involves minimizing ignorance about the true unobservable structure by minimizing a measure of distance between the preferred specified structure and the true underlying structure. Practically this reduces to selecting between similarly dimensioned structures based on a likelihood criterion. A systems AIC criterion is employed in order to discriminate between these competing systems. The robustness of these systems estimates is checked by specifying alternative excess return generating equations and comparing estimates. Non normality of point estimates of volatility can be reduced via suitable transformations. These specification issues are discussed in section three.

Empirical examples are presented in section five where this structural selection is undertaken when the processes are observed at 15 minute intervals. An extension of the data employed in Gannon (1994) is used in this paper. These empirical examples employ data where transmission effects between derivative assets and underlying assets in the medium sized market of Australia should be important. As well, spillover effects from the U.S. market onto these markets is tested. Alternative mean generating equations are specified to generate resultant volatility estimates to check for differences in parameter estimates and standard errors obtained from Full Information Maximum Likelihood (FIML) estimation. Because the signal to noise ratio is extremely low for the first moment equations at these observation intervals the invariance to mean equation mis-specification holds. As well, restrictions imposed on the matrix of endogenous volatilities are also imposed on the transformed reduced form variance/covariance matrix in the estimation process. This variance/covariance matrix captures interactions between the volatility of the volatility measures entering these systems. The dominant system is then augmented to allow structural disturbances, obtained from first round maximum likelihood estimation, to enter as lagged Moving Average (MA) terms in second round estimation. Alternative volatility spillover measures from U.S. markets are then tested within a variable addition framework.

2. MULTIVARIATE GARCH AND DYNAMIC SIMULTANEOUS VOLATILITY STRUCTURES

Let us start by reconsidering the univariate GARCH \((p', q')\) process introduced in Bollerslev (1986)

\[ \varepsilon_t | \psi_{t-1} \sim N(0, h_t), \]
\[ h_i = \omega_0 + \sum_{i=1}^{q'} \alpha_i \varepsilon_{i-1}^2 + \sum_{j=1}^{p'} \beta_j h_{i-j} \]  

where \( \psi_{i-1} \) denotes the \( \sigma \)-field generated by all information through time \( t-1 \). \( \omega_0 > 0, \alpha_i \geq 0, \beta_j \geq 0, \) and \( p' > 0 \) only if \( q' > 0 \). If \( \alpha(1) + \beta(1) < 1, \{\varepsilon_i\} \) is covariance-stationary with \( E(\varepsilon_i^2) = \omega_0 / [1 - \alpha(1) - \beta(1)] \) and \( E(\varepsilon_i \varepsilon_j) = 0 \) for \( s \neq t \). For \( p' = 0 \), equation (1) reduces to an ARCH(q') process.

Now, following Bollerslev (1988), and rewriting equation (1) as

\[
\varepsilon_i^2 = h_i + \nu_i = \omega_0 + \sum_{i=1}^{m} (\alpha_i + \beta_i) \varepsilon_{i-i}^2 - \sum_{j=1}^{p'} \beta_j \nu_{i-j} + \nu_i
\]

where

\[ \nu_i = (\eta_i^2 - 1) h_i, \quad \eta_i^{id} \sim N(0,1), \]

\( m = \max\{p' q'\} \). When \( m = q' = p' \), equations (2) and (3) may be viewed as general ARMA structures for a non-negative discrete time process, \( \varepsilon_i^2 \geq 0 \) a.s. for all \( t \), and \( E(\varepsilon_i^2) = \omega_0 (\sum_{i=1}^{m} (\alpha_i + \beta_i))^{-1} \). The disturbance term \( \nu_i \) is heteroscedastic but not serially correlated. This development can also be extended to a multivariate GARCH structures. The following exposition is simplified to that for a diagonal multivariate vector GARCH (VEC) structure which excludes cross product terms for \( (\varepsilon_i \varepsilon_j)_{i=m} \) and terms for \( (h_{ij})_{i=m} \) to keep the notation compact.

Let us now reconsider the dynamic simultaneous equations structure modified for a set of volatility equations. For the dynamic simultaneous equations system the unrestricted structural form can be written as

\[
\beta \varepsilon^2_i = \Gamma_r \varepsilon^2_i + \ldots + \Gamma_{r-r} \varepsilon^2_{i-r} + \Gamma_o x_i + \ldots + \Gamma_s x_{i-s} + \nu_i.
\]

In this system let us represent the T observations on the G endogenous volatilities by the \((T \times 1)\) vectors \( \varepsilon^2_1, \ldots, \varepsilon^2_T \); the K \((P + R)\) lagged endogenous volatilities and exogenous variables by the \((T \times 1)\) vectors \( \varepsilon^2_{i-1}, \ldots, \varepsilon^2_{i-p}, x_1, \ldots, x_r \); and the G error variables by the \((T \times 1)\)
vectors \( v_1, v_2, \ldots, v_G \). This system is linear in parameters and pre-multiplying both sides of equation (4) by \( \beta^{-1} \) leads to the reduced form

\[
\epsilon_i^2 = \beta^{-1} \Gamma_i' \epsilon_{i-1}^2 + \ldots + \beta^{-1} \Gamma_r' \epsilon_{i-r}^2 + \beta^{-1} \Gamma_0' x_i + \ldots + \beta^{-1} \Gamma_{r-s} x_{i-s} + \beta^{-1} \nu_i \\
= \Pi_i^r \epsilon_{i-1}^2 + \ldots + \Pi_r^r \epsilon_{i-r}^2 + \Pi_0^r x_i + \ldots + \Pi_{r-s}^r x_{i-s} + \omega_i
\]  

(5)

By the linear transformation from \( \nu_i \) to \( \omega_i \), the distribution for the structural disturbances implies the distribution for the reduced form disturbances.

However, depending on the specification of \( \beta \), inclusion or exclusion of \( x_i \), and on the distributional assumptions associated with \( \nu_i \), equation (5) can be mimicked by versions of equation (1).

There are a number of restrictions which can be imposed on the structural system (4) and these are described below and summarised in table 1. To simplify the following notation define a diagonal matrix as D.

**Case 1:**

\[
\beta = D, \Gamma' = D, \Gamma'' = 0
\]  

(6)

If \( \nu_i \) from equation (4) follows a moving average process the ARMA(p,q) volatility structure for \( \epsilon_i^2 \) can be written as

\[
\phi(L)\epsilon_i^2 = \theta(L)\omega_i
\]  

(7)

where \( \omega_i \) is a disturbance and \( \phi(L) \) and \( \theta(L) \) are polynomials in the lag operator defined by

\[
\phi(L) = 1 - \phi_1 L - \ldots - \phi_p L^p,
\]

and

\[
\theta(L) = 1 + \theta_1 L + \ldots + \theta_q L^q.
\]

Stationarity requires that the roots of \( \phi(L) \) lie outside the unit circle, invertibility requires that
the roots of $\theta(L)$ lie outside the unit circle. The GARCH($p',q'$) structure mimics the behaviour of this general structure when $m = q$. The first of two simpler structures arise if $v_t$ is not serially correlated,

$$
\varepsilon_t^2 = \phi_0 + \phi_1 \varepsilon_{t-1}^2 + \phi_q \varepsilon_{t-p}^2 + \omega_t. 
$$

(8)

Stationarity requires that the roots of $\phi(L)$ lie outside the unit circle, as for the ARMA($p,q$) process. Any stationary AR($p$) or ARMA($p,q$) process may be expressed as an infinite moving average. The ARCH($q'$) process mimics the behaviour of this AR($p$) structure.

If $v_t$ is serially correlated, and $\Gamma^* = 0$ a MA($q$) structure describes the behaviour of this process

$$
\varepsilon_t^2 = \theta_0 + \omega_t + \theta_1 \omega_{t-1} + \theta_q \omega_{t-q}. 
$$

(9)

An invertible process can be expressed as an infinite autoregression but no restrictions are required on either $\theta_0$ or on the moving average parameters $\theta_1, \ldots, \theta_q$ to ensure stationarity. However, from equation (2), $\alpha_i \geq 0$ and $p' > 0 IFF q' > 0$ so that there is no GARCH structure which mimics the behaviour of a pure MA($q$) process.

Case 2:

$$\beta = D, \Gamma^* = D, \Gamma^{n} \neq 0$$

(10)

If $v_t$ follows a MA process this is a univariate ARMAX structure. A GARCH($p',q'$) process with exogenous regressors mimics the behaviour of this structure. If $v_t$ is not serially correlated this reduces to an autoregressive process with exogenous regressors. This is mimicked by an ARCH($q'$) process with exogenous regressors.

Case 3:

$$\beta \neq D, \Gamma' \neq D, \Gamma^{n} = 0$$

(11)

If $v_t$ follows an MA process the reduced form is a vector ARMA structure
\[ \varepsilon_t^2 = \phi_0 + \phi_1 \varepsilon_{t-1}^2 + \ldots + \phi_p \varepsilon_{t-p}^2 + \omega_t + \Theta \omega_{t-1} + \ldots + \Theta_q \omega_{t-q} \]  

where \( \omega_t \) is a vector of disturbances. A Multivariate GARCH (MGARCH(\( p',q' \))) process mimics the behaviour of this structure. If \( \nu_t \) is not serially correlated the reduced form is a vector autoregression which can be represented by a MARCH(\( q' \)) process.

**Case 4:**

\[ \beta \neq D, \Gamma \neq D, \Gamma'' \neq 0 \]  

If \( \nu_t \) follows a MA process this model is a vector ARMAX structure. The reduced form for this structure may be written as:

\[ A(L) \varepsilon_t^2 = B(L) x_t + \Theta(L) \omega_t \]  

and is known as the ARMAX(\( r,s,q \)) structure. A MGARCH(\( p',q' \)) process with exogenous regressors mimics the behaviour of this structure. If \( \nu_t \) is not serially correlated the reduced form for this structure reduces to a vector autoregressive distributed lag AD(\( r,s,i \)) structure which can be represented by a MARCH(\( q' \)) process with exogenous regressors.

All of these structures can be considered as restricted forms of a general dynamic simultaneous volatility structure. It is clear that all of these specifications do not pertain to any underlying structural form second moment equations and the only restrictions which enter relate to the dynamic lag structure or complete exclusion for all lags.

**Case 5:**

If stochastic exogenous variables do not enter the system and the disturbance term is not serially correlated, lagged endogenous volatilities may then be classified with the exogenous variables as *predetermined*. Then the predetermined variables play exactly the same role as the exogenous variables in the contemporaneous simultaneous equations system. This means that lagged endogenous volatilities may be subject to exclusion restrictions in exactly the same way as exogenous variables. In the absence of exclusion restrictions, on the predetermined variables in the structural form, this system collapses to the vector AD(\( r,s,i \))
specification in the reduced form.

The restricted system can be augmented by inclusion of lagged structural disturbances generated from first round systems maximum likelihood estimates in second round estimation. These de-trended MA terms help account for mis-specification in the structural part of the system and provide more efficient estimates. The concept of a ‘locally’ identifiable structure can then be considered for competing equally dimensioned structures in which the classical matrix rank and order conditions were satisfied.

3. EXCESS RETURNS AND VOLATILITY TRANSFORMATIONS

Consider a first order autoregressive process for the spot asset price return (a D-AR(1) representation)

\[ s_t = \rho_1 s_{t-1} + e_t \]  

where \( s_t \) is the return on the asset (the difference in the natural logarithm of the spot asset price levels or the difference in the spot asset price levels) between time \( t \) and \( t-1 \), \( \rho_1 \) is the first order serial correlation coefficient between \( s \) at time \( t \) and \( t-1 \), and \( e_t \) is an assumed homoskedastic disturbance term, \( \sigma^2_e \), and \(-1 < \rho_1 < 1\). This equation provides an approximation to the time series behaviour of the spot asset price return.

An alternative specification to equation (15) is a simple autoregressive equation for the level of the spot asset price \( S \) (or natural logarithm of the levels) at time \( t \) and \( t-1 \), (a L-AR(1) representation)

\[ S_t = \phi_1 S_{t-1} + a_t \]  

where \( a_t \) is an assumed homoskedastic disturbance term, \( \sigma^2_a \), and the value of \( \phi_1 \) may be greater or less than one.

When \( \rho_1 \) in equation (15) is zero and \( \phi_1 \) in equation (16) is one and no higher order
serial correlation is present in either of equations (15) and (16) then these two equations are equivalent representations. It follows that shocks generated from either equation are equivalent. As $\rho_1$ increases in absolute value then shocks generated from these two equations become more dissimilar. As the observation interval approaches zero non-trading induced effects in cash market indices and bid ask bounce and order splitting in futures price and stock price processes becomes more severe. Then shocks generated from equations (15) and (16) will be progressively dissimilar. But these effects need to be severe to distort systems estimates obtained from power transformations of squared excess returns series (see Gannon (1996)).

Extreme non-normality of endogenous volatilities can be avoided by employing suitable transformations of the squared excess residual returns. This approach then helps in generating structural disturbances that are approximately normally distributed.

If the underlying distribution is subject to extreme shocks the natural logarithm of the constructed measure of volatility will be more robust to these shocks than measures employed in GARCH and absolute value variance specifications. If these shocks affect parallel processes in a similar manner then extreme weighting on measures of contemporaneous volatility transmission can be misleading. This effect can be reduced by employing a measure robust to these extremes.

A flexible measure of volatility can be derived following the specification in Box and Cox (1964)

$$\sigma_{it}^2 = V^\delta = [((\varepsilon_{it})^2)^\delta - 1]^{1/\delta}$$  \hspace{1cm} (17)

with $\sigma_{it}^2$ the volatility of $i$th asset (derivative) at time $t$ and $\varepsilon_{it}^2$ defined from any assumed excess returns equation. This transformation is employed to generate volatility measures as variables entering structural equations. When $\delta = 1$ or $0.5$ we have the squared excess return translated by one and the absolute value of the squared excess return, respectively, and when $\delta \to 0$ the measure approaches the natural logarithm of the squared excess return. These transformations allow the generated series to mimic the behaviour of volatility measures employed in the GARCH specification of Bollerslev (1986), the absolute value variance specification of Schwert (1989) and the stochastic volatility specification defined in Taylor (1986), respectively.
The issue of identification within sets of volatility equations is addressed in section 4. Restricted dynamic systems of volatility equations and volatility structures based on contemporaneous simultaneous systems of equations are considered. These systems are quite different from multivariate GARCH and multivariate stochastic variance structures. Formal structural systems explicitly account for endogenous volatility and endogenous volume of trade effects. The estimators for structural contemporaneous systems explicitly account for implied restrictions on the variance/covariance matrix of estimated disturbances.

4. SIMULTANEOUS VOLATILITY STRUCTURES

The focus is on three equation systems drawn from the Structural Vector Autoregressive Time Series (SVAR) class of models, Giannini (1992) and the traditional Simultaneous Equations literature, Hsiao (1983).

SVAR systems have received some attention in the applied macroeconomics literature. There are essentially three ways of ‘structuralizing’ the vector autoregressive (VAR) system and these are discussed in Giannini (1992) and in references cited within this monograph. A K-type SVAR imposes restrictions within the endogenous and lagged endogenous variable matrix. A geometrically declining lag structure is imposed in such a way that identification of structural parameters is assured. A C-type SVAR imposes restrictions on the variance/covariance matrix to enable identification. An AB-type structure maintains restrictions on endogenous and lagged endogenous variables as well as separate restrictions on the variance/covariance matrix to enable identification. In this subsection a variant of a K-type SVAR, modified for a set of volatility equations, is considered.

If the dynamic structure of the time paths in a SVAR system is short then traditional simultaneous equations structures are less restrictive systems. This will often be the case when data is observed at high frequency. Contemporaneous effects are relatively more dominant, under these circumstances, than lagged effects. It is also likely the case that extra diagonal terms are irrelevant. This means that complete subsets of equations can be defined and estimated.

These systems can be augmented to include dummy, trend and other part continuous intervention variables. The approach of Magnus and Neudecker (1988) can be adopted in that these variables sharpen point estimates in the structural part of the system and do not hinder identification of parameters of interest. However, when a full set of these variables are not
included in the system short run stochastic effects lead to less efficient estimates.

Then the system can be augmented by inclusion of lagged structural disturbances obtained from first round systems maximum likelihood estimates in second round estimation. Then these de-trended MA terms help account for mis-specification in the structural part of the system and provide more efficient estimates. The approach in Harvey (1990) is adopted in this latter treatment. Then the concept of a ‘locally’ identifiable structure is considered for competing equally dimensioned structures in which the classical matrix rank and order conditions were satisfied.

This approach implies selecting that structure is which sufficiently close to the true structure. This approach reduces to minimizing mis-specification. This general approach can then be seen as one in which the dynamics in the system are adequately accounted for so that measures of correlation between contemporaneous volatilities and variables in the system are identifiable. When observations are generated by a simultaneous volatility structure, their distribution is determined by the reduced form. Even if conditional normality does not strictly hold for the disturbances the parameter estimates will still be asymptotically normally distributed and maximization of the quasi log likelihood will still give consistent estimates. Full information methods will still be asymptotically efficient under conditions discussed in White (1982), and Gourieroux et al (1984).

4.1 Simultaneous Volatility Systems

For simplicity, let the set of volatility relations be represented by the following structural form:

$$\beta(\sigma_t^2) + \Gamma x_t = v_t, \quad t=1,\ldots,T.$$  \hspace{1cm} (18)

where

- $(\sigma_t^2)$ is a Gx1 vector of observed endogenous volatilities/variables;
- $x_t$ is a Kx1 vector of observed exogenous variables and/ or lagged endogenous volatilities/ variables;
- $\beta$ is a GxG matrix of coefficients;
- $\Gamma$ is a GxK matrix of coefficients;
\( \mathbf{v}_t \) is a Gx1 vector of unobserved disturbances;

We assume that:

- \( \beta \) is non-singular.
- \( \lim_{T \to \infty} \sum_{t=1}^T x_t x_t' / T \) exists and is non-singular.
- \( E(\mathbf{v}_t) = 0, E(\mathbf{v}_t x_t') = 0 \), and

\[
E(\mathbf{v}_t \mathbf{v}_s') = \begin{cases} 
\sum_{t}^n & \text{if } t = s, \\
0 & \text{otherwise}.
\end{cases}
\]

The assumption \( \lim_{T \to \infty} \sum_{t=1}^T x_t x_t' / T \) exists and is non-singular rules out the existence of non-stationary regressors.

Multiplying through by a \( G \times G \) non-singular matrix \( F \), the new structure may be written as:

\[
(F \beta)(\sigma_t^2) + (F \Gamma)x_t = \omega_t,
\]

where \( \omega_t = F \mathbf{v}_t \).

When the transformation matrix \( F = \beta^{-1} \) the transformed structure becomes

\[
\sigma_t^2 = \Pi x_t + \omega_t, \quad (19)
\]

where

\[
\beta \Pi + \Gamma = 0, \text{ or } \Pi = -\beta^{-1} \Gamma
\]

with \( \Pi \) the matrix of reduced form coefficients and

\[
\omega_t = \beta^{-1} \mathbf{v}_t,
\]

\[
E(\omega_t \omega_s') = \begin{cases} 
\beta^{-1} \sum_{t}^n \beta^{-1} & \text{if } t = s, \\
0 & \text{otherwise}.
\end{cases}
\]

Equation (19) is the ‘reduced form’ of the ‘structural system’ (18).

Without further restrictions no equation is identified since the system (18) can be premultiplied by any non-singular constant matrix \( F \), and the new equations will be
observationally equivalent to (19). Sufficient restrictions must be imposed on the transformation matrix, $F$, to ensure an identifiable structure. The classical matrix rank and order conditions for two equation systems identification are set out in Hsiao (1983), when sufficient restrictions are imposed on $F$. These conditions are defined for three equation systems in Gannon (1994). Full matrix rank identification conditions are necessary and sufficient to show lack of identification in the system as a whole. A reduced rank matrix condition relates to the unnormalized case. The order condition simply determines whether an equation is identified or overidentified after the matrix rank condition has been satisfied.

Now consider accounting for the short run deviations from the long run structural paths subsumed in the structural part of the system.

To see how these variables may enter these structural systems impose a univariate MA representation on structural disturbances obtained from first round estimates. These processes can be viewed from the perspective of the dynamic simultaneous equations system in Harvey (1990) with the further imposition of structural restrictions on the long run parameters of the system. For ease of exposition write this structural form as was defined by equation (4)

For the dynamic simultaneous volatility system

$$\beta \sigma_i^2 = \Gamma_1 \sigma_{i-1}^2 + \ldots + \Gamma_2 \sigma_{i-2}^2 + \Gamma_0 x_i + \ldots + \Gamma_s x_{t-s} + v_t$$

the system is linear in parameters and defining the matrix polynomial

$$A(L) \sigma_i^2 = \beta - \Gamma_1 L - \ldots - \Gamma_s L^s$$

and

$$B(L) = \Gamma_0 + \Gamma_1 L + \ldots + \Gamma_s L^s$$

the final form is written as

$$\sigma_i^2 = A^{-1}(L) B(L) x_i + A^{-1}(L) v_t$$

If $v_t$ is specified to be a multivariate MA process,
\[ v_t = \theta_u(L) u_t \]  
\[ z_t = \theta_z(L) \xi_t \]

and if some exogenous variables, \( z_t \), are stochastic and generated by a multivariate MA process,

where \( \xi_t \) is a vector of serially uncorrelated disturbances with mean zero and covariance matrix \( \Omega_z \), then the autoregressive final form is written as

\[
\begin{align*}
|A(L)| \sigma_i^2 &= A^*(L) B_1(L) x_t + A^*(L) B_2 z_t + A^*(L) v_t \\
|A(L)| \sigma_i &= A^*(L) B_1(L) x_t + A^*(L) v_i^\# 
\end{align*}
\]

Then \( v_i^\# \) may be decomposed into a set of univariate MA processes. Then each endogenous volatility/variable has a transfer function representation in which only the elements of \( x_t \) appear as explanatory variables in the long run structural part of the system. If exclusion or other restrictions are imposed on \( \beta \) then these restrictions are also imposed on the reduced form disturbance vector.

### 4.2 Locally Identifiable Structures

The discussion that follows is concerned with identification of a structure that is ‘close’ to the true structure. This will sometimes require comparison of mis-specified structures. What is important is selecting a structure that dominates competing structures. This approach to identification can then be seen to be equivalent to selecting that structure which minimizes the effects of possible mis-specification.

White (1982) notes that when \( F \) contains the true structure \( G \) (i.e., \( G(u) \equiv F(u, \theta_0) \) for some \( \theta \) in \( \Theta \) ) the MLE for \( \theta_0 \) is consistent under regularity conditions provided in Wald (1949) and Le Cam (1953). Then the parameter vector which minimizes the Kullback-Leibler (1951) information criterion (KLIC) minimizes ignorance about the true structure. This is
given in:

\[ I(g : f, \theta) = \int \log g(u) dG(u) - \int \log f(u, \theta) dG(u) \]

Assuming \( \mathbb{E}(\log g(U_i)) \) exists and \( \| \log f(u, \theta) \| \leq m(u) \) for all \( \theta \) in \( \Theta \) where \( m \) is integrable with respect to \( G \), ensures \( I(g : f, \theta) \) is defined.

Assuming \( I(g : f, \theta) \) has a unique minimum at \( \theta_* \) in \( \Theta \) ensures \( \theta_* \) is globally identifiable and is equivalent to the matrix rank condition for the true simultaneous equations structure.

\( \theta_* \) is locally identifiable if for some open neighborhood \( \mathcal{B} \subset \Theta, I(g : f, \theta) \) has a unique minimum at \( \theta_* \).

If the matrix rank condition holds for competing structures then alternatives to misspecification testing are required. This problem falls outside the theoretical development in White.

Hsiao (1983) follows a different treatment by considering cases where the ‘true’ structure is unknown and considers conditions for two structures to be observationally equivalent. The following treatment is in terms of structures that are not observationally equivalent.

The joint density of \( (\sigma_1^2, ..., \sigma_N^2) \) can be derived through the structural relation and the density of the \( \nu_i \)'s.

The density of \( \sigma_i^2 \) can be written in terms of the reduced form.

A sufficient condition for two structures to be observationally equivalent is:

There exists a non-singular matrix \( F \) such that

\[ [\bar{\beta}, \bar{\Gamma}] = F[\beta, \Gamma] \]  \hspace{1cm} (25a)

and

\[ \bar{\Sigma} = F\Sigma'F' \]  \hspace{1cm} (25b)

Therefore, the identification problem in these structures is only concerned with imposition of sufficient restrictions on \( \beta \) and \( \Gamma \) so that sufficient restrictions are imposed on the elements of \( F \). It follows that alternative restrictions on \( F \) can provide identifiable structures. For two
structures to be observationally equivalent they must have identical reduced form parameter matrix and variance/covariance matrix. The key to separation of seemingly similar structural forms often reduces to different reduced form variance/covariance matrices.

Within a set of volatility equations this condition reduces to differences in the variance/covariance matrix (i.e., the volatility of the volatility). This holds when \( \Sigma' \) (the structural form variance/covariance matrix) and therefore \( \Omega \) (the reduced form variance/covariance matrix) are non singular.

The approach in this paper considers the model as consisting of all possible structures in which the parameters of interest are identifiable. Defining \( L_1(.) \), \( L_2(.) \)... as separate likelihoods obtained from distinct equally dimensioned structures at the respective maximum of the joint density function then the following conditions for local identification can be defined.

**Condition 1:**

If \( L_1(.) \) of one structure does not admit \( L_2(.) \) of any other possible structure in the model, within some neighbourhood, then this structure cannot be observationally equivalent to any other structure of the model.

**Condition 2:**

The structure, obtained from the model, which attains the maximum of the likelihood function of all identifiable structures of like dimension, obtained from the model, and which does not admit the maximum of the likelihood of any other identifiable structure within some semi neighbourhood is a ‘locally identified structure’. That is, if \( L_1(.) > L_2(.) \) then the first structure attains a smaller information ignorance measure. If \( L_1(.) \) is larger than any other \( L(.) \) obtained from these identifiable structures of the model the structure \( L_1(.) \) is locally identified.

Condition 1 only defines possibly identifiable structures that are not observationally equivalent. Condition 2 admits a preferred structure obtained from a sequence of possible identifiable structures.

Consider three equation specifications of the structure (18) again but represent these structures with only one lag on the dynamics although the caveat that exclusion implies exclusion at all lags is implied. In the structural part of the systems \( \sigma_i^2 \) may represent
endogenous volatilities, \( y_t \) may represent an endogenous variable, \( \sigma^2_{y_{t-1}} \) may represent lagged endogenous volatilities, \( y_{t-1} \) lagged endogenous variables and \( x_t \) other variables deemed exogenous at time \( t \), with each equation containing a distinct structural disturbance term \( v_t \).

These structural forms are identifiable according to the classical matrix rank and order conditions. These conditions only consider distinguishing separate parameter points from an unrestricted model. In order to distinguish between these ‘structures’ the transformation matrices (\( F \)) need be distinct. For different \( F \) the transformed reduced form parameter matrices and transformed variance/covariance matrices will be distinct. It follows that a ‘locally identified’ structure can be defined in terms of conditions 1 and 2. If the autoregressive final form is written down then these structures can be seen to be separate.

Impose the restrictions implied by K-type SVAR structures first.

**SVAR(1)**

\[
\beta_{13} = \beta_{21} = \beta_{32} = \gamma_{13} = \gamma_{21} = \gamma_{32} = 0
\]

so that the structural system is,

\[
\begin{align*}
\beta_{11} \sigma^2_{y_t} + \beta_{12} \sigma^2_{x_t} + y_t \sigma^2_{x_{t-1}} + \gamma_{11} \sigma^2_{x_{t-1}} + \gamma_{12} \sigma^2_{x_{t-2}} + \gamma_{13} = v_{1t} \\
+ \beta_{22} \sigma^2_{y_t} + \beta_{23} y_t + y_t \sigma^2_{y_{t-1}} + \gamma_{22} \sigma^2_{y_{t-1}} + \gamma_{23} y_{t-1} = v_{2t} \\
\beta_{31} \sigma^2_{y_t} + \beta_{33} y_t + y_t \sigma^2_{y_{t-1}} + \gamma_{31} \sigma^2_{y_{t-1}} + \gamma_{33} y_{t-1} = v_{3t}
\end{align*}
\]

(27)

For the unnormalized case all equations are identified by the rank condition as

\[
\text{rank} \left( A \phi^\prime \right) = G - 1 \quad \text{(see Gannon (1994) p. 1060)}.
\]

In all equations \( K = 3, K_j = 2 \) and \( G_j = 2 \) so that \( K - K_j = G_j - 1 \) so that all equations are exactly identified by the order condition.

**SVAR(2)**

\[
\beta_{12} = \beta_{23} = \beta_{32} = \gamma_{12} = \gamma_{23} = \gamma_{31} = 0
\]

so that the structural system is,
\[
\begin{align*}
\beta_{11} \sigma_{1t}^2 + \beta_{13} y_t + \gamma_{11} \sigma_{1t-1}^2 + \gamma_{13} y_{t-1} &= v_{1t} \\
\beta_{21} \sigma_{2t}^2 + \beta_{22} \sigma_{2t}^2 + \gamma_{21} \sigma_{2t-1}^2 + \gamma_{22} \sigma_{2t-1}^2 &= v_{2t} \\
&+ \beta_{32} \sigma_{2t}^2 + \beta_{33} y_t + \gamma_{32} \sigma_{3t-1}^2 + \gamma_{33} y_{t-1} &= v_{3t}
\end{align*}
\] (29)

For the unnormalized case all equations are identified by the rank condition as
\[ \text{rank}(A \phi_g') = G - 1 \] (see Gannon (1994) p. 1061).

In all equations \( K = 3, K_i = 2 \) and \( G_i = 2 \) so that \( K - K_i = G_i - 1 \) so that all equations are exactly identified by the order condition.

The reduced forms are distinct from each other because the transformation matrices \( F \) are distinct. These structures are separate and cannot be non-nested forms of either one or the other. It follows that these structural forms are distinct and separate.

The respective reduced form and final form for the SVAR(1) are identical since only lagged endogenous volatilities and variables enter the structural systems (i.e., \( x \) variables do not appear in the final form). This same feature holds for the SVAR(2) as well. Since the \( F \) matrices for the SVAR(1) and SVAR(2) are distinct it follows that the mean time paths generated from these separate reduced forms will be distinct. Short run deviations from the long run time paths generated from the autoregressive final forms will also be distinct. This follows from the observation that the multivariate MA processes, \( A^{-1}(L)v_t = A^{-1}(L)\theta_u(L)u_t \), from the respective final forms are generated from different sets of structural disturbances, \( v_t \). Then the univariate MA terms, \( A^*(L)v_t^\circ \), obtained from the autoregressive final forms will be distinct.

Now consider the three equation structural simultaneous volatility system where lagged endogenous volatilities, lagged endogenous variables and variables deemed exogenous in separate equations at time \( t \) enter. The system includes a new exogenous variable \( x_4 \).

**SIMULT** (CONTEMPORANEOUS SIMULTANEOUS)
\[
\beta_{12} = \beta_{23} = \beta_{32} = \gamma_{12} = \gamma_{24} = \gamma_{34} = 0
\] (30)
\[ \begin{align*}
\beta_{11} \sigma_{1t}^2 + \beta_{13} y_t + \gamma_{11} \sigma_{1t-1}^2 + \gamma_{13} y_{t-1} + \gamma_{14} x_{4t} &= v_{1t} \\
\beta_{21} \sigma_{2t}^2 + \beta_{22} \sigma_{2t-1}^2 + \gamma_{21} \sigma_{2t-1}^2 + \gamma_{22} \sigma_{2t-1}^2 + \gamma_{23} y_{t-1} &= v_{2t} \\
\beta_{31} \sigma_{3t}^2 + \beta_{33} y_t + \gamma_{31} \sigma_{3t-1}^2 + \gamma_{32} \sigma_{3t-1}^2 + \gamma_{33} y_{t-1} &= v_{3t}
\end{align*} \] (31)

Noting that changing rows in a matrix does not alter the rank of the matrix, for the unnormalized case all equations are identified by the rank condition as \( \text{rank}(A\theta_g) = G - 1 \) (see Gannon (1994) p. 1062).

In all equations \( K = 4, K_i = 3 \) and \( G_i = 2 \) so that \( K - K_i = G_i - 1 \) so that all equations are exactly identified by the order condition.

In this structure the reduced form and final form are different. This structure is distinct from either of the SVAR(1) or SVAR(2). In this structure \( x_4 \) in the first equation, \( y_{t-1} \) in the second equation and \( \sigma_{2t-1}^2 \) in the third equation are considered exogenous to those equations since these variables cannot be written in autoregressive form for the respective equations. These variables explain the systematic component in the mean paths not explained by the autoregressive structure. These are the \( x_i \) components in the final form described in equation (21). Short run deviations from the long run time paths generated from the autoregressive final forms will also be different from either of the SVAR(1) and SVAR(2). This follows from the observation that the multivariate MA processes, \( A^{-1}(L)v_t = A^{-1}(L)\theta_u(L)u_t \), from this final form are generated from quite different sets of structural disturbances, \( v_t \). Then the univariate MA terms, \( A^*(L)v_t^u \), obtained from the autoregressive final forms will be separate and distinct.

A number of systems in which all equations of interest are identifiable have now been presented. Assuming that variables entering these systems are integrated of appropriate order, long run parameters implied by these systems will be identifiable. As well, the parameter estimates will be asymptotically normally distributed even if the structural disturbances are not strictly normally distributed.

Now consider three alternative specifications of the volatility systems. In the following examples the natural logarithm of the squared excess return i.e., equation (17) when \( \delta \to 0 \) is employed as the volatility measure in the systems estimates. The preferred systems from the Australian cash index and index futures markets are augmented with competing alternative variables to account for volatility spillover effects from U.S. futures.
markets onto these markets the following trading day. These information effects, which originate in North American trading time, are often captured in the volatility of financial asset price processes trading in North American trading time. These effects often transfer (spillover) onto other financial asset markets which open after the close of normal North American trading time. Whether this effect is significant is an empirical issue.

5 EMPIRICAL EXAMPLE

In this example a three equation system measuring transmission between the Australian All Ordinaries Index (AOI) volatility and the Share Price Index (SPI) futures volatility and futures volume of trade (VOL) is estimated. Contemporaneous volume of trade effects are endogenously embedded within the system. Mis-specification of the excess returns equation is only important at 5 minute intervals. At 15 minute and longer observation intervals volatility estimates are essentially the same (see Gannon (1996)).

This Australian data has been synchronously sampled from real time feeds from pit trades from the Sydney Futures Exchange (SFE) and online records for the AOI (updated each minute). This data is obtained from a set sampled in an identical manner to that used to generate the results reported in Gannon (1994). This sampled data from this market features a similar trading process in the major U.S. cash index and pit traded index futures markets at the current time. We extend the analysis and subject the models to robustness and specification checks in this paper. The sample corresponds to the June, September and December 1992 nearest SPI contracts (four contract expirations per year) between 10.00am to 12.30pm and from 2.00pm to 4.00pm when both markets are simultaneously active. The futures contract is based on the AOI which is a value weighted index of the largest 276 market capitalized listed stocks trading on the Australian Stock Exchange (ASX). SPI prices are quoted in the same form as the AOI with a minimum fluctuation of 0.1 index point equal to $10.00AUD per contract. The SPI commences trading at 9.50am and ceases normal day trading at 4.10pm (overnight screen trading ceases at 2.00am the following morning) so that the first and last ten minutes of normal market trade are not included. Random opening times for stocks traded on the ASX are alphabetically staggered by four, three minute intervals between 9.00am till 9.09 am. These market features imply that opening market U.S. volatility spillover effects may be difficult to detect and the market open on the ASX needs to be stratified. Volume of trade is the accumulated number of SPI futures contracts traded within
the 15 minute interval (i.e., from 10.00am up to 10.15am etc). Overnight and lunchtime (close to open) returns are deleted from the sample.

The autoregressive parameter estimates obtained from a D-AR(1) specification of the returns processes observed at 30, 15 and 5 minute intervals are reported in table 2. Results are for all three sampling periods for the AOI and SPI (i.e., data sampled corresponding to the three SPI futures contracts last three months to expiration in 1992). As the observation interval reduces to 5 minutes substantial first order serial correlation is detected in the AOI. The autoregressive parameter for the SPI returns moves into the positive region as the differencing interval reduces suggesting order splitting and non-trading induced effects. At five minute intervals preliminary simultaneous systems and GARCH(1,1) estimates obtained from volatilities generated via L-AR(1) and D-AR(1) first moment equations were quite different. These results are not reported but can be obtained from the authors on request. However, these estimates and asymptotic standard errors were almost identical at 15 and 30 minute intervals so that the former are reported.

Augmented Dickey Fuller (ADF) statistics of volatility and volume for 15 minute intervals are reported in table 3. A L-AR(1) equation generated the shocks (excess returns) and natural logarithms of the squared shocks for the SPI and AOI generated these volatility measures (and results almost identical from a D-AR(1) equation). All volatility and volume series are well below the non-stationary boundary.

The systems AIC, for the restrictions defined in equations (26), (28) and (30) generating the three equation systems defined in equations (27), (29) and (31) i.e., the SVAR1, SVAR2 and SIMULT (Simultaneous), respectively, are reported in table 4. A L-AR(1) structure for the excess returns equations generated these results. Volatility systems without and with MA terms are reported. The superiority of the simultaneous system without or with MA terms is clear. The lag structure for the MA terms is the same as that defined for the full systems estimates reported in table 6 i.e., 1 for both the SPI and AOI volatility equations and 2 for the volume equation. This lag structure is sufficient to ensure that there is...
The leading volume term is not a perfect foresight variable in a rational expectations
context. This variable should be important in modelling the behaviour of informed traders in a relatively thin market.

In addition, the system is augmented with the following variables to test for systematic intra-day effects:

**EXPIRE** - A trending variable taking the value \( (T-t)/T \) for each 15 minute interval for each of the three SPI contracts three months to expiration. \( T \) is the sample size (number of 15 minute intervals) for each of the four contracts and \( t \) is the interval count for each contract, in ascending order. This variable is bounded between 1 and 0.

**OPEN** - A dummy variable taking the value 1 for the first interval of each trading day (10.00am to 10.15am), zero otherwise.

In table 5 results are reported from first round FIML estimation where the residual returns have been generated from both D-AR(1) and L-AR(1) first moment equations (equations (15) and (16)). The natural logarithm of the squared residual returns are then employed as the measures of volatility (equation (17)). There does not appear to be much difference between parameter estimates or standard errors by employing alternative first moment generating equations, at least for this data sampled at 15 minute intervals.

In table 6 results of FIML estimation are recorded without and with MA terms. The excess returns are generated from a L-AR(1) equation. A minimum lag length for the MA terms was chosen so that structural disturbance autocorrelation was insignificant for all equations across all sampling intervals. This is not an optimal choice as over fitting for some sampling intervals may be imposed in second round estimates. These systems are augmented with the following variables to test for systematic intra-day effects:

**VOLS&P** - A measure of overnight volatility based on the S&P500 futures volatility in the last 15 minutes of normal market trade. This value is generated as the squared logarithmic return and inserted for the period 10.00am to 10.15am, zero otherwise. Given that market trading on the SPI begins at 9.50am this variable may simply account for some persistence in
overnight volatility. This measure is also available following the 2.00am close of SYCOM (Sydney Computerised Overnight Market) during 1992. All series are matched so that U.S. market volatility leads Australian market volatility with records deleted when matching trades were not available for all series.

Alternative measures of U.S. futures market volatility are generated from open to close squared logarithmic returns and also the standard deviation of intra-day 15 minute returns. The first and last 5 minutes of normal market trade was excluded from the sample prior to calculating all S&P500 volatility measures. All three measures are allowed to enter for all observation intervals within the trading day as well as open only. These alternative measures and specifications are not significant nor are they important when included in all three equations within these systems. Estimates based on VOLSP are reported only as the log likelihood was marginally higher when this alternative measure was included.

VOLAUD - A measure of intraday (lunchtime) volatility based on the AUD/USD volatility between 12.30pm and 2.00pm. This value is the squared logarithmic return and inserted for the period 2.00pm to 2.15pm, zero otherwise.

WEND - A dummy variable taking a value of 1 for the first trading interval (10.00am to 10.15am) for each weekly market open and a value of zero otherwise.

For each equation the system is augmented by the following variables:

SPI EQUATION - CONSTANT, EXPIRE, VOLSP, VOLAUD, WEND.
AOI EQUATION - CONSTANT, OPEN, WEND.
VOLUME EQUATION - CONSTANT, WEND.

From this latter table it is clear that overnight volatility spillover effects are not significant in these specifications for the data observed within this sample period. As well, there does not appear to be any significant spillover effect from trading in currency markets on post lunchtime trading. Possibly the closed nature of the pit coupled with strong intra-day interactions between these variables entering the structural part of the system dominate any residual volatility spillover effect. Nevertheless these are some important features in the results reported in table 6.

Within the SPI volatility equation expiration effects are generally important but only
the estimate for the June quarter has a negative sign and therefore in agreement with the Samuelson hypothesis. There is strong contemporaneous correlation with volume. LAGVOL, LDVOL and LAGSPI help to model the long run dynamics in SPI volatility. The MA terms helps explain short run deviations from the long run time paths subsumed in the structural part of the model for the June and December contracts.

The open effect has the expected positive sign in the AOI volatility equation. Contemporaneous SPI volatility is highly significant for the three sample periods as are lags of SPI and AOI volatility. Lag volume is only important for the June quarter. The MA terms do help model short run deviations from the long run paths for these three sample periods.

Contemporaneous SPI volatility is highly significant in the volume equation and with the lagged endogenous variables help model the dynamics in this equation. Both MA terms are important for the June and December sample periods in this equation.

6 CONCLUSIONS

Structural systems of volatility equations have received almost no attention in the theoretical or empirical literature. In this paper an attempt is made to motivate the utility of employing such systems by formalizing the approach adopted by Schwert (1989). For financial data sampled intra-day this is feasible since the signal to noise ratio is very low in autoregressive returns equations so that mean and variance equations can be estimated separately. Mis-specification of weak dynamics in the mean equations does not affect parameter estimates and efficiency of systems of volatility equations when the sampling interval is not too fine. If these volatility measures and other endogenous variables such as volume of trade are also approximately $I(0)$ variables then the system of volatility equations can be treated as a standard systems estimator. The utility of these systems lies in the imposition of sensible restrictions which can then provide point estimates that have meaning. Clearly this approach is different from that followed within the class of multivariate stochastic volatility estimators.

The focus is to specify and estimate some structures where the parameters in the structural part of the system can be identified. By inclusion of other deterministic dummy, trend and relevant part continuous variables the efficiency of these systems estimators is improved with the added inference obtainable from estimates of these other parameters. These other variables help filter unexpected shocks to these volatility processes. If measures
on these other variables are not available, inclusion of lagged structural disturbances, obtained from first round estimates, can help filter these stochastic effects.

These empirical results from this Australian dataset suggest that serial correlation in returns processes would need be severe to distort parameter estimates from the systems of volatility equations.

For this Australian dataset the failure to quantify spillover effects is possibly the result of employing a measure of overnight volatility as an input to the first observation intraday. The first 10 minutes of trading in the SPI was excluded. As well, early trading may contain effects from position setting and early noise trading which mitigates capturing the spillover effect. These results hold whether the volatility measure is constructed from the last 15 minutes or the open to close prices for the S&P500 futures. Strong unidirectional volatility spillover effects were detected from the SPI to the AOI at these high frequencies. Specification of the excess returns equation does not alter this result. Contemporaneous effects to and from SPI volatility and volume help condition these variables in these equations by capturing these trading patterns. This measure of market activity is very significant in this example. The need to specify two lags for the MA terms for the volume equation suggests stronger serial correlation in this process relative to the volatility measures employed. Then more structure may be needed in volume equations.

Given that the reduced forms and final forms of these estimated systems can be obtained, additional insight into the behaviour of these volatility processes and equations can be gained through multi step estimation and simulation analysis.
BIBLIOGRAPHY


Harvey, A. (1990), The Econometric Analysis of Time Series, Philip Allan.


## TABLE 1
SIMULTANEOUS AND GARCH VOLATILITY STRUCTURES

<table>
<thead>
<tr>
<th>SIMULTANEOUS</th>
<th>GARCH</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Case 1</strong></td>
<td></td>
</tr>
<tr>
<td>$\beta=D, \Gamma'=D, \Gamma''=0$</td>
<td></td>
</tr>
<tr>
<td>ARMA ($p, q$)</td>
<td>$\nu_t$ follows MA process</td>
</tr>
<tr>
<td>AR ($p$)</td>
<td>$\nu_t$ is not serially correlated</td>
</tr>
<tr>
<td>MA ($q$)</td>
<td>$\nu_t$ follows MA process</td>
</tr>
<tr>
<td><strong>Case 2</strong></td>
<td></td>
</tr>
<tr>
<td>$\beta=D, \Gamma'=D, \Gamma''\neq0$</td>
<td></td>
</tr>
<tr>
<td>ARMAX ($p, q$)</td>
<td>$\nu_t$ follows MA process</td>
</tr>
<tr>
<td>AR ($p, s_i$)</td>
<td>$\nu_t$ not serially correlated</td>
</tr>
<tr>
<td><strong>Case 3</strong></td>
<td></td>
</tr>
<tr>
<td>$\beta \neq D, \Gamma' \neq D, \Gamma''=0$</td>
<td></td>
</tr>
<tr>
<td>vector ARMA ($p,q$)</td>
<td>$\nu_t$ follows MA process</td>
</tr>
<tr>
<td>VAR</td>
<td>$\nu_t$ not serially correlated</td>
</tr>
<tr>
<td><strong>Case 4</strong></td>
<td></td>
</tr>
<tr>
<td>$\beta \neq D, \Gamma' \neq D, \Gamma''\neq0$</td>
<td></td>
</tr>
<tr>
<td>Vector ARMAX ($r,s_i,q$)</td>
<td>$\nu_t$ follows MA process</td>
</tr>
<tr>
<td>vector AD ($r, s_i$)</td>
<td>$\nu_t$ not serially correlated</td>
</tr>
<tr>
<td>vector ARMAX ($r,s_i,p,q$)</td>
<td>$\nu_t$ follows ARMA ($p, q$)</td>
</tr>
<tr>
<td><strong>Case 5</strong></td>
<td></td>
</tr>
<tr>
<td>Restrictions on $\beta', \Gamma', \Gamma''$</td>
<td></td>
</tr>
<tr>
<td>Recursive models</td>
<td>$\beta$ triangular, $\Omega$ diagonal</td>
</tr>
<tr>
<td>Structural models</td>
<td>$x_t$ nonstochastic, $\nu_t$ not serially correlated</td>
</tr>
<tr>
<td>Dynamic models</td>
<td>$\nu_t$ follows MA process</td>
</tr>
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### TABLE 2
SUMMARY STATISTICS FOR THE SPI AND AOI FOR 1992
AUTOREGRESSIVE PARAMETERS FROM A D-AR(1) EQUATION

<table>
<thead>
<tr>
<th>INTERVAL</th>
<th>SPI</th>
<th>AOI</th>
<th>AOI</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>JUNE</td>
<td>SEPTEMBER</td>
<td>DECEMBER</td>
</tr>
<tr>
<td>30 MIN</td>
<td>-0.0496</td>
<td>-0.0420</td>
<td>-0.0150</td>
</tr>
<tr>
<td></td>
<td>(-1.179)</td>
<td>(-1.008)</td>
<td>(-0.358)</td>
</tr>
<tr>
<td></td>
<td>0.0487</td>
<td>0.1113</td>
<td>0.0881</td>
</tr>
<tr>
<td></td>
<td>(1.146)</td>
<td>(2.689)</td>
<td>(2.103)</td>
</tr>
<tr>
<td>15 MIN</td>
<td>-0.1448</td>
<td>-0.0264</td>
<td>-0.0212</td>
</tr>
<tr>
<td></td>
<td>(-4.878)</td>
<td>(-0.898)</td>
<td>(-0.709)</td>
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<tr>
<td></td>
<td>0.0495</td>
<td>0.0907</td>
<td>0.0963</td>
</tr>
<tr>
<td></td>
<td>(1.646)</td>
<td>(3.100)</td>
<td>(3.218)</td>
</tr>
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<td>-0.0112</td>
<td>0.0599</td>
<td>0.0795</td>
</tr>
<tr>
<td></td>
<td>(-0.648)</td>
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<td>(4.595)</td>
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<td></td>
<td>0.3475</td>
<td>0.3051</td>
<td>0.3494</td>
</tr>
<tr>
<td></td>
<td>(21.37)</td>
<td>(18.88)</td>
<td>(21.61)</td>
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</table>

**First order autoregressive parameter estimate from a D-AR(1) OLS regression.**
<table>
<thead>
<tr>
<th>INTERVAL</th>
<th>SPI</th>
<th>AOI</th>
<th>VOL</th>
</tr>
</thead>
<tbody>
<tr>
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<td>-29.64</td>
<td>-18.73</td>
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<tr>
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<td>(.000)</td>
<td>(.000)</td>
<td>(.000)</td>
</tr>
<tr>
<td>MARCH</td>
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<tr>
<td></td>
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<td>(.000)</td>
<td>(.000)</td>
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<tr>
<td>JUNE</td>
<td>-31.02</td>
<td>-20.95</td>
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</tr>
<tr>
<td></td>
<td>(.000)</td>
<td>(.000)</td>
<td>(.000)</td>
</tr>
<tr>
<td>SEPTEMBER</td>
<td>-29.54</td>
<td>-25.01</td>
<td>-12.83</td>
</tr>
<tr>
<td></td>
<td>(.000)</td>
<td>(.000)</td>
<td>(.000)</td>
</tr>
<tr>
<td>DECEMBER</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

ADF REGRESSION WITH TREND: LAG LENGTH SET TO MAXIMUM SUBJECT TO 10% SIGNIFICANCE ON EXTREME LAG.
### TABLE 4

**AUSTRALIAN SYSTEMS AIC FOR 15 MINUTE VOLATILITIES FROM A L-AR(1) NO MA MA**

<table>
<thead>
<tr>
<th>SYSTEM</th>
<th>JUN.</th>
<th>SEPT.</th>
<th>DEC.</th>
<th>JUN.</th>
<th>SEPT.</th>
<th>DEC.</th>
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</thead>
<tbody>
<tr>
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<td>21944</td>
<td>22414</td>
<td>21514</td>
<td>21880</td>
<td>22302</td>
<td>21438</td>
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<td>SVAR2</td>
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This version of the AIC is evaluated as $-2 \times \text{(Log Likelihood)} + 2(p)$ where $p$ is the number of freely estimated parameters for exogenous variables and lagged MA terms in the respective systems. Given that the dimension of the endogenous variable matrices are the same across systems then the number of implied restrictions on the variance/covariance matrices are the same across systems. For comparison across systems preference is given when the AIC is minimized.
### TABLE 5
**SPI, AOI & VOLUME: L-AR(1) FIML & D-AR(1) FIML ESTIMATES 15 MINUTE INTERVAL**

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<th>DECEMBER</th>
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<td>------------</td>
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ASYMPTOTIC BHHH T STATISTICS IN SMALL PRINT. FIML ESTIMATES OBTAINED VIA GAUSS NEWTON ITERATIONS.
### TABLE 6
SPI, AOI & VOLUME: L-AR(1) FIML & L-AR(1) FIML(MA) ESTIMATES 1992: 15

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**VOLUME EQUATION**

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ASYMPTOTIC BHHH T STATISTICS IN SMALL PRINT. FIML ESTIMATES OBTAINED VIA GAUSS NEWTON ITERATIONS.