Competition and the Hold-up Problem: a Setting with Non-Exclusive Contracts

Guillem Roig*

Abstract

This article offers a solution to the "hold-up" problem in a bilateral investment game. Without the existence of a centralized grand-contract, a buyer signs non-exclusive contracts with many sellers, and the equilibrium investment profile depends on the level of competition in the trading outcome. I a common agency game where both sides of the market undertake investment, full efficiency is only implemented when the trading outcome is the most competitive. Due to the strategic complementarity of investments, payoffs are generally not monotone with the bargaining position, and lower competitive outcomes may generate larger aggregate surpluses.

Keywords: Hold-up; Bilateral investment; Common agency; Competition; Welfare.

JEL Classification Numbers: D44; L11.

*Toulouse School of Economics. Contact information: guillemroig182@gmail.com. This paper is based on the first chapter of my thesis. I am thankful to David-Pérez Castrillo, Zhijun Chen, Jacques Crémer, Natalia Fabra, Simona Fabrizi, Olga Gorelkina, Inés Macho-Standler, Steffen Lippert, Gerard Llobet, John Panzar, Martin Pollrich, Santiago Sánchez Pagués, and Simone Sepe. I am also indebted to the comments received at Australasian Theory Workshop, Deakin 2015; Auckland University; the UAB micro seminar; the games and behavior UB seminar; the CREIP seminar and the participants of the Jamboree ENTER conference, Brussels 2013.

1 Introduction

Trading partners often undertake investments to increase potential gains from trade. Consider an insurer who researches on contingencies to better suit the special needs of his client, or a seller who reduces the production cost of an intermediate good specific to a downstream producer. Because investments are sunk at the trading stage, the investing party fears opportunistic behavior by his counterpart, which result into inefficient investment decisions. Fisher Body, a manufacturer of body cars, refused to locate his body plants adjacent to General Motors assembly facilities, a necessary move for production efficiency.

The existence of the "hold-up" problem is traced to incomplete contracts, i.e., the inability of parties to write contracts depending on all relevant and publicly available information.[1] The economic literature has focused on two different approaches to solve the problem. On the one hand, the organizational approach, relating to the theory of the firm, establishes the provisions for asset ownership and the allocation of residual rights of control. On the other hand, the long-term contract approach designs contractual provisions aiming to relax potential conflicts of interests between trading parties.

The solutions explored by the literature have either emphasized on a microeconomic bilateral relationships within a single buyer and seller pair taken in isolation, or have assumed that the whole economy can be ruled within a single grand-contract. This article relaxes the grand-contracting approach and models an environment where investment contracts cannot be enforced. I explore how the introduction of competition to one side of the market gives incentives to undertake profitable investment decisions, and I provide a new perspective on the interaction between organizations and markets.

In my model, a single buyer trades with many sellers for the provision of an homogenous input. One of the sellers knows a technology enabling him to reduce the cost of input production. The

[1] If investment is verifiable or enforceable ex-post, it is in the interest of the contractual parties to write compensation schemes linked to investment, Grossman & Hart (1986), Grout (1984), Hart & Moore (1988) and Williamson (1985).

buyer can also invest to improve her valuation for the input by adapting her production process. An application fitting the model is the provision of military or medical supplies to governments in and environment where economic institutions do not allow for the design or enforcement of ex-ante contracts, or when a government cannot commit to trade exclusively with a single provider. The model then proposes a normative analysis to the design of trading relationships to incentivize profitable investment from both sides of the market.

In the provision of military and medical supplies, neither part of the market has the whole bargaining power. Modeling the bargaining procedure in a common agency game with investment is a daunting task. Following the existing literature, I consider an analog of a first price auction in which, to compete for the buyer, each seller offers a menu of trading contracts. I restrict attention to nonlinear price trading contracts, where a trading contract consists of a pair specifying an amount of input and a transfer. With the available trading contracts, the buyer selects the best contract from each one of the sellers.

With the two requirements that characterize an equilibrium in a common agency game: "*bilateral efficiency*", each seller's trading contracts maximize the gains from trade between the buyer and himself; and "*individual excludability*", the buyer obtains the same equilibrium payoffs after excluding any seller from trade; the trading surplus is divided between the buyer and the sellers, and the payoff of each seller measures his contribution to the surplus. The equilibrium transfer for each seller equals to the loss of the trading surplus originated when the buyer excludes him from trade, which depends on the latent contracts or out-of-equilibrium trading contracts offered by the rest of the sellers. The number of available latent contracts determine the outside option available to the buyer. With fierce competition, i.e, a large number of rival sellers submit latent contracts to compete for the excluded seller, the available outside option for the buyer becomes large and the equilibrium transfers of the seller is minimized.

Trading partners invest efficiently only when competition for the trading contracts becomes the largest. When the bargaining position of each seller is minimized, investments do not effect the outside option available to the buyer, and each seller appropriates his marginal contribution of the trading surplus. With softer competition, investment decisions are not efficient. Investment decisions influence the outside option available to the buyer, affecting the bargaining position of the sellers. The investing seller over-invests as his investment endogenously increases his bargaining position with the buyer. Nonetheless, the impact that investments have on the buyer's outside option diminishes with the number of sellers the buyer establishes trade with. When the number of sellers is arbitrarily large, regardless of the level of competition, the equilibrium investment profile tends to efficiency.

I explore which is the sellers' most preferred equilibrium. Because the equilibrium investment profile depends on sellers' competition with trading contracts, sellers not always prefer situations with the least competitive equilibrium outcome. A lower seller's bargaining position incentivizes the investment of the buyer. How sensitive is the equilibrium trading allocation to investment influences the results. Since relative productive efficiency changes with the investment of the seller, a larger seller's investment translates into a reduction of the amount traded by the competing sellers. If the effect turns out to be small, all sellers prefer a less competitive equilibrium outcome gr anting them a more favorable bargaining position. Otherwise, different sellers prefer different bargaining positions.

Strategic complementarity of investment may lead to larger welfare with low competitive equilibrium outcomes. Investment inefficiencies created to one side of the market may restore efficiency to the other side, leading to larger potential gains from trade. When the seller over-invests he reduces the bargaining position of the competing sellers giving larger incentives for the buyer to invest. Lower competitive outcomes may lead to higher levels of welfare. Results suggest, a competition authority should be careful examining competition in an industry where ex-ante investments are important. Promoting competitive outcomes may fail to maximize the potential welfare generated in a market.

In the next section I discuss the related literature. In section 3, I introduce the set-up of the model and proceed to solve it backwards. In section 4.1 I study the properties of the equilibrium allocation and I characterize the equilibrium payoffs in section 4.2. I obtain the equilibrium investment profile in section 4.3. In section 5, I compare equilibria and section 6 concludes. All proofs are in the appendix.

2 Literature

This article builds on the literature of markets and contracts. Instead of considering the impossibility of contracting over some states of nature or actions, this literature limits the number of parties that can be part of the same contract. In its most recent set-up, trading contracts are non-exclusive and a common agent can freely sign multiple bilateral trading contracts with different parties.[2] Bernheim & Whinston (1986) first considered a model of contracting between one agent and multiple principals. The authors take a group of principals aiming to provide incentives to a common agent, and characterize necessary and sufficient conditions to achieve an efficient outcome. In a trading environment, Segal (1999) demonstrate that in the absence of direct externalities, the equilibrium trading outcome is unique and efficient. No externalities exist when the principals' payoffs depend only on their own trade with the agent. Even in a bidding game, where multiple principals propose trading contracts to the common agent, and inefficiencies may arise from the coexistence of multiple offers, efficiency remains.

While in the absence of direct externalities, a unique and efficient trading outcome exists, Chiesa & Denicolò (2009) demonstrate multiplicity of the equilibrium payoffs.[3] In a common agency framework, the "threat" of being replaced by his competitors determines equilibrium payoff of the principals. This "threat" pins down to what latent or out-of-equilibrium trading contracts the competing principals submit. Chiesa & Denicolò (2009) characterize the maximum sellers' payoff compatible for a non-cooperative notion of equilibrium, which is given by the "threat" of any principal to

[2]Earlier papers center on the study of exclusive contracts: Akerlof (1970), Rothschild & Stiglitz (1976) and Aghion & Bolton (1987) Biglasier & Mezzetti (1993, 2000).

[3]The set of equilibrium payoffs is a semi-open hyper-rectangle. Martimort & Stole (2009) show multiplicity of equilibria in a public common agency game and use asymmetric information as a tool for equilibrium refinement.

be unilaterally replaced by one of his competitors. By defining the degree of competition by the number of sellers who compete by submitting latent contracts, I obtain a sub-set of the equilibrium payoffs in Chiesa and Denicolò (2009). The lowest available payoff for sellers coincides with the "truthful" equilibrium. In a "truthful" equilibrium, each seller obtains his marginal contribution to the surplus.

Chiesa & Denicolò (2012) undertake comparative statics of different equilibria, and state that the Pareto dominant equilibrium for the sellers is where the rent of the buyer minimizes. In their framework, potential gains from trade stay invariant with distribution, and sellers always prefer an equilibrium where the portion of the gains from trade favors them the most. I introduce a previous stage where both sides of the market undertake specific investment. With this previous stage, I compare equilibria with regard to welfare. To the best of my knowledge, this paper is the first to consider welfare analysis in a common agency game with complete information. In my model, the redistribution of the gains from trade has implications on the investment decisions of the parties and on the final size of those gains.

This paper also relates to the "hold-up" literature from Klein, Crawford & Alchian (1978) and Williamson (1979, 1983), were the "hold-up" problem arises because parties are unable to bargain over specific investment. Investment is unverifiable. In my model the "hold-up" problem comes from the lack of contract enforceability. The "hold-up" literature concludes that in the absence of ex-ante contracts, investments stay inefficiently low under any possible bargaining game, Grossman & Hart (1986) and Hart & Moore (1990). The literature studies mechanisms to restore the efficient levels of investment. Mainly, the allocation of property rights or the design of ex-ante contracts Aghion, Dewatripont & Rey (1994), Chung (1991) and Edlin & Reichelstein (1996). In my model, ex-ante contracts cannot be enforced which relates to the literature on competition and the "hold-up" problem as in Cole, Mailath & Postlewaite (2001a, 2001b); Mailath, Postlewaite & Samuelson (2013); Felli & Roberts (2012); Makowski (2004) and Samuelson (2013). Nevertheless, all those models consider a matching mechanism where once investment has been undertaken, agents decide on the trading partner. Investment then works as a mechanism to increase the outside option giving higher incentives to invest. Departing from this literature, I allow the offering part of the market to compete offering trading contracts to the monopolistic side. Trade in my model is non exclusive, and I give a normative analysis on the design of trading relationships in situations where both sides of the market undertake specific investment.

3 Model

I consider a bilateral investment game where a single buyer trades with many ex-ante identical sellers. Sellers are indexed by $i \in N = \{1, ..., N\}$ and produce an homogeneous input.

The game consist of two stages played sequentially. In stage one, specific investment takes place. Here, only seller $i = 1$ invests in a cost-reducing technology, which allows to reduce his production costs. The amount of investment is a continuous variable $\sigma \geq 0$, with a convex cost $\psi(\sigma)$. The buyer undertakes investment to enhance her valuation from the total amount traded. She takes a binary decision on whether or not to invest $b \in \{0, 1\}$, and incurs to a fixed costs of K. By investing, I consider that the buyer adapts her production process to the homogeneous input provided by the sellers. I assume that investing parties do not have any budget constraint; they are not financially restrained on the amount of investment they can take.

In stage two, each seller trades with the common buyer. Following Chiesa & Denicolò (2009), I consider a bidding game where sellers simultaneously submit a menu of trading contracts $M_i \subset \Re^+$. I restrict attention to trading contracts nonlinear prices, hence, a typical trading contract is a pair $m_i = (x_i, T_i)$, where $x_i \geq 0$ represents the quantity seller i supplies and $T_i \geq 0$ stands for the transfer requested by the seller. Because trade is voluntary, each seller offers the null contract $m_i^0 = (0, 0)$ in equilibrium. The buyer chooses a single trading contract from each seller. To guarantee the existence of an optimal choice for the buyer, I require the menus of trading contracts M_i to be a compact set Γ. Formally, with the menu profile of trading contract $\mathbf{M} = (M_1, M_2, ..., M_N) \in \Gamma^N$, a strategy for the buyer is a function $\mathcal{M}(\mathbf{M}) : \Gamma^N \to (\Re^+)^N$ such that $\mathcal{M}(\mathbf{M}) \in \times_{i=1}^N M_i$ for all

$\mathbf{M} \in \Gamma^N$.

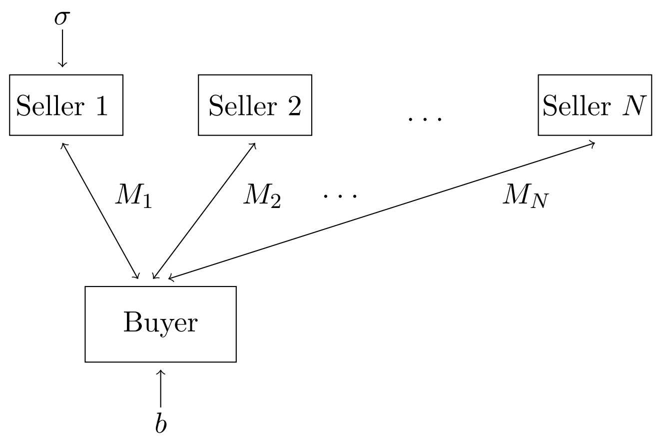

Figure 1: Bilateral investment game with N competing sellers.

The model belongs to private and delegated common agency. The model is private since a seller cannot condition payments on the quantities others trade, and delegated because the buyer can trade with any subset of sellers. Information in the game is complete and the equilibrium concept is sub-game perfect Nash (SPNE). Investment is observable but not contractible, because a third party cannot enforce it.

3.1 Payoffs and trading surplus

The payoffs of the buyer and the sellers are quasi-linear in transfers.[4] The buyer obtains

$$\Pi(\mathcal{M} \mid b) = U\left(X \mid b\right) - \sum_{i=1}^{N} T_i - K \times b, \tag{3.1}$$

where $X = \sum_{i=1}^{N} x_i$ represents the total quantity traded. The payoff for seller 1 is

$$\pi_1(\mathcal{M} \mid \sigma) = \pi_1(\mathcal{M}_1 \mid \sigma) = T_1 - C\left(x_1 \mid \sigma\right) - \psi(\sigma), \tag{3.2}$$

[4] All parties have a constant marginal utility of money, allowing to reduce the complexity of the problem and focusing the analysis on welfare comparison.

and

$$\pi_i(\mathcal{M}) = \pi_i(\mathcal{M}_i) = T_i - C\left(x_i\right), \quad \text{for all } i \neq 1. \tag{3.3}$$

for the rest of the sellers.

For a given investment profile (b, σ), the maximum trading surplus is

$$TS^*(b, \sigma) = \max_{x_1, \ldots, x_n} \left[U(x_1 + \ldots + x_n \mid b) - C_1(x_1 \mid \sigma) - \sum_{i \neq 1} C_i(x_i) \right], \tag{3.4}$$

where $\mathbf{x}^* = (x_1^*, \ldots, x_N^*)$ stands for the vector of quantities that solves the problem. For later use, I denote by $X_{-H}^* = \sum_{i \notin H} x_i^*$, for $H \subset N$, the sum of the previous quantities without taking the quantities of the subset of sellers in H. I finish by stating the assumptions of the utility and costs functions. Subscripts denote partial derivatives. I denote the utility of the buyer when she does not invest $U(X \mid b = 0)$ by $U(X)$.

1. $U_x(\cdot) > 0$, $U_{xx}(\cdot) < 0$, $U(X \mid b = 1) > U(X)$ and $U_x(X \mid b = 1) > U_x(X)$.

2. $C_x(\cdot) > 0$, $C_{xx}(\cdot) > 0$, $C_\sigma(\cdot) < 0$, $C_{x\sigma}(\cdot) < 0$, $\psi_\sigma(\sigma) > 0$ and $\underbrace{C_{\sigma\sigma}(\cdot) > 0 \ \psi_{\sigma\sigma}(\sigma) > 0}_{\text{Not too large}}$.

3. $\lim_{X \to 0} U_x(\cdot) = +\infty$, $\lim_{X \to \infty} U_x(\cdot) = 0$, $\lim_{x_i \to 0} C_x(\cdot) = 0$ and $\lim_{x_i \to \infty} C_x(\cdot) = +\infty$.

4 Analysis

I solve the model backwards to obtain the sub-game perfect Nash equilibrium (SPNE). I begin with the equilibrium of the trading game played in stage two. After describing the properties of the equilibrium trading allocation, I characterize a subset of the equilibrium transfers. Departing from the existing literature, I do not put any restrictions on the number of sellers submitting latent contracts and I obtain and characterize a subset of the equilibrium payoffs of Chiesa and Denicolò (2009). Later, I solve stage one of the game and obtain the equilibrium investment profile. Finally, I rank equilibria with regard to Pareto dominance and welfare.

4.1 Equilibrium trading allocation

The equilibrium allocation in the trading game depends on the investment undertaken at stage one. I then proceed to characterize the equilibrium allocation for a given vector of investment.

With the failure of the grand-contracting approach, potential inefficiencies may arise due to externalities among sellers. However, because the production cost of each seller depends only directly on the amount of input he produces, the trading contracts submitted by all other sellers do not directly affects his individual payoff. This is clear in the payoff equations (3.2) and (3.3) where the payoff of any seller i does not depend on the whole strategy profile of the buyer. Hence, the model does not have direct externalities, only contractual externalities among the sellers which arise from the buyer's marginal willingness to pay for the good, which depends on the total amount traded.[5]

Absent direct externalities, given the menu of trading contracts of the competing sellers, each seller effectively plays a bilateral trading game with the buyer where he has the whole bargaining power of the remaining potential surplus. When submitting a trading contract each seller i maximizes the potential gains from trade that can be generated between him and the buyer. For any quantity traded X_{-i} with the rest of the sellers

$$
\begin{aligned}
\Pi(\mathcal{M} \mid b) + \pi_i(\mathcal{M}_i \mid \sigma) &= U\left(X_{-i} + x_i^* \mid b\right) - \sum_{j \neq i} T_j - C(x_i^* \mid \cdot) \\
&> U\left(X_{-i} + \hat{x}_i \mid b\right) - \sum_{j \neq i} T_j - C(\hat{x}_i \mid \cdot); \text{ for any } \hat{x}_i \geq 0,\ \forall i \in N,
\end{aligned}
$$

and seller i does not profit by deviating from the efficient trading amount x_i^*. This holds true for every seller $i \in N$. This result derives from "*bilateral efficiency*" which fully characterize the equilibrium allocation of the game.[6] The efficient allocation constitutes a Nash equilibrium, defined

[5] Inefficient equilibria arise if the buyer has to purchase a pre-set total quantity as in Krishna & Tranaes (2002).

[6] For an exhaustive analysis see Bernheim & Whinston (1996) and Segal (1999).

by the system of equations

$$\begin{aligned} U_x(X^* \mid b) &= C_x(x_1^* \mid \sigma) \qquad \text{for } i = 1, \\ U_x(X^* \mid b) &= C_x(x_i^*) \qquad \quad \text{for } i \neq 1, \end{aligned} \tag{4.1}$$

where, for a given investment profile, the marginal utility of consumption equals the marginal costs of production. From the first-order condition, I obtain the following intuitive lemma

Lemma 1. *In the equilibrium trading allocation:*

i) for a given investment of the buyer, an increase on the investment by seller 1 *rises the amount of trade between the buyer and himself, but decreases the amount of trade with all other sellers. The total amount traded increases.*

$$\frac{dx_1^*}{d\sigma} > 0; \quad \frac{dx_j^*}{d\sigma} < 0 \ \text{ for all } \ j \neq 1 \quad \text{and} \quad \frac{\partial}{\partial \sigma} X^* > 0.$$

ii) For a given investment of seller 1*, the amount of trade by each seller increases with the investment of the buyer.*

$$x_i^*(1, \sigma) > x_i^*(0, \sigma) \qquad \forall i \in N.$$

The higher the investment undertaken by seller 1, the more efficient he becomes with respect to the other sellers and the buyer substitutes trading from any other sellers to seller 1. Yet, this substitution effect is of second order. Because the economy in aggregate becomes more efficient, the total amount of trade increases. For a given investment of the seller, the relative efficiency among sellers stays the same, and when the buyer invests she trades a larger amount with all the sellers.

The crowding-out effect that the investment of seller 1 has on the equilibrium quantity traded by the rest of the sellers is crucial at the investment stage. I then introduce the following definition

Definition 1. ***(Allocative sensitivity)*** *I call $dx_j^*/d\sigma$ for $j \neq 1$ the allocative sensitivity, corresponding to the crowding-out of the equilibrium trading allocation of sellers $j \neq 1$ from an increase* *of investment of seller* 1.

In the model, sellers produce completely homogeneous products. Nevertheless, the degree of product substitutability has a strong effect on the crowding-out effect that investment creates on the equilibrium allocation. With homogenous products, the buyer is able to substitute production from sellers and the allocative sensitivity is big. In my model, the degree of substitutability depends on the primitives of the economy.

4.2 Equilibrium transfers

The literature of markets and contracts establishes that the maximum transfer for any seller depends on the "threat" that the buyer excludes him from trade. This "threat" is directly related the latent trading contracts that competing sellers offer to the buyer. Latent contracts are out-of-equilibrium contracts never accepted by the buyer, yet they impose a constraint on the equilibrium transfer of the sellers. In the absence of any restrictions on these latent contracts, Chiesa & Denicolò (2009) show multiplicity in the equilibrium transfers and characterize the equilibrium strategies arising when only one of the sellers submits one of those latent contracts.

In my model, I characterize a sub-set of the equilibrium transfers from Chiesa & Denicolò (2009), by not imposing any restriction on the number of sellers who submit latent contracts. The "truthful" equilibrium, where each seller obtains his marginal contribution to the trading surplus, arises when all sellers in the market submit latent contract to compete after the exclusion of a seller.[7]

I begin the analysis with the following definition:

Definition 2. ***(Competing sellers)*** *A group of sellers $j \in J$ who offer latent contracts to compete after exclusion of a seller.*

When any seller i offers his equilibrium trading contract, $m_i^* = \{x_i^*, T_i^*\}$, he takes into consid-

[7] Truthful strategies are assumed in Bernheim & Whinston (1986), Bergemann & Välimäki (2003), Dixit, Grossman & Helpman (1997), Spence (1976) and Spulber (1979). A strategy is called to be "truthful" to a given action if it truly reflects the principal's marginal preference for another action relative to the given action. In a private common agency, truthfulness means that each principal can ask payments that differ from his true valuations of the proposed trades only by a constant.

eration how much the buyer is able to generate with the rest of the sellers. The outside option available to the buyer depends on the latent contracts offered by the competing sellers. When a set of sellers $J \subset N$ for $i \notin J$ submit latent contracts to compete after the exclusion of seller i, given the equilibrium contracts and the latent contracts of sellers $j' \in J$, by bilateral efficiency, the latent contract of seller j, $\tilde{m}_j = \{\tilde{x}_j, \tilde{T}_j\}$ for $j \in J$ and $j \neq j'$, must satisfy

$$\tilde{x}_j = \underset{x_j \geq 0}{\arg\max} \left[U \left(X^*_{-\{J_i,i\}} + \sum_{j' \in (J \setminus \{j\})} \tilde{x}_{j'} + x_j \,\middle|\, b \right) - C_j(x_j \mid \cdot) \right] \tag{4.2}$$

Hence, $\tilde{x}_j(b, \sigma \mid J)$ is the trading quantity that seller j submits in his latent contract. The amount of trade that sellers $j' \in J$ submit in their latent contracts are equally obtained. The following lemma compares the amount of trade in equilibrium against the one included in the latent contracts. The result is used later in the paper.

Lemma 2. *For any investment profile (b, σ) and a set of sellers in J, the aggregate trading quantity offered with the latent contracts is smaller than the aggregate equilibrium trading quantity*

$$X^*(b, \sigma) > X^*_{-\{J,i\}}(b, \sigma) + \sum_{j \in J} \tilde{x}_j(b, \sigma \mid J), \quad \textit{for any} \;\; J \subset N.$$

The individual trading quantity in the latent contract for any seller $j \in J$ is larger than their equilibrium trading quantity, and it is decreasing with the number of sellers in J.

$$\tilde{x}_j(b, \sigma \mid J') > \tilde{x}_j(b, \sigma \mid J) > x^*_j(b, \sigma); \quad \forall j \in J, J' \textit{ and } J' \subset J.$$

From the convexity of the cost function, the amount of trade with seller i is always larger than the increase in the quantity traded with the set of sellers in J. The individual trading quantity that any seller $j \in J$ submits with the latent trading contract is also bigger than his efficient quantity. Because latent trading contracts are aimed at excluding seller i, they have to offer a larger quantity of trade as compensation for the trade not realized from exclusion of seller i

Latent contracts characterize the available outside option of the buyer after excluding any seller i from trade, and this is analytically given by:

$$V_J\left(X^*_{-\{J,i\}} \mid b,\sigma\right) = \left[U\left(X^*_{-\{J,i\}} + \sum_{j\in J}\tilde{x}_j \middle| x_i = 0, b\right) - \sum_{j\in J} C_j(\tilde{x}_j \mid \cdot)\right]. \tag{4.3}$$

To obtain the equilibrium transfers, any equilibrium in a common agency game must satisfy "*individual excludability*". The buyer excludes one given seller and still obtain her equilibrium payoffs. Accordingly, for a given investment profile (b,σ) and latent trading contracts $\tilde{m}_j = \{\tilde{x}_j(b,\sigma \mid J), \tilde{T}_j\}$ for $j \in J$,

$$U(X^* \mid b) - \sum_i T_i^* = U\left(X^*_{-\{J,i\}} + \sum_{j\in J}\tilde{x}_j(J) \;\middle|\; b\right) - \sum_{j\in N\setminus\{J,i\}} T_j^* - \sum_{j\in J}\tilde{T}_j \tag{4.4}$$

The left hand side represents the equilibrium payoff of the buyer. The right hand side stands for the payoff of the buyer from excluding seller i, and accepting the latent trading contracts from sellers $j \in J$. For ease of notation, I have neglected the vector of investment on the trading allocation.

In equilibrium, the set of sellers in $j \in J$ have to be indifferent between supplying their equilibrium offers and the latent trading contracts

$$T_j^* - C_j(x_j^*) = \tilde{T}_j - C_j(\tilde{x}_j) \quad \text{for } j \in J,$$

and summing over the number of sellers belonging to the set J,

$$\sum_{j\in J}\left[T_j^* - C_j(x_j^*)\right] = \sum_{j\in J}\left[\tilde{T}_j - C_j(\tilde{x}_j)\right]. \tag{4.5}$$

Combining expression (4.5) and (4.4), results into the equilibrium transfer

$$T_i^*(b,\sigma \mid J) = U(X^* \mid b) - U\left(X^*_{-\{J,i\}} + \sum_{j\in J}\tilde{x}_j(J) \,\middle|\, b\right) + \sum_{j\in J}\left[C_j(\tilde{x}_j(J)) - C_j(x_j^*)\right]$$
$$= \left(U(X^* \mid b) - \sum_{j\in J} C_j(x_j^* \mid \cdot)\right) - V_J\left(X^*_{-\{J,i\}} \mid b,\sigma\right).$$

The convexity of the cost function makes the equilibrium transfer $T_i^*(J)$ weakly decreasing in the set $J : J \supseteq J' \implies T_i^*(J') \geq T_i^*(J)$.[8] The more sellers submitting latent trading contracts, the larger the trading surplus they generate with the buyer. The outside option available to the buyer increases and the bargaining position of the seller decrease.

The following proposition, states the equilibrium payoffs in the trading game.

Proposition 1. *i) For a given set of sellers in J and an investment profile (b,σ), the sellers' equilibrium payoffs are*

$$\pi_1(b,\sigma \mid J) = \underbrace{TS^*(b,\sigma) - \tilde{TS}_{-1}(b \mid J)}_{\textit{Contribution to the surplus}} - \psi(\sigma); \quad \textit{for } i = 1, \tag{4.6}$$

$$\pi_i(b,\sigma \mid J) = TS^*(b,\sigma) - \tilde{TS}_{-i}(b,\sigma \mid J); \quad \textit{for } i \neq 1. \tag{4.7}$$

The equilibrium payoff of the buyer is

$$\Pi(b,\sigma \mid J) = TS^*(b,\sigma) - \sum_i \left(TS^*(b,\sigma) - \tilde{TS}_{-i}(b,\sigma \mid J)\right) - K \times b, \tag{4.8}$$

where $\tilde{TS}_{-i}(b,\sigma \mid J)$ is the maximal trading surplus that can be generated without seller i and a set of sellers in J submitting latent trading contracts.

ii) $\tilde{TS}_{-i}(b,\sigma \mid J) > \tilde{TS}_{-i}(b,\sigma \mid J')$ for $J' \subset J$. Moreover, for $J \subset N \setminus \{i\}$ each seller obtains more than his marginal contribution of the trading surplus.

In equilibrium each seller obtain his contribution to the surplus, relating to the loss of the trading surplus originated from his exclusion. Seller's loss from exclusion comes from buyer's available

[8] In general the inequality is strict if J' is not equal to J.

outside option, determined by the degree of competition in the trading contracts. The most competitive equilibrium arises when all competing sellers submit latent trading contracts to compete after the exclusion of a given seller i, i.e., $J = N \setminus \{i\}$, for all $i \in N$. Each seller's bargaining position is minimized, obtaining only his marginal contribution to the surplus. In this equilibrium, the trading gains stay evenly distributed to all players.

In an equilibrium where a lower number of sellers submit latent trading contracts, i.e., $J \subset N \setminus \{i\}$, the equilibrium outcome becomes less competitive. The bargaining position of the sellers increase; each seller appropriates more than his marginal contribution to the surplus. The distribution of the gains from trade favors the sellers in detriment of the buyer. I proceed to state the notion of "intensive" competition.

Definition 3. ***(Competition)*** *An equilibrium outcome is more competitive the lower the bargaining position of the sellers. A more competitive equilibrium implies a larger number of sellers in set J.*

Example 1. *Consider an equilibrium of the trading game where three sellers constitute the set of sellers who submit latent trading contracts. For any seller $i = 4, ..., N$, the set of rival sellers who submit latent trading contracts to compete after exclusion of seller i is $J_i = \{1, 2, 3\}$. Without loss of generality, I consider seller $i = 4$ the one who also submits a latent trading contract to compete after exclusion of any seller belonging to the set J_i. Therefore, in this equilibrium, the sets for sellers offering latent trading contracts are $J_1 = \{2, 3, 4\}$, $J_2 = \{1, 3, 4\}$, $J_3 = \{1, 2, 4\}$, and $J_i = \{1, 2, 3\}$ for $i = 4, ..., N$. While the cardinality of the set is three, the identity of the sellers submitting latent trading contracts is $i = 1, 2, 3, 4$.*

4.3 Investment profile

I begin characterizing the efficient investment profile and I proceed with equilibrium. Efficient investment serves as a benchmark to allow comparisons with equilibrium. The decision to invest depends on the competitiveness of the equilibrium outcome, affecting the bargaining position of the sellers and the gains that the investing parties appropriate.

4.3.1 Efficient investment

The efficient vector of investment maximizes welfare: trading surplus minus investment costs. Buyer and seller 1 invest efficiently when they appropriate all the gains coming from investment. The solution of this system of equations uniquely characterizes the efficient investment:

$$\psi_\sigma(\sigma_{\mathbf{E}}) = -C_\sigma\left(x_1^*(b, \sigma_{\mathbf{E}}^b) \mid \sigma_{\mathbf{E}}^b\right), \qquad \forall\, b; \tag{4.9}$$

$$K \begin{cases} \leq TS^*(1, \sigma_{\mathbf{E}}^1) - TS^*(0, \sigma_{\mathbf{E}}^0) - \left(\psi(\sigma_{\mathbf{E}}^1) - \psi(\sigma_{\mathbf{E}}^0)\right) \equiv \hat{K}_{\mathbf{E}} & \text{then } b = 1 \\ > \hat{K}_{\mathbf{E}} & \text{then } b = 0, \end{cases} \tag{4.10}$$

where, underscripts on functions stand for partial derivatives, and the upperscript on the investment of seller 1 represents the investment of the buyer. Accordingly, $\sigma_{\mathbf{E}}^1$ stands for the efficient investment of the seller when the buyer invests in equilibrium, $b = 1$.

Seller 1 invests until the marginal reduction on his production costs equals his marginal cost of investment. Similarly, the buyer invests if the fixed cost of investment K stays lower than the increase on welfare arising from her investment, represented by the threshold $\hat{K}_{\mathbf{E}}$. A characteristic of the efficient investment profile, that carries over in equilibrium, is the strategic complementarity of investments. The more one party invests, the higher the incentives of the other party to increase investment. This comes from a variant of super-modularity. Lemma 1 demonstrates that the investment of one party always increases the total amount of trade, and through trade increase, the value of investment from one party increases the marginal return of the other's party investment. The assumption on the cost functions, guarantees a unique investment profile.

4.3.2 Equilibrium investment

Sellers' bargaining position and the subsequent partition of the profits from investment determine the incentives to invest in equilibrium. In the analysis, I consider both the "intensive" and "extensive" degree of competition. "Intensive" competition takes into account how many sellers submit latent trading contracts. "Extensive" competition considers how the number of active sellers the buyer establishes trade with alters investment incentives.

4.3.3 Intensive competition

Equilibrium investment decisions are best-response actions. The following definition states an equilibrium in the investing game.

Definition 4. *The pair of investments* (b^e_J, σ^e_J) *constitutes an equilibrium, if and only if:*

$$b^e_J \in \underset{b\in\{0,1\}}{\arg\max}\ \Pi\left(b, \sigma^e_J \mid J\right),$$
$$\sigma^e_{J_1} \in \underset{\sigma\geq 0}{\arg\max}\ \pi_1\left(b^e_J, \sigma \mid J_1\right).$$

Equilibrium payoffs depend on the number of competing sellers submitting latent trading contracts, and investment decisions reflect each parties' appropriation of the gains originated from investment. When the outcome of the trading game becomes the most competitive, each seller obtains his marginal contribution of the trading surplus. Seller's 1 investment does not affect the outside option available to the buyer; competing sellers generate out-of equilibrium constant gains from trade. Seller 1 exclusively appropriates the increase of the trading surplus originated from investment and invests efficiently. Because in the most competitive equilibrium sellers' bargaining position is minimized, the set of parameters where the buyer invests efficiently is large. Full investment efficiency cannot be implemented with less competitive trading outcomes. When sellers' bargaining position increases, each seller obtains more than his marginal contribution to the surplus, distorting the incentives to invest efficiently. This discussion introduces the following proposition:

Proposition 2. *The efficient investment profile is implementable if and only if the outcome of the trading game is the most competitive, i.e.,* $J_i = N \setminus \{i\}$ *for all* $i \in N$.

Conditional on the buyer taking the efficient investment decision, proposition 2 states existence of the efficient investment profile. Corollary 1 presents the result when the buyer fails to invest efficiently:

Corollary 1. *When buyer's investment fails to be efficient in the most competitive equilibrium, underinvestment emerges to both sides of the market.*

Corollary 1 characterizes the "hold-up" problem. Both, seller 1 and the buyer underinvest in equilibrium. While seller 1 takes efficient unilateral decisions, underinvestment arises from investments' strategic complementarity. Buyer's failure to invest reduces potential gains from trade, translating into lower investment incentives for seller 1.

Next proposition states the investment of seller 1 when the equilibrium outcome is not the most competitive:

Proposition 3. *When the trading outcome is not the most competitive, $J_i \subset N \setminus \{i\}$ for all $i \in N$, for a given investment of the buyer, the magnitude of seller 1 over-investment*

$$\gamma(J_1) = - \sum_{m \notin \{J_1,1\}} \left(\int_{X^*}^{X^*_{-\{J_1,1\}} + \sum_{j \in J_1} \tilde{x}_j(J_1)} U_{xx}(\tau) d\tau \right) \frac{dx^*_m}{d\sigma}$$

depends on the level of competition and the allocative sensitivity $dx^*_m/d\sigma$. Over-investment decreases with competition, i.e., $\gamma(J_1') > \gamma(J_1)$ for $J_1' \subset J_1$.

Seller 1 investment is distorted upwards. The investment from seller 1 affects the outside option available to the buyer, generating an endogenous bargaining position that grows with investment. Seller's 1 investment crowds out the equilibrium allocation of competing sellers, diminishing the amount of trade from the sellers not submitting latent trading contracts. Trade reductions constraint the gains from trade that can be generated out-of-equilibrium, reducing the outside option available to the buyer. The higher is the allocative sensitivity, the larger is seller's 1 bargaining position, increasing his incentives to invest. Hence, seller 1 effectively appropriates all the direct gains from his investment and part of the payoffs from the competing sellers.

With regard to the equilibrium investment profile, when the equilibrium outcome is not the most competitive, the next corollary states that investment inefficiencies may arise to both sides of the market.

Corollary 2. *When the buyer invest efficiently, seller* 1 *over-invests.*

i) When buyer's investment is not efficient, investment inefficiencies are two-sided:

A) the buyers underinvests, and

B) seller 1 *over or underinvest depending on the magnitude of the allocative sensitivity. Overinvestment arises in equilibrium if*

$$-\frac{dx_m^*}{d\sigma} > \frac{\int_{x_1^*(1,\sigma_E^1)}^{x_1^*(0,\sigma_E^0)} C_{x\sigma}(\tau)d\tau}{(N \setminus \{J_1, 1\}) \times \int_{X^*(0,\sigma_{J_1}^0)}^{X^*_{-\{J_1,1\}}(0,\sigma_{J_1}^0)+\sum_{j\in J}\tilde{x}_j(0,\sigma_{J_1}^0|J_1)} U_{xx}(\tau)d\tau} = \lambda(J_1).$$

Results state how the equilibrium investment profile depends on competition over trading contracts. To compare equilibrium investment profiles, I explicitate buyer's investment decision. The buyer invests if the gains obtained from her investment

$$\hat{K}(J) \equiv TS^*(1, \sigma^1(J)) - TS^*(0, \sigma^0(J)) - \sum_{i\in N} \left(T_i^1(J) - T_i^0(J)\right),$$

are larger than her fixed investment costs K. The first part of the right hand side stand for the surplus gains coming from buyer's investment. The second part represent changes on the sellers' equilibrium transfers.

With a fixed investment of seller 1, buyer's investment threshold increases with competition. Sellers' bargaining position shrinks, and a larger portion of the trading surplus goes to the buyer. Investment complementarity counterbalances buyer's reduced bargaining position arising from a less competitive equilibrium. Larger seller's investments diminishes equilibrium transfers of the competing sellers, benefiting the buyer. The countervailing effect is of second order with a small allocative sensitivity. Increases on seller's investment from reduced competition is limited, and the buyer has higher incentives to invest with more competitive equilibria. Results change with a large allocative sensitivity. Lower competitive equilibria boost seller's investment, reducing the bargaining position of competing seller with respect to the buyer. Because seller 1 investment makes him more efficient, he offers large out-of-equilibrium trading quantities, constraining the equilibrium transfers of the competing sellers. Larger buyer's investment incentives may arise in low competitive equilibria. Results are summarized in the lemma:

Lemma 3. *The evolution of buyer's investment threshold $\hat{K}(J)$ with competition depends on the magnitude of the allocative sensitivity.*

a) With a small allocative sensitivity, the buyer's investment threshold monotonically increases with competition, i.e., $\hat{K}(J) > \hat{K}(J')$ for $J' \subset J$.

b) With a large allocative sensitivity, buyer's investment threshold fails to be monotone with competition.

To illustrate the results, I graphically represent the equilibrium investment profile as a function of competition. Points further away from the origin of the horizontal axis represent higher competitive equilibria.[9] On the upper part of figure 2, pictures a) and b) represent the equilibrium investment for seller 1. The level of competition and the degree of allocative sensitivity determine seller's 1 investment. With a small allocative sensitivity, picture a), seller's investment is less sensitive to competition than with a large allocative sensitivity, picture b). Seller's 1 bargaining position increases with the allocative sensitivity giving him more incentives to invest. Discontinuous jumps on seller's investment decisions arise from buyer's investment. Picture c), shows a monotone investment threshold of the buyer. Investment complementarity explains the discreet jump downwards on the investment of the seller represented in picture a). Picture d) represents a non-monotone buyer's investment threshold, where intermediate levels of competition refrain her from investing. With lower competitive equilibria, seller's investment effect kicks-in, and the buyer has higher incentives to invest.

[9] The figure aims at giving a simple illustration of the results and the lines represented do not stand for computed equilibrium.

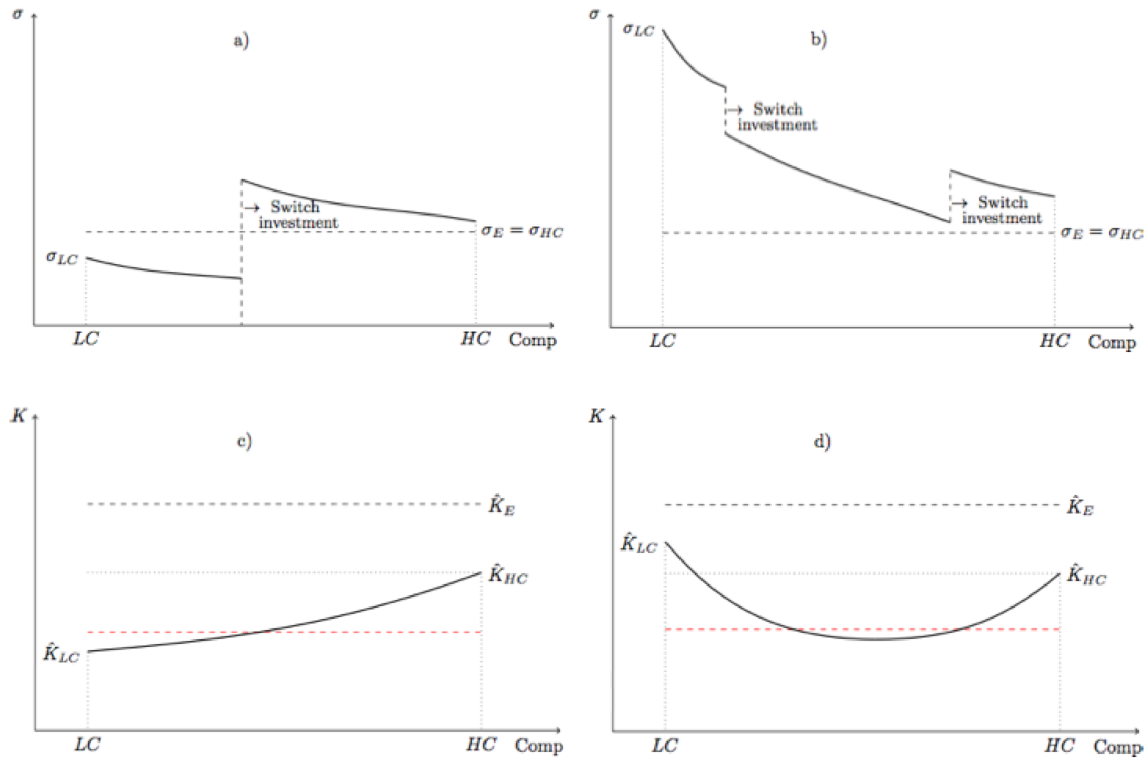

Figure 2: Equilibrium investment profile of seller 1 and buyer depending on competition. Buyer's fixed cost of investment is represented by the dashed red line in pictures c) and d). Left figures represent moderate allocative sensitivity. Right pictures stand for a large allocative sensitivity.

4.3.4 Extensive competition

Competition on the trading contracts determine the outside option available to the buyer. Yet, the number of sellers the buyer establishes trade with affects competition. After exclusion of a given seller, the outside option available to the buyer increases with the number of trading partners. The trading quantity from the excluded seller can be easily substituted away with the rest of the sellers. Larger buyer's available outside option decreases sellers' bargaining position, affecting the incentives to invest.

Figure 3 illustrates unilateral investment decisions as a function of the number of trading partners. With one seller, bilateral monopoly takes place. The seller has the whole bargaining power "holding-up" the buyer completely. While seller's investment is efficient, the buyer never invests. A positive buyer's investment threshold occurs with more than one seller. Sellers compete for the

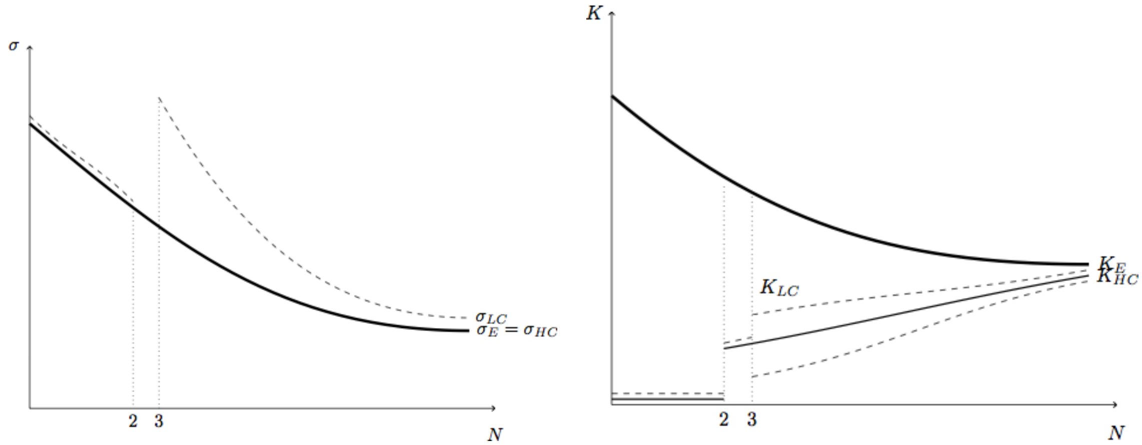

Figure 3: Unilateral investment decisions as function of number of sellers. Seller 1 unilateral investment decision stand on the left picture. The right picture represents the buyer's investment threshold. The thick solid line stands efficiency, the solid line represents the most competitive equilibrium and the dashed line the least competitive equilibrium.

trading contracts and the buyer appropriates part of the benefits coming from investment. With a larger number of sellers, competition intensifies. For a given level of competition on the trading contracts, each sellers' bargaining position reduces. When the number of sellers tends to infinite, each seller only appropriates his marginal contribution to the surplus. The buyer also takes the efficient investment decision. In the limit, she appropriates the whole gains coming from her investment, and invests efficiently no matter her fixed costs of investment. The following proposition states the result.

Proposition 4. *Full efficiency is implemented when the number of sellers tends to infinity.*

5 Comparison of equilibria

I start by examining which equilibria gives larger payoffs to the sellers. Later, departing from surplus distribution, I compare equilibria in terms of welfare.

5.1 Pareto optimality

With a given investment profile, sellers prefer situations with less competitive outcomes. Chiesa & Denicolò (2009, 2012), state that the equilibrium minimizing the rent of the buyer is Pareto dominant for the sellers. Results differ in my model. Investment decisions depend on the level of competition and sellers are heterogenous on their production costs. For seller 1, a larger bargaining position may affect the investing decision of the buyer. For the rest of sellers, in addition to buyer's incentives to invest, the investment of seller 1 determines their equilibrium trading allocation affecting their equilibrium payoffs. The proposition presents the result:

Proposition 5. *With a monotone buyer's investment threshold:*

i) the least competitive equilibrium is Pareto dominant for the sellers if the investment decision of the buyer is equilibrium invariant,

ii) otherwise, Pareto dominance is attained with an intermediate level of competition.

When buyer's investment threshold fails to be monotone, the least competitive equilibrium is never Pareto dominant for the sellers,

iii) while this is the most preferred for seller 1*,*

iv) the rest of sellers prefer a more competitive trading outcome.

Figure 4 and 5 interpret the results graphically. Points further away from the origin of the horizontal axis represent higher competitive equilibria. In Figure 4, the allocative sensitivity is small, and buyer's incentive to invest increase with competition. Switches of buyer's investment can only

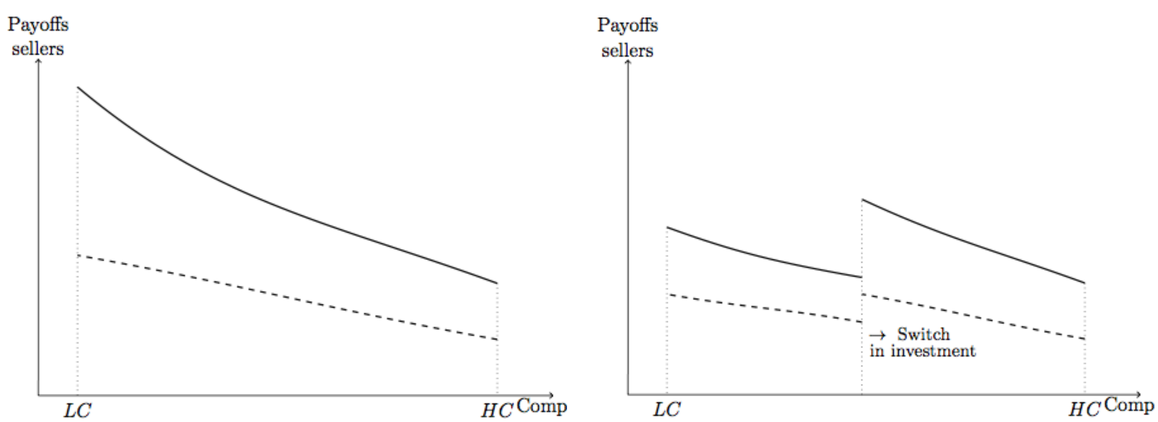

Figure 4: Sellers' payoff as a function of competition with a small allocative sensitivity. The black line represents the payoff of seller 1 and the dashed line stands for the non-investing sellers.

happen in low competitive equilibria where she may decide not to invest. Sellers' preferences over competition are aligned. More favorable surplus partitions dominate changes on the equilibrium allocation emerging from a larger seller's investment.

Figure 5 represents a large allocative sensitivity. An increase in the level of investment from seller 1 generates a significant reduction in the equilibrium trading allocation for the competing sellers. Preferences over competitive outcome are not aligned. Larger discrepancies emerge from buyer's investment, whose investment incentives decrease with competition.

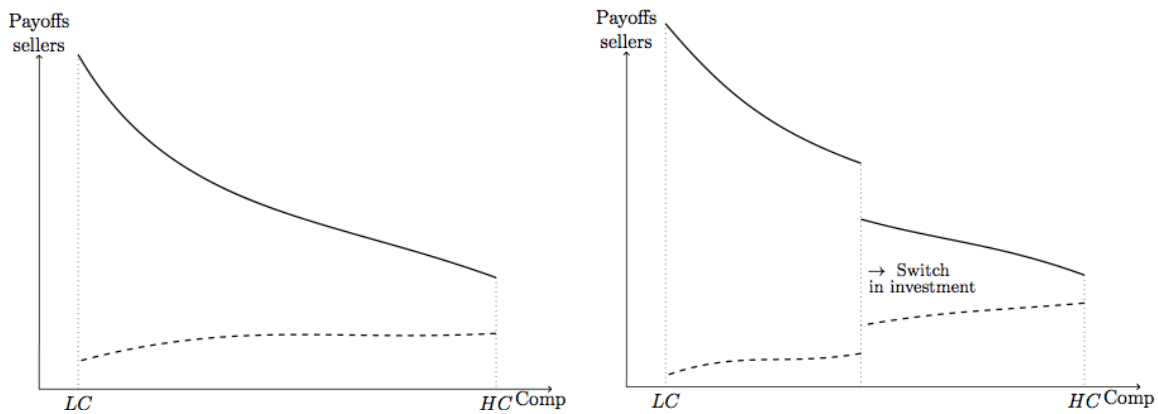

Figure 5: Sellers' payoff as a function of competition with a big allocative sensitivity. The black line represents the payoff of seller 1 and the dashed line stands for the non-investing sellers..

5.2 Welfare

I departs from distributional issues and rank equilibria according to welfare. Welfare equals to the trading surplus minus the costs of investment

$$W^*(b,\sigma) = TS^*(b,\sigma) - K \times b - \psi(\sigma).$$

Proposition 2 and corollary 2 state that ex-ante inefficiencies are more prompt to emerge with mild competition for trading contracts. The most competitive equilibrium, in general, generates larger welfare. Yet, investments decisions are strategic complements. A less competitive equilibrium may bring larger welfare if investment inefficiencies created to one side of the market restore the efficient investment decision of the other side. This happen only when seller's 1 investment inefficiencies are of second order compared to the gains in welfare emerging from buyer's investment. The following proposition states the result.

Proposition 6. *When in the most competitive equilibrium, the investment decision of the buyer is not efficient and buyer's investment threshold is not monotone with competition, welfare may be maximized with an intermediate level of competition. Otherwise, the largest welfare is obtained with the highest level of competition.*

Figure 6 graphically interprets the result. Welfare monotonically increases with competition when the investment of the buyer is equilibrium invariant. Inefficiencies only emerge from seller's investment and they are larger the milder the competition on trading contracts become. Welfare presents jumps when the investment decision of the buyer depends on competition. Large welfare may be attained with an intermediate level of competition only if the decision of the buyer switches from non-investing to investing with less competitive equilibria. The left picture in figure 6 illustrates this case. When the buyer takes the efficient investment decision in the most competitive equilibrium, any reduction on the level of competition translates into a lower level of welfare.

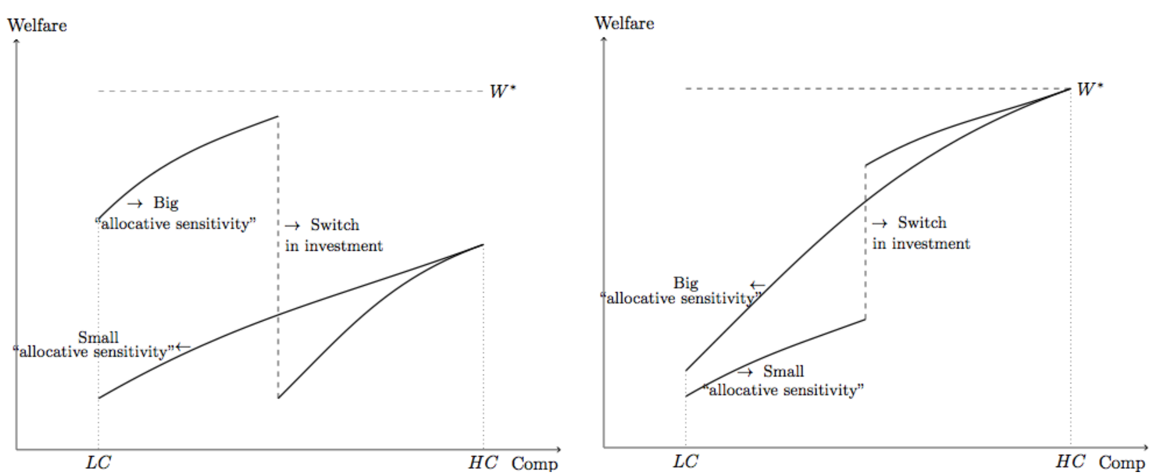

Figure 6: Welfare as function of competition in the trading outcome. The figure on the right illustrates the implementation of efficient investment.

6 Conclusion

Introducing competition to the side of the market offering trading contracts mitigates the "hold-up" problem without the introduction of ex-ante contracts. Full ex-ante efficiency is achieved when sellers compete fiercely with their trading contracts. Seller's bargaining position minimizes when all sellers submit latent trading contracts. In this equilibrium, each seller appropriates his marginal contribution to the surplus and investment decisions do not alter the outside option available to the buyer. In any other equilibrium, buyer's available outside option depends on seller's 1 investment, distorting the incentives to invest efficiently.

The equilibrium played in the trading game not only redistributes rents between sellers and the buyer, but also determines the potential gains from trade. Previous analysis state that a very competitive equilibrium is not attractive for the part of the market offering trading contracts. Tacit coordination can be achieved to reduce competition and obtain more favorable surplus partitions. This paper demonstrates that a higher competitive equilibrium generally displays more efficient investment and larger welfare. The result depends on the crowding-out effect that seller's investment has on the equilibrium allocation of the competing sellers.

Results are robust to different model settings. An extra layer of complexity emerges when all sellers can reduce their production cost with investment. The investment decisions of the buyer among the sellers remains strategic complements, while the investment decisions among sellers become strategic substitutes. Strategic substitutability among sellers' investment is of second order. Investment complementarity emerges and full efficiency is still implementable in the most competitive equilibrium. An environment without a monopolistic buyer, where sellers can sign multiple bilateral contracts with different buyers, is harder to study. A buyer differentiates and creates an indirect externality to the other buyers by investing. Despite the complexity of the equilibrium trading contracts, I conjecture that the competitive advantage induces the buyer to over-invest.

Several articles, Bernheim & Whinston (1986), Martimort & Stole (2009) and Klemperer & Meyer (1989) address the question of equilibrium selection. In this article the selection question of equilibrium selection is of great importance due to welfare effects. More work must be done on equilibrium refinement. In the present work, I select equilibria from Chiesa & Denicolò (2009) by not restricting the number of competing sellers who can submit latent trading contracts. If ex-ante investment decisions works as a mechanism for equilibrium selection is left for a topic of future research.

References

[1] Aghion, P. and P. Bolton. (1987): "Contracts as a Barrier to Entry", *American Economic Review*, 77(3), 388-400.

[2] Aghion, P., M. Dewatripont, and P. Rey, (1994): "Renegotiation Design with Unverifiable Information", *Econometrica*, 62, 257-282.

[3] Akerlof, G. (1970): "The Market for 'Lemons': Quality Uncertainty and the Market Mechanism", *The Quarterly Journal of Economics*, vol. 84, pp 488-500.

[4] Bernheim, B., and M. Whinston (1986): "Common Agency", *Econometrica*, Vol. 54, No. 4, 923-942.

[5] ——— (1986): "Menu Auctions, Resource Allocation and Economic Influence", *Quarterly Journal of Economics*, 101, 1-31.

[6] Biglasier, G., and C. Mezzetti (1993): "Principals Competing for an Agent in the Presence of Adverse Selection and Moral Hazard", *Journal of Economic Theory*, No. 61, 302-330.

[7] Chiesa, G., and V. Denicolò (2009): "Trading with a Common Agent under Complete Information: A Characterization of Nash Equilibria", *Journal of Economic Theory*, 144, 296-311.

[8] ——— (2012): "Competition in Non-linear Pricing, Market Concentration and Mergers", *Economic Letters*, 117, 414-417.

[9] Chung, P. (1991): "Incomplete Contracts, Specific Investment and Risk Sharing", *Review of Economics Studies*, 58, 1031-1042.

[10] Cole, H. L., G. J. Mailath, and A. Postlewaite (2001a): "Efficient Non-Contractible Investments in Large Economies", *Journal of Economic Theory*, 101, 333-373.

[11] ——— (2001b): "Efficient Non-Contractible Investments in Finite Economies," *Advances in Theoretical Economics* 1, Iss. 1, Article 2.

[12] Edlin, A., and S. Reichelstein, (1996): "Hold-ups, Standard Brach Remedies, and Optimal Investment", *The American Economic Review*, 86, 478-282.

[13] Felli, L., and K. Roberts (2012): "Does Competition Solve the Hold-Up Problem?" *Theoretical Economics Discussion Paper TE/01/414, STICERD*, London School of Economics.

[14] Grossman, S., and O. Hart (1986): "The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration", *The Journal of Political Economy*, 94, 691-719.

[15] Hart, O., and J. Moore (1999): "Foundations of Incomplete Contracts", *Review of Economics Studies*, 66, 115-138.

[16] Klein, B., R. Crawford, and A. Alchian (1978): "Vertical Integration, Appropriable Rents and the Competitive Contracting Process", *Journal of Law and Economics*, 21, 297-326.

[17] Klemperer, P and M. Meyer (1989): "Supply Function Equilibria in Oligopoly with Demand Uncertainty", *Econometrica* 57, 1243-1277.

[18] Mailath, G. J., Postlewaite, A. and L. Samuelson (2013): "Permutation Values and Investments in Matching Markets".

[19] Makowski, L. (2004): "Pre-Contractual Investment Without The Fear Of Holdups: The Perfect Competition Connection", Unpublished working paper.

[20] Martimort, D. and L. Stole (2011): "Selecting Equilibria in Common Agency Games", *Journal of Economic Theory* Volume 144, 604-634.

[21] McLeod, W. B., and J. M. Malcomson (1993): "Investments, Holdup and the Form of Markets Contracts", *American Economic Association*, 83, 811-837.

[22] Nöldeke, G., and L. Schmidt (1995): "Option Contracts and Renegotiation: a Solution to the Hold-up Problem", *RAND Journal of Economics*, 26, 163-179.

[23] Segal, I. (1999): "Contracting with Externalities", *The Quarterly Journal of Economics*, 114, 337-388.

[24] Schmitz, P.W. (2010): "The Hold-up Problem and Incomplete Contracts: A Survey of Recent Topics in Contract Theory".

[25] Williamson, O. (1979): "The Transaction-Cost Economics: the Governance of Contractual relations", *Journal of Law and Economics*, 22, 233-261.

[26] ——— (1983): "Credible Commitment: Using Hostages to Support Exchange", *The American Economic Review*, 73, 519-540.

A Appendix

Lemma 3. *The total gains from trade are larger with a higher investment of seller 1, that is,* $TS^*(b, \sigma\prime) > TS^*(b, \sigma)$ *for* $\sigma\prime > \sigma$ *and any* b.

Proof. I consider the case where $b = 0$ but the case where $b = 1$ is analogous.

$$
\begin{aligned}
TS^*(0, \sigma) &= U(X^*(0, \sigma)) - C(x_1^*(0, \sigma) \mid \sigma) - \sum_{i \neq 1} C_i(x_i^*(0, \sigma)) \\
&< U(X^*(0, \sigma)) - C(x_1^*(0, \sigma) \mid \sigma\prime) - \sum_{i \neq 1} C_i(x_i^*(0, \sigma)) + U(X^*(0, \sigma\prime)) - U(X^*(0, \sigma\prime)) \\
&< U(X^*(0, \sigma)) - U(X^*(0, \sigma\prime)) + TS^*(0, \sigma\prime) \\
\implies \quad & TS^*(0, \sigma\prime) - TS^*(0, \sigma) > U(X^*(0, \sigma\prime)) - U(X^*(0, \sigma)) = \int_{X^*(0,\sigma)}^{X^*(0,\sigma\prime)} U_x(\tau) d\tau > 0,
\end{aligned}
$$

where the first strict inequality comes from the lower cost of production due to larger investment and the second from efficiency. The strict inequality in the last line is due to lemma 1, where I showed that $X^*(0, \sigma\prime) > X^*(0, \sigma)$ for any $\sigma\prime > \sigma$. □

Lemma 4. *Total gains from trade increase with buyer's investment, i.e,* $TS^*(1, \sigma^1) > TS^*(0, \sigma^0)$.

Proof. This only states that the potential gains from trade are larger with larger amounts of investment.

$$
\begin{aligned}
TS^*(1, \sigma^1) &= U(X^*(1, \sigma^1) \mid b = 1) - C(x_1^*(1, \sigma^1) \mid \sigma^1) - \sum_{i \neq 1} C_i(x_i^*(1, \sigma^1)) \\
&= U(X^*(1, \sigma^1) \mid b = 1) - U(X^*(1, \sigma^1)) + U(X^*(1, \sigma^1)) - C(x_1^*(1, \sigma^1) \mid \sigma^1) - \sum_{i \neq 1} C_i(x_i^*(1, \sigma^1)) \\
&\geq U(X^*(1, \sigma^1) \mid b = 1) - U(X^*(1, \sigma^1)) + TS^*(0, \sigma^0) \\
\implies \quad & TS^*(1, \sigma^1) - TS^*(0, \sigma^0) \geq U(X^*(1, \sigma^1) \mid b = 1) - U(X^*(1, \sigma^1)) > 0.
\end{aligned}
$$

The first inequality comes from lemma 3 and the last strict inequality comes by the assumption that $U(X^* \mid b = 1) - U(X^*) > 0$. □

Lemma 5. *The increase on the total gains from trade by any non-investing seller are higher when the buyer is investing and the level of competition in the trading game is the largest, i.e.,* $\bar{J}$:

$$
TS^*(1, \sigma^1) - \tilde{TS}_{-i}(1, \sigma^1 \mid \bar{J}_1) \geq TS^*(0, \sigma^0) - \tilde{TS}_{-i}(0, \sigma^0 \mid \bar{J}_1) \quad for \quad i \neq 1.
$$

Proof. Here I state that the contribution that a seller have on the trading surplus is larger if the buyer invests. I will make explicit use of lemma 4. Observe that the previous expression is equivalent to $TS^*(1,\sigma^1) - TS^*(0,\sigma^0) \geq \tilde{TS}_{-i}(1,\sigma^1 \mid \bar{J}_1) - \tilde{TS}_{-i}(0,\sigma^0 \mid \bar{J}_1)$ and by lemma 4 I know that the lower bound of the expression on the left is $\underline{D} = U(X^*(1,\sigma^1) \mid b=1) - U(X^*(1,\sigma^1))$. I proceed by obtaining the upper bound of the difference $\tilde{TS}_{-i}(1,\sigma^1 \mid \bar{J}_1) - \tilde{TS}_{-i}(0,\sigma^0 \mid \bar{J}_1)$.

$$
\begin{aligned}
&\tilde{TS}_{-i}(1,\sigma^1 \mid \bar{J}_1) = U\left(\sum_{j\neq 1} \tilde{x}_j(1,\sigma^1 \mid \bar{J}) \mid b=1\right) - C\left(\tilde{x}_1(1,\sigma^1 \mid \bar{J}) \mid \sigma^1\right) - \sum_{j\neq i,1} C_j\left(\tilde{x}_j(1,\sigma^1 \mid \bar{J})\right) \\
&\leq U\left(\sum_{j\neq i} \tilde{x}_j(1,\sigma^1 \mid \bar{J}) \mid b=1\right) - C\left(\tilde{x}_1(1,\sigma^0 \mid \bar{J}) \mid \sigma^0\right) - \sum_{j\neq i,1} C\left(\tilde{x}_j(0,\sigma^0 \mid \bar{J})\right) \\
&= U\left(\sum_{j\neq i} \tilde{x}_j(1,\sigma^1 \mid \bar{J}) \mid b=1\right) - U\left(\sum_{j\neq i} \tilde{x}_j(1,\sigma^1 \mid \bar{J})\right) + U\left(\sum_{j\neq i} \tilde{x}_j(1,\sigma^1 \mid \bar{J})\right) \\
&- C\left(\tilde{x}_1(1,\sigma^0 \mid \bar{J}) \mid \sigma^0\right) - \sum_{j\neq i,1} C_j\left(\tilde{x}_j(0,\sigma^0 \mid \bar{J})\right) \\
&\leq U\left(\sum_{j\neq i} \tilde{x}_j(1,\sigma^1 \mid \bar{J}) \mid b=1\right) - U\left(\sum_{j\neq i} \tilde{x}_j(1,\sigma^1 \mid \bar{J})\right) + \tilde{TS}_{-i}(0,\sigma^0 \mid \bar{J}_1) \\
&\implies \quad \tilde{TS}_{-i}(1,\sigma^1 \mid \bar{J}_1) - \tilde{TS}_{-i}(0,\sigma^0 \mid \bar{J}_1) \leq U\left(\sum_{j\neq i} \tilde{x}_j(1,\sigma^1 \mid \bar{J}) \mid b=1\right) - U\left(\sum_{j\neq i} \tilde{x}_j(1,\sigma^1 \mid \bar{J})\right) = \overline{D}.
\end{aligned}
$$

Where the inequalities comes from efficiency. I proceed to show that the difference between the lower and the upper bound is positive $\underline{D} - \overline{D} > 0$ because

$$
\begin{aligned}
\underline{D} - \overline{D} &= U(X^*(1,\sigma^1) \mid b=1) - U(X^*(1,\sigma^1)) - \left[U\left(\sum_{j\neq i} \tilde{x}_j(1,\sigma^1 \mid \bar{J}) \mid b=1\right) - U\left(\sum_{j\neq i} \tilde{x}_j(1,\sigma^1 \mid \bar{J})\right)\right] \\
&= U(X^*(1,\sigma^1) \mid b=1) - U\left(\sum_{j\neq i} \tilde{x}_j(1,\sigma^1 \mid \bar{J}) \mid b=1\right) - \left[U(X^*(1,\sigma^1)) - U\left(\sum_{j\neq i} \tilde{x}_j(1,\sigma^1 \mid \bar{J})\right)\right] \\
&= \int_{\sum_{j\neq i} \tilde{x}_j(1,\sigma^1 \mid \bar{J})}^{X^*(1,\sigma^1)} \left(U_x(\tau \mid b=1) - U_x(\tau)\right) d\tau > 0,
\end{aligned}
$$

which is positive by lemma 2 and by the assumption that $U_x(\tau \mid b=1) > U_x(\tau)$. $\square$

Lemma 6. *The increase on welfare given by seller* 1 *is higher when the buyer is investing, i.e.,*

$$
TS^*(1,\sigma^1) - \tilde{TS}_{-1}(1 \mid \bar{J}_1) - \psi(\sigma^1) \geq TS^*(0,\sigma^0) - \tilde{TS}_{-1}(0 \mid \bar{J}_1) - \psi(\sigma^0).
$$

Proof. I am going to proceed by contradiction. Take the contrary and assume that

$$TS^*(1,\sigma^1) - \tilde{TS}_{-1}(1 \mid \bar{J}_1) - \psi(\sigma^1) < TS^*(0,\sigma^0) - \tilde{TS}_{-1}(0) - \psi(\sigma^0 \mid \bar{J}_1).$$

This implies that the investing seller is worst-off when the buyer is investing and hence he has less incentives to invest. This would imply that $\sigma^1 < \sigma^0$, but this contradicts the fact that investments are strategic complements. $\square$

Lemma 7. *For any $J_i \subset N \setminus \{i\}$ and $\bar{J}_i$, and with a small allocative sensitivity, i.e, the level of investment by seller 1 is similar, we have that*

$$\tilde{TS}_{-i}(1,\sigma_E \mid \bar{J}_i) - \tilde{TS}_{-i}(1,\sigma_J \mid J_i) > \tilde{TS}_{-i}(0,\sigma_E \mid \bar{J}_i) - \tilde{TS}_{-i}(0,\sigma_J \mid J_i).$$

Proof. By using the same procedure as in lemma 5 I obtain:

$$\begin{aligned}
&\tilde{TS}_{-i}(1,\sigma_E \mid \bar{J}_i) - \tilde{TS}_{-i}(1,\sigma_J \mid J_i) > \tilde{TS}_{-i}(0,\sigma_E \mid \bar{J}_i) - \tilde{TS}_{-i}(0,\sigma_J \mid J_i) \\
&\geq U\left(\sum_{j\neq i} \tilde{x}_j(\bar{J}) \mid 1\right) - U\left(X^*_{-\{J,1\}} + \sum_{j\in J} \tilde{x}_j(J) \mid 1\right) - \left[U\left(\sum_{j\neq i} \tilde{x}_j(\bar{J})\right) - U\left(X^*_{-\{J,1\}} + \sum_{j\in J} \tilde{x}_j(J)\right)\right] \\
&= \int_{X^*_{-\{J,1\}} + \sum_{j\in J} \tilde{x}_j(J)}^{\sum_{j\neq 1} \tilde{x}_j(\bar{J})} \left(U_x(\tau \mid b=1) - U_x(\tau)\right) d\tau > 0,
\end{aligned}$$

and this is positive by lemma 2 and by assumption $U_x(X \mid b=1) > U_x(X)$. $\square$

Lemma 8. *When the buyer invests in J' but not in J and $J' \subset J$, the non-investing seller is always better in a more competitive equilibrium. For any $J' \subset J$ it has to be that*

$$TS^*(0,\sigma_J^0) - \tilde{TS}_{-i}(0,\sigma_J^0 \mid J) > TS^*(1,\sigma_{J'}^1) - \tilde{TS}_{-i}(1,\sigma_{J'}^1 \mid J'), \quad \forall\, i \neq 1.$$

Proof. I proceed by contradiction, consider the contrary

$$TS^*(0,\sigma_{\mathbf{J}}^0) - \tilde{TS}_{-i}(0,\sigma_{\mathbf{J}}^0 \mid J) < TS^*(1,\sigma_{\mathbf{J}'}^1) - \tilde{TS}_{-i}(1,\sigma_{\mathbf{J}'}^1 \mid J'),$$

but then it has to be the case than the investment threshold for J' is lower than when J, i.e., $\hat{K}(J') < \hat{K}(J)$. However, this implies that if the buyer decides to invest in J' she also has to invest in J, and I reach a contradiction. $\square$

Lemma 9. *For a given investment profile (b, σ) the amount that each seller trades with the buyer decreases with the number of active sellers, but the aggregate level of trade is higher.*

$$x_i^*(N+1) < x_i^*(N) \quad \forall i \in N \quad \textit{and} \quad X^*(N+1) > X^*(N).$$

Proof. The results comes directly from the concavity of the utility function and the convexity of the cost function. In order to ease notation, I do not consider investment. For an number of $N+1$ active sellers, the amount traded in equilibrium needs to satisfy

$$U_x\left(\sum_{i=1}^{N+1} x_i^*(N+1)\right) = C_x\left(x_i^*(N+1)\right).$$

I prove the claim by contradiction, assume that $x_i^*(N+1) \geq x_i^*(N)\ \forall\ i \in N$, and since $N+1 > N$, I have that $\sum_{i=1}^{N+1} x_i^*(N+1) > \sum_{i=1}^{N} x_i^*(N)$ and by the concavity of the utility function $U(\cdot)$ and optimality, it has to be the case that

$$C_x\left(x_i^*(N+1)\right) = U_x\left(\sum_{i=1}^{N+1} x_i^*(N+1)\right) < U_x\left(\sum_{i=1}^{N} x_i^*(N)\right) = C_x\left(x_i^*(N)\right) \quad \forall i \in N,$$

but the convexity of $C_x(\cdot)$ implies that $x^*(N+1) < x^*(N)$, which leads to a contradiction. The previous also implies that $X^*(N+1) > X^*(N)$. $\square$

Lemma 10. *The function $V_{J_i}\left(X^*_{-\{J_i,i\}}\right)$ is well defined, strictly increasing and strictly concave in $X^*_{-\{J_i,i\}}$. The maximizer $\tilde{x}_j\left(X^*_{-\{J_i,i\}}\right)$ for $j \in J_i$ is decreasing in $X^*_{-\{J_i,i\}}$.*

Proof. This is the general case of Chiesa & Denicolò (2009) for any set J_i. That the function $V\left(X^*_{-\{J_i,i\}}\right)$ is well defined follows from the Inada conditions. By the envelop theorem I obtain $V_x\left(X^*_{-\{J_i,i\}}\right) > 0$ and $V_{xx}\left(X^*_{-\{J_i,i\}}\right) < 0$, which implies that the function is strictly increasing and strictly concave. By the implicit function theorem, I find that:

$$\frac{\partial \tilde{x}_j\left(X^*_{-\{J_i,i\}}\right)}{\partial X^*_{-\{J_i,i\}}} = \frac{U_{xx}(\cdot)}{C_{xx}(\cdot) - U_{xx}(\cdot)} < 0.$$

$\square$

B Appendix

Proof of lemma 1: I start by showing how the investment of seller 1 affects the equilibrium allocation. I consider the case where the buyer decides not to invest, i.e., $b = 0$ but the proof is analogous for $b = 1$. Differentiating the first-order conditions given in (4.1) for x_j^* and $j \neq i$ with respect to σ I obtain

$$U_{xx}(X^*) \times \sum_{h=1}^{N} \frac{dx_h^*}{d\sigma} = C_{xx}(x_j^*) \times \frac{dx_j^*}{d\sigma}. \tag{B.1}$$

Because the left hand side is independent of j I find that all $dx_j^*/d\sigma$ have the same sign. Now suppose also that $dx_1^*/d\sigma$ has that same sign. Then also the sum has that same sign and since $U_{xx}(\cdot) < 0$ and $C_{xx}(\cdot) > 0$ this leads to a contradiction. Now suppose $dx_1^*/d\sigma < 0$. The other signs therefore have to be positive. By (B.1) I find that $\sum_{h=1}^{N} dx_h^*/d\sigma < 0$. But the first-order condition for x_1^*, differentiated with respect to σ is

$$U_{xx}(X^*) \times \sum_{h=1}^{N} \frac{dx_h^*}{d\sigma} = C_{xx}(x_1^* \mid \sigma) \times \frac{dx_1^*}{d\sigma} + C_{x\sigma}(x_1^* \mid \sigma), \tag{B.2}$$

which would then have a positive left hand side and a negative right hand side due to $C_{x\sigma}(\cdot) < 0$ - a contradiction.

Thus I have shown the first and the second part of point i) of the lemma. Again by (B.1) the last claim follows from $\partial X^*/\partial\sigma = \sum_{h=1}^{N} dx_h^*/d\sigma$ and the level of the allocative sensitivity is implicitly characterized in expression (B.1). I proceed by analyzing the effect that the investment of the buyer has on the equilibrium allocation. Again, I am going to make use of the conditions for the equilibrium allocation represented in equation (4.1), and for a fixed investment of seller 1 I obtain

$$C_x(x_1^* \mid \sigma) = U_x(X^* \mid b = 1) > U_x(X^*) = C_x(x_1^* \mid \sigma) \qquad \text{for} \quad 1,$$

$$C_x(x_j^*) = U_x(X^* \mid b = 1) > U_x(X^*) = C_x(x_j^*) \qquad \text{for} \quad j \neq 1.$$

The strict inequality is by assumption and by the convexity of the cost function I obtain the result.

Proof of lemma 2: I have to show that $X^*(b, \sigma) > X^*_{-\{J_i,i\}}(b, \sigma) + \sum_{j \in J_i} \tilde{x}_j(b, \sigma \mid J_i)$. I am going to consider the case where there buyer is not investing, i.e., $b = 0$ but the proof is analogous for the case when the buyer invests $b = 1$. Also, consider any set of sellers $J_i \subset N$. With the same investment profile, I know that $\sum_{h \neq J_i, i} x_h^* = X^*_{-\{J_i,i\}}$, and the expression above is equivalent to $\sum_{j \in J_i} x_j^* + x_i^* > \sum_{j \in J_i} \tilde{x}_j(J_i)$.

Therefore and from the Inada conditions I have that $x_i^* > 0$ if $\sum_{j\in J_i}\left(x_j^* - \tilde{x}_j(J_i)\right) > 0$ I am done. Observe that for a given investment profile, if the above is true, it has to be true for any $j \in J_i$, hence $x_j^* > \tilde{x}_j(J_i)$. If the contrary occurs, $x_j^* < \tilde{x}_j(J_i)$, then from the equilibrium allocation I have

$$U_x\left(X^*_{-\{J_i,i\}} + \sum_{j\in J_i}\tilde{x}_j(J_i)\right) = C_x(\tilde{x}_j(J_i)) > C_x(x_j^*) = U_x(X^*),$$

and by concavity of $U(\cdot)$ I prove the claim. The previous also implies that for any $j \in J_i$ I have $\tilde{x}_j(J_i) > x_j^*$. Using the same procedure I can easily prove that for any $J_i' \subseteq J_i$ I obtain

$$X^*_{-\{J_i,i\}} + \sum_{j\in J_i}\tilde{x}_j(J_i) \geq X^*_{-\{J_i',i\}} + \sum_{j\in J_i'}\tilde{x}_j(J_i'),$$

and by using the same argument as before, I get $\tilde{x}_j(J_i') \geq \tilde{x}_j(J_i)$.

Proof of proposition 1: The equilibrium transfer of seller 1 depends on the number of sellers belonging to the set J_1 and for a given investment profile this is equal to

$$T_1(J_1 \mid b,\sigma) = U\left(X^* \mid b\right) - \left(\max_{\{x_j\}_{j\in J_1}}\left[U\left(X^*_{-\{J_1,1\}} + \sum_{j\in J_1} x_j \mid \tilde{x}_1 = 0, b\right) - \sum_{j\in J_1} C_j(x_j)\right] + \sum_{j\in J_1} C_j(x_j^*)\right).$$

Operating further I obtain

$$\begin{aligned}
T_1(J_1 \mid b,\sigma) &= U(X^* \mid b) - \sum_{j\in J_1} C_j(x_j^*) - \left[U\left(X^*_{-\{J_1,1\}} + \sum_{j\in J_1}\tilde{x}_j(J_1) \mid \tilde{x}_1 = 0, b\right) - \sum_{j\in J} C_j(\tilde{x}_j(J_1))\right]\\
&= U(X^* \mid b) - \sum_{j\in J_1} C_j(x_j^*) - \left[U\left(X^*_{-\{J_1,1\}} + \sum_{j\in J_1}\tilde{x}_j(J_1) \mid \tilde{x}_1 = 0, b\right) - \sum_{j\in J_1} C_j(\tilde{x}_j(J_1))\right]\\
&+ \left[\sum_{j\notin J_1,1}\left(C_j(x_j^*) - C_j(x_j^*)\right)\right] + \left[C(x_1^* \mid \sigma) - C(x_1^* \mid \sigma)\right]\\
&= TS^*(b,\sigma) - \left[U\left(X^*_{-\{J_1,1\}} + \sum_{j\in J_1}\tilde{x}_j(J_1) \mid \tilde{x}_1 = 0, b\right) - \sum_{j\in J_1} C_j(\tilde{x}_j(J_1)) - \sum_{j\neq J_1,1} C_j(x_j^*)\right]\\
&+ C(x_1^* \mid \sigma)\\
&= TS^*(b,\sigma) - \tilde{TS}_{-1}(b \mid J_1) + C(x_1^* \mid \sigma).
\end{aligned}$$

Introducing this equilibrium transfer to the payoffs of seller 1 in expression (3.2) I get the equilibrium payoffs stated in the proposition. The payoff of the buyer is

$$
\begin{aligned}
\Pi\left(b,\sigma \mid J\right) &= U\left(X^{*} \mid b\right) - \sum_{i} T_{i}^{e}(J_{i} \mid b,\sigma) - K \times b \\
&= U\left(X^{*} \mid b\right) - \left[\sum_{i} TS^{*}(b,\sigma) - \tilde{TS}_{-i}(b \mid J_{i}) + C(x_{i}^{*} \mid \cdot)\right] - K \times b \\
&= U\left(X^{*} \mid b\right) - \sum_{i} C_{i}(x_{i}^{*} \mid \cdot) - \left[\sum_{i} TS^{*}(b,\sigma) - \tilde{TS}_{-i}(b \mid J_{i})\right] - K \times b \\
&= TS^{*}(b,\sigma) - \sum_{i}\Big(TS^{*}(b,\sigma) - \tilde{TS}_{-i}(b,\sigma \mid J_{i})\Big) - K \times b.
\end{aligned}
$$

I proceed to show point (ii), i.e, $\tilde{TS}_{-i}\left(b,\sigma \mid J_{i}\right) > \tilde{TS}_{-i}\left(b,\sigma \mid J_{i}'\right)$ for $J_{i}' \subset J_{i}$. I take $b = 0$ and the payoffs of seller 1

$$
\begin{aligned}
\tilde{TS}_{-1}(0 \mid J_{1}') &= U\left(X_{-\{J_{1}',1\}}^{*} + \sum_{j \in J_{1}'} \tilde{x}_{j}(J_{1}')\right) - \sum_{j \in J_{1}'} C_{j}(\tilde{x}_{j}(J_{1}')) - \sum_{j \neq J_{1}',1} C_{j}(x_{j}^{*}) \\
&= U\left(X_{-\{J_{1}',1\}}^{*} + \sum_{j \in J_{1}'} \tilde{x}_{j}(J_{1}')\right) - \sum_{j \in J_{1}'} C_{j}(\tilde{x}_{j}(J_{1}')) - \sum_{j \neq J_{1}',1} C_{j}(x_{j}^{*}) \\
&\quad + \left[U\left(\sum_{j \neq 1} \tilde{x}_{j}(J_{1})\right) - U\left(\sum_{j \neq 1} \tilde{x}_{j}(J_{1})\right)\right] \\
&\leq U\left(X_{-\{J_{1}',1\}}^{*} + \sum_{j \in J_{1}'} \tilde{x}_{j}(J_{1}')\right) - U\left(\sum_{j \neq i} \tilde{x}_{j}(J_{1})\right) + \tilde{TS}_{-1}(0 \mid J_{1}) \\
\Longrightarrow \tilde{TS}_{-1}(0 \mid J_{1}) - \tilde{TS}_{-1}(0 \mid J_{1}') &\geq U\left(\sum_{j \neq 1} \tilde{x}_{j}(J_{1})\right) - U\left(X_{-\{J_{1}',1\}}^{*} + \sum_{j \in J_{1}'} \tilde{x}_{j}(J_{1}')\right) \\
&= \int_{X_{-\{J_{1}',1\}}^{*} + \sum_{j \in J_{1}'} \tilde{x}_{j}(J_{1}')}^{\sum_{j \neq 1} \tilde{x}_{j}(J_{1})} U_{x}(\tau) d\tau > 0,
\end{aligned}
$$

The first inequality comes from efficiency and the last strict inequality comes from lemma 2. This result can be applied to any seller $i \in N$. Because each seller obtains his marginal contribution whenever $\bar{J}_{i} = N \setminus \{i\}$, then it immediate to see that for any other $J_{i}' \subset \bar{J}_{i}$ any seller gets more than his marginal contribution to the trading surplus.

Proof of proposition 2: To show existence of efficiency in the equilibrium investment profile, I pay attention to seller's 1 investment. I later show that there always exists a region of the fixed cost of investment of the buyer where she always takes the efficient investment.

I first show the "if" part of the proposition. From proposition 1, I obtain that the payoff of seller 1 in the most competitive equilibrium is equal to

$$\pi_1(b, \sigma \mid \bar{J}_1) = TS^*(b, \sigma) - \tilde{TS}_{-1}(b \mid \bar{J}_1) - \psi(\sigma).$$

The term $\tilde{TS}_{-1}(b \mid \bar{J}_1)$ does not depend on the amount invested σ. Therefore using $TS^*(b, \sigma)$ given in the main text, and by the envelope-theorem, the first-order condition for the seller 1 is given by

$$\psi_\sigma(\sigma) = -C_\sigma\left(x_1^*(b, \sigma^b)|\sigma^b\right), \quad \forall b;$$

which is the same expression obtained in (4.9). Because seller 1 receives the marginal contribution of the trading surplus, he becomes the residual claimant and invests efficiently.

To show the "only if" part, I take any $J_1 \subset N \setminus \{1\}$. Now the equilibrium payoff of seller 1 is

$$\pi_1(b, \sigma \mid J_1) = TS^*(b, \sigma) - \tilde{TS}_{-1}(b, \sigma \mid J_1),$$

and calculating the first order condition and applying the envelope theorem I obtain that the equilibrium investment profile is

$$\psi_\sigma(\sigma) = -C_\sigma(x_1^*(b, \sigma^b)|\sigma^b) - \frac{\partial\left(\tilde{TS}_{-1}(b, \sigma \mid J_1)\right)}{\partial\sigma},$$

where the extra term depends on the investment of the seller 1 from the allocation that remains unchanged $X^*_{-\{J_1,1\}}(b, \sigma)$. As a result, $\partial\left(\tilde{TS}_{-1}(b, \sigma \mid J_1)\right)/\partial\sigma \neq 0$ and this creates a distortion of the investment of the seller. Hence, the efficient investment profile is only implementable when the trading outcome is the most competitive.

Proof of corollary 1: The investment decision of seller 1 is as in proposition 2 and the investment of the buyer is given by:

$$K \begin{cases} \leq TS^*(1, \sigma^1_{\mathbf{E}}) - TS^*(0, \sigma^0_{\mathbf{E}}) - \kappa(\bar{J}) \equiv \hat{K}(\bar{J}) & \text{then } b = 1 \\ > \hat{K}(\bar{J}) & \text{then } b = 0, \end{cases}$$

where the term $\kappa(\bar{J})$ is the difference in the payoff of the sellers when the buyer decides to invest and it is equal to

$$\kappa(\bar{J}) \equiv \pi_1(1, \sigma^1_{\mathbf{E}} \mid \bar{J}_1) - \pi_1(0, \sigma^0_{\mathbf{E}} \mid \bar{J}_1) + \sum_{i \neq 1} \left[\pi_i(1, \sigma^1_{\mathbf{E}} \mid \bar{J}_i) - \pi_i(0, \sigma^0_{\mathbf{E}} \mid \bar{J}_i)\right]$$
$$= TS^*(1, \sigma^1) - \tilde{TS}_{-1}(1 \mid \bar{J}_1) - TS^*(0, \sigma^0) + \tilde{TS}_{-1}(0 \mid \bar{J}_1)$$
$$+ \sum_{i \neq 1} \left[TS^*(1, \sigma^1) - \tilde{TS}_{-i}(1, \sigma^1 \mid \bar{J}_i) - TS^*(0, \sigma^0) + \tilde{TS}_{-i}(0, \sigma^0 \mid \bar{J}_i)\right].$$

The magnitude $\kappa(\bar{J})$ represents how much the sellers benefit from the investment of the buyer and those are the gains that cannot be appropriated by the latter. By making an explicit use of the lemmas in appendix A I show that the appropriation of the gains by the sellers is bigger than the cost of investment $\kappa(\bar{J}) > \psi(\sigma^1_{\mathbf{E}}) - \psi(\sigma^0_{\mathbf{E}})$. I split $\kappa(\bar{J})$ into two parts

$$A = \sum_{i \neq 1} \left[TS^*(1, \sigma^1) - \tilde{TS}_{-i}(1, \sigma^1 \mid \bar{J}_i) - TS^*(0, \sigma^0) + \tilde{TS}_{-i}(0, \sigma^0 \mid \bar{J}_i)\right]$$

and

$$B = TS^*(1, \sigma^1) - \tilde{TS}_{-1}(1 \mid \bar{J}_1) - TS^*(0, \sigma^0) + \tilde{TS}_{-1}(0 \mid \bar{J}_1).$$

In lemma 5, I show that $A > 0$ and in lemma 6 I show that $B > \psi(\sigma^1_{\mathbf{E}}) - \psi(\sigma^0_{\mathbf{E}})$.

Hence, the investment threshold below which the buyer invests is lower compared to efficiency $\hat{K}(\bar{J}) < \hat{K}_{\mathbf{E}}$. Thus, because the buyer cannot appropriate all the gains coming from her investment, she underinvests whenever the fix cost of investment lies between $K \in \left(\hat{K}(\bar{J}), \hat{K}_{\mathbf{E}}\right)$. Finally, since investments are strategic complements implies that seller 1 also underinvests in equilibrium.

Proof of proposition 3: From proposition 2 I know that the seller's 1 investment fails to be efficient whenever $J_1 \subset N \setminus \{1\}$. Here, I show that there exist over-investment and I characterize the magnitude, Without loss of generality I consider that the investment of the buyer to be $b = 0$. I take the first order condition of the equilibrium payoffs from seller 1 with respect to investment, and by applying the envelope condition I obtain

$$\begin{aligned}\psi_\sigma(\sigma) &= -C_\sigma(x_1^*(b,\sigma) \mid \sigma) - \sum_{m \neq J_1,1}\left(U_x\left(X^*_{-\{J_1,1\}} + \sum_{j\in J_1}\tilde{x}_j(J_1) \mid b\right) - C_x(x_j^*)\right) \times \frac{dx_m^*}{d\sigma} \\ &= -C_\sigma(x_1^*(b,\sigma)|\sigma) - \sum_{m \neq J_1,1}\left(U_x\left(X^*_{-\{J_1,1\}} + \sum_{j\in J_1}\tilde{x}_j(J_1) \mid b\right) - U_x(X^*)\right) \times \frac{dx_m^*}{d\sigma},\end{aligned} \tag{B.3}$$

where the transformation in the second line is due to the fact that, at the equilibrium allocation, marginal benefit equals marginal cost, i.e. $U_x(X^*) = C_x(x_j^*)$, $\forall j \in N$. Comparing this condition with efficiency (4.9), I see that the difference is the additional term is

$$\gamma(J_1) \equiv -\sum_{m \neq J_1,1}\left(U_x\left(X^*_{-\{J_1,1\}} + \sum_{j\in J_1}\tilde{x}_j(J_1) \mid b\right) - U_x(X^*)\right) \times \frac{dx_m^*}{d\sigma},$$

and by applying the fundamental theorem of calculus I obtain

$$\gamma(J_1) \equiv -\sum_{m \neq J_1,1}\left(\int_{X^*}^{X^*_{-\{J_1,1\}}+\sum_{j\in J_1}\tilde{x}_j(J_1)} U_{xx}(\tau)d\tau\right) \times \frac{dx_m^*}{d\sigma} > 0,$$

and the whole expression is positive. By lemma 2 and the concavity of the utility function $U(\cdot)$, I know that the part in brackets is positive. By lemma 1 I know that the amount traded with the sellers that are not investing is decreasing with the amount invested by seller 1. Therefore, this term is strictly positive which means that the seller 1 over-invests and its magnitude depends on the allocative sensitivity that the investment of seller 1 creates to the rest of the sellers.

To show that the degree of over-investment decreases with the number of sellers in J_1, I use a continuous approximation and I show that $\partial\gamma(J_1)/\partial J_1 < 0$. Hence, by applying the Leibniz rule of differentiation to the previous expression I obtain

$$\begin{aligned}\frac{\partial\gamma(J_1)}{\partial J_1} &= \left(\int_{X^*}^{X^*_{-\{J_1,1\}}+\sum_{j\in J_1}\tilde{x}_j(J_1)} U_{xx}(\tau)d\tau\right) \times \frac{dx_m^*}{d\sigma} \\ &\quad - U_{xx}\left(X^*_{-\{J_1,1\}} + \sum_{j\in J_1}\tilde{x}_j(J_1)\right) \times \underbrace{\frac{\partial\left(X^*_{-\{J_1,1\}} + \sum_{j\in J_1}\tilde{x}_j(J_1)\right)}{\partial J_1}}_{(+)} \times \frac{dx_m^*}{d\sigma} < 0,\end{aligned} \tag{B.4}$$

and the sign is due to lemma 2.

Proof of corollary 2: The first point is shown in proposition 3. To show point A) I take the investment decision of the buyer, where for any $J \subset N \setminus \{i\}$ this is:

$$K \begin{cases} \leq TS^*(1, \sigma^1_{\mathbf{J}}) - TS^*(0, \sigma^0_{\mathbf{J}}) - \kappa(J) \equiv \hat{K}(J) & \text{then } b = 1 \\ > \hat{K}(J) & \text{then } b = 0, \end{cases} \tag{B.5}$$

where the extra term $\kappa(J)$ is the difference in the payoff of the sellers when the buyer invests. Again this represents how much the sellers benefit from the investment of the buyer and those benefits can not be appropriate by the latter.

$$\begin{aligned} \kappa(J) &\equiv \pi_1\left(1, \sigma^1_{\mathbf{J}} \mid J_1\right) - \pi_1\left(0, \sigma^0_{\mathbf{J}} \mid J_1\right) + \sum_{i \neq 1}\left[\pi_i\left(1, \sigma^1_{\mathbf{J}} \mid J_i\right) - \pi_i\left(0, \sigma^0_{\mathbf{J}} \mid J_i\right)\right] \\ &= TS^*(1, \sigma^1) - \tilde{TS}_{-1}(1, \sigma^1 \mid J_1) - TS^*(0, \sigma^0) + \tilde{TS}_{-1}(0, \sigma^0 \mid J_1) \\ &+ \sum_{i \neq 1}\left[TS^*(1, \sigma^1) - \tilde{TS}_{-i}(1, \sigma^1 \mid J_i) - TS^*(0, \sigma^0) + \tilde{TS}_{-i}(0, \sigma^0 \mid J_i)\right]. \end{aligned}$$

The buyer underinvests, under some fixed cost of investment, if the threshold of investment is below efficiency, that is,

$$\begin{aligned} \hat{K}(J) \leq \hat{K}_E \iff\ & TS^*(1, \sigma^1_{\mathbf{J}}) - TS^*(0, \sigma^0_{\mathbf{J}}) - \kappa(J) \leq TS^*(1, \sigma^1_{\mathbf{E}}) - TS^*(0, \sigma^0_{\mathbf{E}}) - \left(\psi\left(\sigma^1_{\mathbf{E}}\right) - \psi\left(\sigma^0_{\mathbf{E}}\right)\right) \\ \iff\ & \psi\left(\sigma^1_{\mathbf{E}}\right) - \psi\left(\sigma^0_{\mathbf{E}}\right) \leq TS^*(1, \sigma^1_{\mathbf{E}}) - \tilde{TS}_{-1}(1, \sigma^1_{\mathbf{J}} \mid J_1) - \left(TS^*(0, \sigma^0_{\mathbf{E}}) - \tilde{TS}_{-1}(0, \sigma^0_{\mathbf{J}} \mid J_1)\right) \\ & + \sum_{i \neq 1}\left[TS^*(1, \sigma^1_{\mathbf{E}}) - \tilde{TS}_{-i}(1, \sigma^1_{\mathbf{J}} \mid J_i) - \left(TS^*(0, \sigma^0_{\mathbf{E}}) - \tilde{TS}_{-i}(0, \sigma^0_{\mathbf{J}} \mid J_i)\right)\right]. \end{aligned}$$

By using the same procedure as in lemma 5 I can show that the last part in brackets is positive. Therefore, I only need to verify that

$$TS^*(1, \sigma^1_{\mathbf{E}}) - \tilde{TS}_{-1}(1, \sigma^1_{\mathbf{J}} \mid J_1) - \left(TS^*(0, \sigma^0_{\mathbf{E}}) - \tilde{TS}_{-1}(0, \sigma^0_{\mathbf{J}} \mid J_1)\right) \geq \psi\left(\sigma^1_{\mathbf{E}}\right) - \psi\left(\sigma^0_{\mathbf{E}}\right).$$

Here I apply lemma 7 that states $\tilde{TS}_{-1}(1, \sigma^1_{\mathbf{J}} \mid J) > \tilde{TS}_{-1}(1 \mid \bar{J}_1) - \tilde{TS}_{-1}(0 \mid \bar{J}_1) + \tilde{TS}_{-1}(0, \sigma^0_{\mathbf{J}} \mid J_1)$, and by introducing this the the previous expression I have that

$$
\begin{aligned}
TS^*(1,\sigma^1_{\mathbf{E}}) - \tilde{TS}_{-1}(1,\sigma^1_{\mathbf{J}} \mid J_1) - \Big(TS^*(0,\sigma^0_{\mathbf{E}}) - \tilde{TS}_{-1}(0,\sigma^0_{\mathbf{J}} \mid J_1)\Big) &> TS^*(1,\sigma^1_{\mathbf{E}}) \\
&- \Big[\tilde{TS}_{-1}(1 \mid \bar{J}_1) - \tilde{TS}_{-1}(0 \mid \bar{J}_1) + \tilde{TS}_{-1}(0,\sigma^0_{\mathbf{J}} \mid J_1)\Big] - \Big(TS^*(0,\sigma^0_{\mathbf{E}}) - \tilde{TS}_{-1}(0,\sigma^0_{\mathbf{J}} \mid J_1)\Big) \\
&= TS^*(1,\sigma^1_{\mathbf{E}}) - \tilde{TS}_{-1}(1 \mid \bar{J}_1) - \Big(TS^*(0,\sigma^0_{\mathbf{E}}) - \tilde{TS}_{-1}(0 \mid \bar{J}_1)\Big) > \psi\left(\sigma^1_{\mathbf{E}}\right) - \psi\left(\sigma^0_{\mathbf{E}}\right),
\end{aligned}
$$

where the last inequality comes by lemma 6. Therefore, the investing threshold in equilibrium is lower that efficiency.

To show point B) I need to compare the right hand side of the expression determining the investment of the seller 1 in equilibrium (B.3) evaluated at $b = 0$, with the right hand side of expression determining the efficient investment (4.9) evaluated at $b = 1$.

$$
Rhs^{J_1}(b=0) = -C_\sigma(x_1^*(0,\sigma^0_{\mathbf{J}}) \mid \sigma) - \sum_{m \neq J_1,1} \left(\int_{X^*}^{X^*_{-\{J_1,1\}} + \sum_{j \in J_1} \tilde{x}_j(J_1)} U_{xx}(\tau) d\tau \right) \frac{dx^*_m}{d\sigma}.
$$

$$
Rhs^{E}(b=1) = -C_\sigma(x_1^*(1,\sigma^1_{\mathbf{E}}) \mid \sigma),
$$

and I will have that the efficient investment is higher if

$$
\begin{aligned}
& Rhs^{E}(b=1) > Rhs^{J_1}(b=0) \\
& \Longrightarrow -C_\sigma(x_1^*(1,\sigma^1) \mid \sigma) > -C_\sigma(x_1^*(0,\sigma^0) \mid \sigma) - \sum_{m \neq J_1,1} \left(\int_{X^*}^{X^*_{-\{J_1,1\}} + \sum_{j \in J_1} \tilde{x}_j(J_1)} U_{xx}(\tau) d\tau \right) \frac{dx^*_m}{d\sigma} \\
& \Longrightarrow \int_{x_1^*(1,\sigma^1)}^{x_1^*(0,\sigma^0)} C_{x\sigma}(\tau) d\tau > - \sum_{m \neq J_1,1} \left(\int_{X^*}^{X^*_{-\{J_1,1\}} + \sum_{j \in J_1} \tilde{x}_j(J_1)} U_{xx}(\tau) d\tau \right) \frac{dx^*_j}{d\sigma} \\
& \Longrightarrow -\frac{dx^*_m}{d\sigma} > \frac{\int_{x_1^*(1,\sigma^1_{\mathbf{E}})}^{x_1^*(0,\sigma^0_{\mathbf{E}})} C_{x\sigma}(\tau) d\tau}{(N \setminus \{1\} - J_1) \times \int_{X^*(0,\sigma^0_{\mathbf{J}})}^{X^*_{-\{J_1,m\}}(0,\sigma^0_{\mathbf{J}}) + \sum_{j \in J_1} \tilde{x}_j(0,\sigma^0_{\mathbf{J}} \mid J_1)} U_{xx}(\tau) d\tau} = \lambda(J_1).
\end{aligned}
$$

otherwise, the contrary occurs. Therefore, if the allocative sensitivity is large, seller 1 invests more than the efficiency level regardless of the investment decision of the buyer.

Proof of proposition 3: For a given subset of sellers in J, the investment threshold of the buyer is given by

$$
\hat{K}(J) \equiv TS^*(1,\sigma^1(J)) - TS^*\left(0,\sigma^0(J)\right) - \sum_{i \in N} \left(T_i^1(J) - T_i^0(J)\right).
$$

I redefine this threshold as

$$\hat{K}(J) = \aleph(J) - \beth(J).$$

where $\aleph(J) = TS^*(1, \sigma^1(J)) - TS^*\left(0, \sigma^0(J)\right)$ and $\beth(J) = \sum_{i \in N}\left(T_i^1(J) - T_i^0(J)\right)$.

When the allocative sensitivity is very small, I have shown in proposition 3 that the distortion of investment is little and for any $J' \subset J$ I have that $\gamma(J) \approx 0$. Hence, for any $J' \subset J$, I obtain that $\sigma_{J'} \approx \sigma_J$. Then it is immediate to get that $\hat{K}(J') < \hat{K}(J)$ because $\aleph(J') \approx \aleph(J)$ and $\beth(J') > \beth(J)$, which comes directly from lemma 7, but considering the investment of seller 1 fixed, in appendix A. Therefore, the higher the level of competition in the trading game, the more incentives for the buyer to invest. With a similar investment of seller 1, the trading surplus remains constant over the level of competition ex-post and the buyer is better-off when she can appropriate a larger proportion of those gains.

When, the allocative sensitivity is significant, I obtain that the seller's 1 investment changes with competition and for $J' \subset J$ I obtain that $\sigma_{J'} > \sigma_J$. Then, I also obtain that $\aleph(J') > \aleph(J)$ because

$$\frac{\partial \aleph(J')}{\partial \sigma} \approx \frac{\partial(TS^*(1, \sigma^1) - TS^*(0, \sigma^0))}{\partial \sigma} \times (\gamma(J') - \gamma(J)) = \int_{x_1^*(1,\sigma^1)}^{x_1^*(0,\sigma^0)} C_{x\sigma}(\tau) d\tau \times [\gamma(J') - \gamma(J)] > 0.$$

Moreover, the part $\beth(J)$ is now affected by the change of investment and I obtain that $\partial \beth(J)/\partial \sigma < 0$ because

$$\begin{aligned}
\frac{\partial \beth(J')}{\partial \sigma} &\approx \sum_i \left(\int_{x_i^*(1,\sigma^1)}^{\tilde{x}_i(1,\sigma^1|J')} C_{x\sigma}(\tau) d\tau - \int_{x_i^*(0,\sigma^0)}^{\tilde{x}_i(0,\sigma^0|J')} C_{x\sigma}(\tau) d\tau \right) \times [\gamma(J') - \gamma(J)] \\
&= \sum_i \left(\int_{x_i^*(1,\sigma^1)+\tilde{x}_i(0,\sigma^0|J')}^{\tilde{x}_i(1,\sigma^1|J')+x_i^*(0,\sigma^0)} C_{x\sigma}(\tau) d\tau \right) \times [\gamma(J') - \gamma(J)] < 0
\end{aligned}$$

Hence, I obtain that

$$\begin{aligned}
\hat{K}(J') - \hat{K}(J) = &\underbrace{-\left(\beth(J') - \beth(J)\right)}_{(-)} \\
&+ \underbrace{\left[\int_{x_1^*(1,\sigma^1)}^{x_1^*(0,\sigma^0)} C_{x\sigma}(\tau) d\tau + \sum_i \left(\int_{\tilde{x}_i(1,\sigma^1|J')+x_i^*(0,\sigma^0)}^{x_i^*(1,\sigma^1)+\tilde{x}_i(0,\sigma^0|J')} C_{x\sigma}(\tau) d\tau \right) \right]}_{(+)} \times \underbrace{[\gamma(J') - \gamma(J)]}_{(+)}
\end{aligned}$$

Thus, with a big enough allocative sensitivity $[\gamma(J') - \gamma(J)]$ is big enough such that $\hat{K}(J') > \hat{K}(J)$.

Proof of proposition 4: I proceed by construction and I study a situation where the number of active sellers tends to infinity. I consider first the investment decision of seller 1 and later I study the investment threshold of the buyer. Regarding the investment of seller 1, it is without loss of generality to take the case where the outcome in the trading game is the least competitive, that is, whenever the set of sellers in J_1 is a singleton $|J_1| = 1 = \underline{J}_1$.

I have shown in proposition 3 that the investment distortion of the seller in the least competitive equilibrium is given by

$$\gamma(N \mid \underline{J}_1) \equiv - \sum_{m \neq \underline{J}_1, 1} \left(\int_{X^*(N)}^{X^*_{-\{1,\underline{J}_1\}}(N) + \tilde{x}_{\underline{J}_1}(N|\underline{J}_1)} U_{xx}(\tau) d\tau \right) \times \frac{dx^*_m}{d\sigma}. \tag{B.6}$$

The magnitude of this object depends on the number of active sellers N as represented by the bounds of the integral, and the difference is equal to $x^*_{\underline{J}_1}(N) + x^*_1(N) - \tilde{x}_{\underline{J}_1}(N \mid \underline{J}_1) > 0$. I now show that this difference tends to zero when the number of active sellers is arbitrarily large and hence the investment distortion is also zero.

By lemma 9 in appendix A, I have shown that as the number of sellers increase, the per seller amount of trade decreases. In the limit, from the concavity of the utility function, together with the Inada condition, I obtain that for any investment b the individual trading amount tends to zero, that is, $\lim_{N\to\infty} x^*_1(N) \approx 0$. With the equilibrium trading allocation given in expression (4.1) I get

$$\lim_{N\to\infty} [X^*(N)] \approx \infty \to \lim_{N\to\infty} [U_x(X^* \mid b)] \approx 0 \approx C_x(x^*_1(N) \mid \sigma) \to \lim_{N\to\infty} [x^*_1(N)] \approx 0$$

With regard to how the amount $\tilde{x}_{\underline{J}_1}(N \mid \underline{J}_1)$ evolves with the number of sellers, I know that this object is the solution of the value function $V_{J_1}(X^*_{-\{\underline{J}_1,1\}})$, whose properties are stated in lemma 10 in appendix A. From lemma 9, the amount $X^*_{-\{\underline{J}_1,1\}}(N)$ increases, and the change on $\tilde{x}_{\underline{J}_1}(N \mid \underline{J}_1)$ is decreasing and equals

$$\frac{d\tilde{x}_{\underline{J}_1}(N \mid \underline{J}_1)}{dX^*_{-\{\underline{J}_1,1\}}} = \frac{U_{xx}(\cdot)}{C_{xx}(\cdot) - U_{xx}(\cdot)}.$$

By totally differentiating the first order condition of the equilibrium trading allocation, the change on the efficient amount is also decreasing and equals

$$\frac{dx^*_{\underline{J}_1}(N)}{dX^*_{-\{\underline{J}_1,1\}}(N)} = \frac{U_{xx}(\cdot)}{C_{xx}(\cdot)}.$$

But the decrease on the first amount dominates

$$\frac{U_{xx}(\cdot)}{C_{xx}(\cdot) - U_{xx}(\cdot)} < \frac{U_{xx}(\cdot)}{C_{xx}(\cdot)} \Longrightarrow C_{xx}(\cdot) < C_{xx}(\cdot) - U_{xx}(\cdot) \Longrightarrow U_{xx}(\cdot) < 0,$$

and whenever the number of sellers tend to infinity I obtain that $\lim_{N\to\infty}\left[\tilde{x}_{\underline{J}_1}(N \mid \underline{J}_1) \approx x^*_{\underline{J}_1}(N)\right]$. Therefore, when the number of sellers tend to infinity the difference between the upper and the lower integrand of (B.6) tends to zero, and the distortion of investment is also zero. This happens for any set of sellers belonging to $J_1 \subset N$.

I now show that the investment thresholds of the buyer converges to efficiency when the number of active sellers tend to infinity. For any $J \subset N$ the investment threshold of the buyer is

$$\hat{K}(J) \equiv TS^*(1, \sigma^1_{\mathbf{J}}) - TS^*(0, \sigma^0_{\mathbf{J}}) - \kappa(J).$$

Because seller's 1 investment tends to efficiency with an infinite number of sellers, the first part of the investment threshold tends to efficiency as represented in (4.9)

$$\lim_{N\to\infty}\left[TS^*(1, \sigma^1_{\mathbf{J}}) - TS^*(0, \sigma^0_{\mathbf{J}})\right] \approx TS^*(1, \sigma^1_{\mathbf{E}}) - TS^*(0, \sigma^0_{\mathbf{E}}).$$

Moreover, the appropriation of the gains from trade, coming from an investment of the buyer, by the non investing sellers tends to zero, because

$$\begin{aligned} TS^*(1, \sigma^1_E) - TS^*(0, \sigma^0_E) &\approx \lim_{N\to\infty}[\tilde{TS}_{-i}(1, \sigma^1_E \mid J_i) - \tilde{TS}_{-i}(0, \sigma^0_E \mid J_i)] \\ \Longrightarrow TS^*(1, \sigma^1_E) - \tilde{TS}_{-i}(1, \sigma^1_E \mid J_i) &- \Big(TS^*(0, \sigma^0_E) - \tilde{TS}_{-i}(0, \sigma^0_E \mid J_i)\Big) \approx 0 \end{aligned}$$

and this result comes from the fact that $\lim_{N\to\infty}[x^*_i(N) \approx 0]$. By the same argument, the appropriation of the trading surplus by seller 1 has to be very close to the difference of investment costs $\psi(\sigma^1_{\mathbf{E}}) - \psi(\sigma^0_{\mathbf{E}})$. Hence, I finally obtain that the investment threshold of the buyer also tends to efficiency $\lim_{N\to\infty}\left[\hat{K}(J)\right] \approx \hat{K}_E$, for any $J \subset N$.

Proof of proposition 5: I begin by considering the case where the allocative sensitivity is small. In this case, I have established in proposition 3 that the investment threshold of the buyer is monotonically increasing with competition. The lower portion of the surplus appropriated by the buyer with lower competition dominates the higher investment of seller 1, i.e., $\hat{K}(J') < \hat{K}(J)$ for any $J' \subset J$. I first consider point i) when the investment of the buyer is equilibrium invariant. Here, to show that seller 1 is better-off with lower levels of competition I only need to verify that his investment increases with lower levels of competition and this is the case since I know that $\gamma(J') > \gamma(J)$, for any $J' \subset J$. For the non-investing sellers, it is easy to see that for any $J' \subset J$, I obtain

$$\begin{aligned} TS^*(b, \sigma^b_{\mathbf{J}'}) - \tilde{TS}_{-i}(b, \sigma^b_{\mathbf{J}'} \mid J') &> TS^*(b, \sigma^b_{\mathbf{J}}) - \tilde{TS}_{-i}(b, \sigma^b_{\mathbf{J}} \mid J) \\ \implies \tilde{TS}_{-i}(b, \sigma^b_{\mathbf{J}} \mid J) - \tilde{TS}_{-i}(b, \sigma^b_{\mathbf{J}'} \mid J') &> TS^*(b, \sigma^b_{\mathbf{J}}) - TS^*(b, \sigma^b_{\mathbf{J}'}) \approx 0 \\ \implies \tilde{TS}_{-i}(b, \sigma^b_{\mathbf{J}} \mid J) - \tilde{TS}_{-i}(b, \sigma^b_{\mathbf{J}} \mid J') &> 0. \end{aligned}$$

The right hand side of the second line is close to zero because with a small allocative sensitivity, the investment of the seller is similar regardless to the equilibrium ex-post $\sigma^b_{\mathbf{J}'} \approx \sigma^b_{\mathbf{J}}$. The third line is positive by point ii) in proposition 1.

When the investment of the buyer depends on the equilibrium played ex-post and with a small allocative sensitivity, from proposition 3 I know that the investment threshold monotonically increases with competition, and hence there must exist a $J' \subset N$ where the buyer invests whenever $J' \subset J$ and does not invest otherwise. Due to complementarity of investment and proposition 3, I know also that $\sigma^1_{\mathbf{J}'} > \sigma^1_{\mathbf{J}}$. Furthermore, because the allocative sensitivity is small, I obtain that the variation $(\gamma(J) - \gamma(J'))$ is small so $\sigma^1_{\mathbf{J}'} > \sigma^0_{\underline{\mathbf{J}}}$ for any $J' \subset J$. This implies that the largest payoff of seller 1 is achieved with an intermediate level of competition.

With regard to the non-investing sellers, I get the same result. Hence, for any $J' \subset J$ I obtain $TS^*(1, \sigma^1_{\mathbf{J}'}) - \tilde{TS}_{-i}(1, \sigma^1_{\mathbf{J}'} \mid J') > TS^*(1, \sigma^1_{\mathbf{J}}) - \tilde{TS}_{-i}(1, \sigma^1_{\mathbf{J}} \mid J)$. Thus, I only need to show that for $J \subset J'$ I get

$$TS^*(1, \sigma^1_{\mathbf{J}'}) - \tilde{TS}_{-i}(1, \sigma^1_{\mathbf{J}'} \mid J') > TS^*(0, \sigma^0_{\mathbf{J}}) - \tilde{TS}_{-i}(0, \sigma^0_{\mathbf{J}} \mid J). \tag{B.7}$$

By lemma 5 and proposition 1 I know that

$$TS^*(1,\sigma_J^1) - \tilde{TS}_{-i}(1,\sigma_J^1 \mid J) > TS^*(0,\sigma_J^0) - \tilde{TS}_{-i}(0,\sigma_J^0 \mid J) \quad \forall\, J \subset N,$$

$$TS^*(0,\sigma_J^0) - \tilde{TS}_{-i}(0,\sigma_J^0 \mid J) > TS^*(0,\sigma_{J'}^0) - \tilde{TS}_{-i}(0,\sigma_{J'}^0 \mid J').$$

By summing up both expressions and letting $J = J'$ in the first inequality then

$$\begin{aligned} TS^*(1,\sigma_{J'}^1) - \tilde{TS}_{-i}(1,\sigma_{J'}^1 \mid J') > & -TS^*(0,\sigma_J^0) + \tilde{TS}_{-i}(0,\sigma_J^0 \mid J) \\ & + 2\left[TS^*(0,\sigma_{J'}^0) - \tilde{TS}_{-i}(0,\sigma_{J'}^0 \mid J')\right], \end{aligned} \tag{B.8}$$

and by summing up expression (B.7) and (B.8) I obtain

$$2\left[TS^*(1,\sigma_{J'}^1) - \tilde{TS}_{-i}(1,\sigma_{J'}^1 \mid J')\right] > 2\left[TS^*(0,\sigma_{J'}^0) - \tilde{TS}_{-i}(0,\sigma_{J'}^0 \mid J')\right]$$

where the last inequality holds by lemma 5.

When the allocative sensitivity is big, by proposition 3 I know that there exist a $J' \subset J$ such that $\hat{K}(J') > \hat{K}(J)$. Point iii) is easy to obtain. Because of investment complementarity, seller 1 always obtains larger payoffs the less competitive the equilibrium outcome is, he is not only able to appropriate a larger partition of the trading surplus, but the investment of the buyer goes in his favor. Contrarily, the non-investing sellers obtain the largest payoffs when the outcome in the trading game is the most competitive. When, there exist a $J' \subset J$, where the buyer decides to invest in any $J'' \subset J'$, I obtain

$$TS^*(0,\sigma_{\mathbf{J}}^0) - \tilde{TS}_{-i}(0,\sigma_{\mathbf{J}}^0 \mid J) > TS^*(1,\sigma_{\mathbf{J}"}^1) - \tilde{TS}_{-i}(1,\sigma_{\mathbf{J}"}^1 \mid J''),$$

which comes from lemma 8 in appendix A. Moreover, when the investment of the buyer is equilibrium invariant, as long as the allocative sensitivity is big enough I will also have that

$$TS^*(b,\sigma_J^b) - \tilde{TS}_{-i}(b,\sigma_J^b \mid J) > TS^*(b,\sigma_{J'}^b) - \tilde{TS}_{-i}(b,\sigma_{J'}^b \mid J'); \quad \text{for any } J' \subset J.$$

And the non-investing sellers are better with a lower partition of the surplus as this also implies a smaller equilibrium investment of seller 1.

Proof of proposition 6: I first consider the case when the allocative sensitivity is small such that $\hat{K}(J) > \hat{K}(J')$ for any $J' \subset J$. This entails that the investment decision of the buyer is only efficient in an equilibrium with a set of sellers in J' if it is also in J. Then, because the investment decision of seller 1 is inefficient in any $J' \subset J$ as stated in proposition 3, I obtain that the highest level of welfare is obtained when competition is the most severe, that is, when $J_i = N \setminus \{i\}$.

With a big allocative sensitivity, I know that the investment threshold of the buyer is not monotonically increasing with the level of competition ex-post, and there exist a $J' \subset J$ such that $\hat{K}(J') > \hat{K}(J)$. In this case, I show that welfare is maximized for J'.

For the case when $J \subset J'$, it is immediate and the argument is the same as in the first paragraph. For any $J \supset J'$, I only need to show that $W^{J'}(\cdot) > W^{\bar{J}}(\cdot)$. Then, I define the difference in welfare

$$D(\cdot) = W^{J'}(1, \sigma^1_{J'}) - W^{\bar{J}}(0, \sigma^0_{E}) = TS^*(1, \sigma^1_{\mathbf{J}'}) - \hat{K}(J') - \psi(\sigma^1_{\mathbf{J}'}) - TS^*(0, \sigma^0_{\mathbf{E}}) + \psi(\sigma^0_{\mathbf{E}}).$$

Because I only want to know if there exists a situation where a less competitive equilibrium does better in terms of welfare, I take the lowest possible value of the fixed investment costs, $K = \hat{K}(\bar{J}) = TS^*(1, \sigma^1_{\mathbf{E}}) - TS^*(0, \sigma^0_{\mathbf{E}}) - \kappa(\bar{J})$. By introducing this in the previous expression, I obtain that the lower bound of the difference is given by

$$\begin{aligned}
\underline{D}(\cdot) &= TS^*(1, \sigma^1_{\mathbf{J}'}) - TS^*(1, \sigma^1_{\mathbf{E}}) + TS^*(0, \sigma^0_{\mathbf{E}}) + \kappa(\bar{J}) - \psi(\sigma^1_{\mathbf{J}'}) - TS^*(0, \sigma^0_{\mathbf{E}}) + \psi(\sigma^0_{\mathbf{E}}) \\
&= TS^*(1, \sigma^1_{\mathbf{J}'}) - TS^*(1, \sigma^1_{\mathbf{E}}) + \kappa(\bar{J}) - \left(\psi(\sigma^1_{\mathbf{J}'}) - \psi(\sigma^0_{\mathbf{E}})\right) \\
&> TS^*(1, \sigma^1_{\mathbf{J}'}) - TS^*(1, \sigma^1_{\mathbf{E}}) + \psi(\sigma^1_{\mathbf{E}}) - \psi(\sigma^0_{\mathbf{E}}) - \left(\psi(\sigma^1_{\mathbf{J}'}) - \psi(\sigma^0_{\mathbf{E}})\right) \\
&= TS^*(1, \sigma^1_{\mathbf{J}'}) - TS^*(1, \sigma^1_{\mathbf{E}}) - \left(\psi(\sigma^1_{\mathbf{J}'}) - \psi(\sigma^1_{\mathbf{E}})\right),
\end{aligned}$$

where the first inequality comes from the proof of corollary 1 in the appendix. Hence, I obtain that the difference is positive whenever the increase in the trading surplus coming from a higher investment of seller 1 is bigger than the investment cost, i.e. $TS^*(1, \sigma^1_{\mathbf{J}'}) - TS^*(1, \sigma^1_{\mathbf{E}}) > \psi(\sigma^1_{\mathbf{J}'}) - \psi(\sigma^1_{\mathbf{E}})$. Therefore, I additionally require that the effect of the investment of seller 1 in the trading surplus is sufficiently big.