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A Stochastic Growth Model with Income Tax Evasion: Implications for Australia*

Ratbek Dzhumashev\textsuperscript{a}, Emin Gahramanov\textsuperscript{b}

\textsuperscript{a}Department of Economics, Monash University, Berwick, Australia
\textsuperscript{b}School of Accounting, Economics and Finance, Deakin University, Burwood, Australia

May 15, 2009

Abstract

In this paper we develop a stochastic endogenous growth model augmented with income tax evasion. Our model avoids some existing discrepancies between empirical evidence and theoretical predictions of traditional tax evasion models. Further, we show that: \textit{i)} productive government expenditures play an important role in affecting economy’s tax evasion rate; \textit{ii)} the average marginal income tax rate in Australia come close to the optimal; and \textit{iii)} the phenomenon of tax evasion is not an excuse for a productive government to advocate an excessive income taxation.

\textbf{JEL Classification:} H26; D91; O41.

\textbf{Key Words:} Tax evasion; Economic growth; Public services.

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\*We are grateful to Aarti Singh, Chris Edmond, Jon Kesselman, Mehmet Ulubaşoğlu, Xueli Tang, Abdul Hayat Muhammad, Debdul Mallick and the seminar participants of the 2009 14th Australasian Macroeconomic Workshop at Deakin University for useful comments on a recent version of the paper. We also thank James Feigenbaum, Shlomo Yitzhaki, Jean-Pierre Laffargue, Nejat Anbarci, Stephen Miller, He-ling Shi and the seminar participants at Monash University for helpful comments on an earlier version of this paper. All remaining errors are our own.

\dagger Corresponding author. E-mail: ratbek.dzhumashev@buseco.monash.edu.au
"The hidden economy is a profound phenomenon of our times; however measured, however defined, our conclusion is common to all the authors who attempted to deal with it: the problem of the hidden economy cannot be dismissed as quantitatively trivial."

– Marrelli (1987)

1 Introduction

Income tax evasion, constituting a sizable share of underground economy, is a chronic problem in virtually every country. For example, in the U.S. the fraction of income underreported is around 10 percent (Andreoni et al. (1998)), and this is comparable with the average estimates for leading European economies and Australia (Gupta (2004), Hepburn (1992), Bajada (1999)). In less developed countries the problem of tax noncompliance is even more severe (Alm et al. (1993), Chen (2003)). It is widely accepted among economists that income tax evasion leads to various forms of welfare, public revenue and distributional losses, as well as serious ethical issues within a society (Cullis and Jones (1998), Giles and Caragata (2001)). Furthermore, Bajada (2003) estimated that the underground economy deepens recessions and increases the volatility of business cycles. It undermines government policies intending to smooth cyclical fluctuations in the economy. Thus, studying income tax evasion phenomenon in more details posits a great interest.

In this paper we develop a stochastic endogenous growth model augmented with income tax evasion and based on the neoclassical assumption of rational choice. We then fully calibrate the model to some of the salient features of Australian economy.

One might argue that by modelling income tax evasion within an expected utility framework we are shooting at our own legs. This is because main theoretical predictions of traditional rational-choice models with tax evasion are inconsistent with existing empirical evidence. The inconsistencies are as follows.

**Inconsistency 1**: Traditional theoretical models of income tax evasion predict that higher marginal tax rates on income encourage tax compliance (Yitzhaki’s (1974) puzzle);\(^1\)

**Inconsistency 2**: Realistically calibrated tax evasion models predict that the extent of noncompliance is huge, which is not what we observe in reality.\(^2\)

The model we develop successfully avoids both inconsistencies, potentially allowing us to seriously consider its theoretical and policy implications. The first important feature of our model is that it does not, unlike most tax evasion models, assume that the revenues collected by the government simply disappear. That would have been in our opinion an unforgivable simplification.

\(^1\)For the sake of tractability we consider a one-sector general-equilibrium model with government-taxpayer interactions within an economic growth framework. For alternative non-growth models with different possibilities for tax evasion in two sectors refer to Kesselman (1989; 1993) and Jung et al. (1994).

\(^2\)Although there are differences in modelling techniques and assumptions used, most empirical studies do suggest a clear positive relationship between taxes and tax evasion (or hidden economy, in general). These studies are Clotfelter (1983), Crane and Nourzad (1990), Alm, Bahl and Murray (1993), Giles and Caragata (2001), to name a few. Nevertheless, in our opinion, Inconsistency 1 is still somewhat questionable since a few studies report a negative association between taxes and tax evasion. Refer to Feinstein (1991) and Geeroms and Wilmots (1985), for example.

\(^3\)For a thorough review of the problem refer to Dhami and al-Nowaihi (2007).
Government spending, financed by the revenues collected, can enhance economic activity indirectly via externalities, or be as a direct input to private production. As was stated by Cowell and Gordon (1988), "... while the government taketh away, it also giveth back, and the latter activity surely exerts some influence on evasion". The quote should be especially relevant for Australia where public spending traditionally is an important component of the economy. Kam and Wang (2008) show that public capital has dominated the process of economic development in Australia. In addition, despite a marked downward trend in the ratio of public investment to GDP, and given a rising cost of public investment, Otto and Voss (1998) showed that there was no systematic evidence of the resource misallocation in Australian public sector. Therefore, it is worth rigorously exploring a possible theoretical link between government spending and tax noncompliance, highlighting the implications for Australia. This is one of the aims of our paper. Further, given that public spending is useful for private production, would it be optimal for the government to set a statutory tax rate higher than the degree of the public spending externality in the presence of tax evasion? According to Chen's (2003) deterministic growth model, the answer is 'yes'. Our model answers 'not necessarily'.

It is worth mentioning, existing studies generally ignored the fact that while tax noncompliance might generate ill-gotten benefits to the taxpayer it also leads to a serious waste of resources in the process, such as "shoe-leather" costs associated with evasion. Tax cheaters may also prefer to shelter illegal gains from domestic financial institutions, and this may forgo interest payments and cause investment misallocation. Costly evasion was previously considered by Usher (1982, 1986), Cowell (1990), Kaplow (1990), Cremer and Gahvari (1994), Yaniv (1999), Chen (2003) and Bayer and Sutter (2008). These models are mainly theoretical, except Bayer and Sutter's study. Bayer and Sutter quantify the impact of various policy instruments on the excess burden of tax evasion in a rigorous experimental game-theoretic framework. Chen's calibration exercise sheds some light on the value of evasion costs in the context of East Asian economies. No such estimate is available for Australia, and more importantly, to what extent these costs can mitigate the taxpayer's evasion incentives is an unanswered question. An insight into such incentives is essential. If costly evasion successfully deters excessive noncompliance, the government might want to exercise some power over it in order to discourage tax crimes. In our model we attempt to pin down the magnitude of tax evasion costs in Australia, rather than assume its value exogenously.

We now emphasize main features and implications of our model.

- **We consider a stochastic growth environment**, where a rise in the evasion and tax rates contribute to the riskiness of the taxpayer's budget constraint by affecting the latter's variance term. In such an environment, a typical risk-averse taxpayer's aspirations towards noncompliance are somewhat mitigated, potentially helping to reconcile Inconsistency 2. Note, in deterministic models the interaction between taxes and evasion only partially capture the riskiness associated with the evolution of the individual's capital profile;

- **Although the deterministic growth model by Chen (2003) is a direct ancestor of ours** in terms of considering productive public sector and evasion costs, our model has two distinct features. First, we are able to theoretically account for the role of public sector spending in explaining tax evasion behavior. This is not possible in Chen's model, (nor in a static model) and

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4 For a thorough review of the evidences on the role of the broad public infrastructure capital refer to Otto and Voss (1995).

5 Bayer and Sutter (2008) assume that fraction of income wasted in the process of tax noncompliance is 4 percent.

6 In Chen's deterministic continuous-time model there is no analytical solution to the evasion rate (refer to his equation (7) with $h_0=0$). Hence, it is not possible to rigorously isolate the mechanism according to which public...
this has not been analyzed in a similar manner previously.\textsuperscript{7} Thus, we contribute to the understanding of the above quote by Cowell and Gordon (1988) regarding the role of public revenues in explaining tax noncompliance. Second, in Chen’s study the welfare-maximizing government must \textit{always} increase the statutory tax rate beyond what is satisfied by the natural efficiency condition of government size (Barro’s (1990) natural efficiency condition).\textsuperscript{8} Based on an Australian example, our model predicts that the conventional output elasticity of public input in production is generally a good rule-of-thumb to determine the growth-maximizing tax rate. That is, even if there is some tax evasion, the government cannot use an efficiency argument as an excuse for a heavier statutory tax burden.

- Whilst most studies analyzed tax evasion phenomenon for the U.S. and Europe, we focus on Australia. To the best of our knowledge, the welfare implications of the government tax policy in the presence of tax evasion has not been analyzed for Australia within a general-equilibrium growth model.\textsuperscript{9} Our study has two key predictions. First, given a realistic calibration, the existing average marginal tax rate in Australia falls short (but only slightly so) of the optimal one. In addition, our model predicts that given costly evasion, one should not expect the noncompliance rate in Australia to rise significantly over about a quarter of the true income.\textsuperscript{10}

Welfare implications of our model are as follows. A cut in the tax rate, on the one hand, raises the mean, after-tax return on taxpayers’ income and tends to stimulate growth via more private capital accumulation. On the other hand, resulting lower tax collections suppress the output-enhancing public spending. Further, a cut in the tax rate reduces the evasion rate (which alone would, \textit{ceteris paribus}, lower private capital holdings), but lower noncompliance would also reduce the magnitude of wasteful resources associated with evasion. Hence, based on the latter argument the net effect on growth will be ambiguous. Moreover, a low evasion rate helps the economy to better capitalize on the government expenditure externalities, which will be growth-enhancing. Finally, because of the stochastic nature of the taxpayer’s budget constraint in our model, both tax and noncompliance rates further distort the taxpayer’s behavior. We see our simple model creates a strong and complicated theoretical ambiguity between fiscal policy and growth. There is, therefore, no \textit{a priori} reason to believe that a given decrease or increase in the tax rate will be welfare-improving. Our benchmark parameterization and sensitivity analysis demonstrate that initially the growth rate rises as the tax rate increases, reaching a maximum at a positive rate, then decreases at higher tax rates. Our calibration exercise suggests the optimal marginal tax revenues can exert any influence on tax evasion. Essentially, in Chen’s model public goods provision simply offers a rationale for taxation.

\textsuperscript{7}To our knowledge, the only other study to consider a role of public spending in non-compliance decision was that of Cowell and Gordon (1988). However, their modeling is different from ours as their static model assumes tax revenues enter the utility of the taxpayer in a particular way.

\textsuperscript{8}This is even true if the cost of administering the tax system in Chen’s model is set to zero to be consistent with our setting (see expression (17) in Chen).

\textsuperscript{9}We want to emphasize our main goal to analyze tax evasion behavior within a realistic but tractable calibrated general-equilibrium model. In the calibration exercise we try to capture the structure of evasion process in Australia by using available parameter values. We are, however, aware of the disagreement on the extent of tax evasion in the country (Breusch (2005)), so our numerical exercise should \textit{not} be treated as an estimation of Australian tax evasion. Comprehensively reconciling the disagreement would go considerably beyond the scope of our study.

\textsuperscript{10}We do not model in this study the costs associated with tax administration. One simple way to do so would be to assume these costs erode a fraction of public revenues, which might suggest higher optimal tax rates in order to finance a given government expenditure. In a more complicated scenario, reducing the cost of administering the tax system by lowering the audit rate should free up some resources which would help to reduce the budgetary burden and could then allow for a \textit{cut} in the tax rate (Kolm (1973), Baldry (1984)).
rate in Australia should be just a few percentage points lower than the present rate. It is worth noting, however, our numerical example is fit to our analytical model which abstracts from some important real-world extensions in order not to lose analytical tractability. Thus, one might treat our numerical results as somewhat illustrative. Indeed, the precise welfare analysis of any tax policy change would obviously remain wide open for empirical debates (and perhaps even more so in an endogenous growth framework). A more realistic welfare study would thus require quite complicated general-equilibrium simulations (but with fewer analytical propositions), encompassing more than one sector and certainly allowing for endogeneity in labour supply decisions.\footnote{For a broad role of tax policies within a general-equilibrium framework, refer, for instance, to Creedy (1997).}

The rest of the paper is organized as follows. We first very briefly revisit a static Allingham-Sandmo-Yitzhaki model of tax evasion to stress its main inconsistencies once more. We show that even if one assumes income-enhancing government spending in the static framework, the evasion rate in the economy is not going to be affected. In the next section it will become clear this is not the case when public spending externalities are introduced in a general equilibrium dynamic model. We start by developing a general model with convex evasion costs. We then temporarily disregard the costs to draw a consistent parallel with static models. Calibrating our model confirms some of the salient features of Australian economy. Further, having analyzed the implications for optimal taxation and the magnitude of evasion costs, we conclude the paper with some critical remarks and caveats.

## 2 Modeling Tax Evasion

### 2.1 A Static model: Inconsistency 1, Inconsistency 2 and the irrelevance of government spending

In static models the output-enhancing government revenue does not play a role in influencing the evasion rate of a representative taxpayer. To see this consider first a simple environment in the spirit of Allingham and Sandmo (1972) and Yitzhaki (1974), where a rational taxpayer with income, \(y\), chooses a fraction of income, \(e \in (0, 1)\), to hide from the government. The latter imposes a constant marginal tax rate, \(\tau \in (0, 1)\), on any declared income. The government detects the taxpayer with probability \(\pi \in (0, 1)\), and upon catching the evader, imposes a penalty on the amount of undeclared taxes. The penalty is given by the parameter \(\theta = 1 + s\), where \(s > 0\) is the surcharge rate. The random return on a unit of evaded tax, \(r\), is 1 with probability \(1 - \pi\), and \(1 - \theta\) with probability \(\pi\). Thus, \(\mathbb{E}[r] = 1 - \pi \theta\) and is assumed to be positive, with \(\mathbb{E}\) standing for the expectation operator.

The taxpayer’s preferences are represented by a standard isoelastic utility function where for the sake of tractability we consider logarithmic preferences.\footnote{Sometimes \(\pi\) is interpreted as the taxpayer’s subjective evaluation of the probability of getting caught. Certainly, objective and perceived detection probabilities do not have to be the same but to be consistent with most studies in this line of research we treat them as if they are.} The taxpayer’s problem is to solve the following program:

\[
\max_{\{e\}} \mathbb{E}[\ln c], \tag{1}
\]
where consumption, \( c \), is a random variable:

\[
    c = (1 - \tau) y + r \tau \epsilon y.
\]

(2)

The solution to the above program yields the optimal evasion profile, given by:

\[
    e = \frac{(\tau - 1)(\pi \theta - 1)}{\tau(\theta - 1)}.
\]

(3)

Expression (3) leads to the following remark.\(^{14}\)

**Remark 1** Existence of income enhancing public input does not have any impact on the evasion rate.

Indeed, note government revenues are equal to

\[
    (1 - \pi)(1 - e) \tau y + \pi (\theta \tau (y - (1 - e) y) + (1 - e) \tau y) = \tau y (1 - e (1 - \pi \theta)) = (1 - \tau e) \tau y.
\]

(4)

Clearly, government revenue collections depend on the noncompliance rate. Now, assume the revenues, once collected, are directed to finance expenditures enhancing the income level, \( y \). Essentially, that means \( y \) is an implicit function of \((1 - \tau e) \tau y\). Since the first-order conditions in the taxpayer maximization problem cancel any effect of income, \( y \), on the choice variable, \( e \), the fact that income is endogenous is irrelevant for the taxpayer in a large economy. Not only will Yitzhaki’s puzzle be unsettled, but essentially two drastically different economies with different effects of fiscal spending will have similar consequences for people’s tax evasion incentives. These results, however, are sensitive to the model specification. Thus, we next consider a continuous-time stochastic model to see that the implications are, indeed, drastically different from the static case.

### 2.2 Tax evasion in a stochastic growth model with productive government spending

#### 2.2.1 A case with a costly and costless evasion

Consider a household-producer who has an access to the following aggregate production function as in Barro (1990):

\[
    y(t) = Ak(t) \left( \frac{g(t)}{k(t)} \right)^{1-\alpha},
\]

(5)

where \( 0 < \alpha < 1 \), and \( g(t) \) is the amount of public services available to each household-producer. That is, we assume the government provides services to private sector without charging user fees, and these public services increase the marginal product of private capital. Examples of productive public spending include the government expenditure on infrastructure, research and development.

\(^{14}\)Here a reader would recall Inconsistency 1 mentioned earlier. One can differentiate (3) with respect to \( \tau \) to confirm it. Intuitively, when fines are imposed on the amount of concealed taxes, a change in the tax rate increases expected penalties and the marginal benefit of cheating proportionally, rendering the substitution effect zero. Recall we are considering logarithmic preferences, which exhibit decreasing absolute risk aversion, meaning that the optimal dollar amount invested in risky assets will be an increasing function of wealth. Since a rise in the tax rate means a negative wealth effect, the taxpayer would want to reduce the optimal amount of undeclared income. With income being constant, the only way to do it is to reduce the fraction to hide, \( e \). Further, for arguments’ sake assuming \( \tau =0.30, \pi =3\%, \) and \( \theta =1.5 \) would lead to an absurdly high evasion rate, confirming Inconsistency 2.
and education, to name just a few. We assume, like Barro (1990) does, the services are not subject to congestion effects. Hence, for the sake of simplicity we do not explicitly model congestion with varying degrees of non-rivalry and non-exclusiveness as in Kam and Wang (2008), for instance.

Note the amount of productive government services can be represented as

\[ g(t) = \tau y(t) = \tau A k(t) \left( \frac{g(t)}{k(t)} \right)^{1-\alpha}, \]

where \( \tau \equiv (1 - \tau e(t)) \) \( \tau \) is the effective tax rate as in (4) and \( c(t) \) is the fraction of income concealed at time \( t \). If \( e(t) = 0 \), the effective tax rate is the same as the statutory tax rate, \( \tau \). From (6) it is easy to deduce that

\[ \left( \frac{g(t)}{k(t)} \right)^{\alpha} = A \tau. \]

Consequently, expression (5) can be presented as

\[ y(t) = A^{\frac{1}{\alpha}} k(t) \tau^{\frac{1-\alpha}{\alpha}}. \]

Assume tax evasion involves costs described by the function \( \xi e^2(t) \), \( \xi > 0 \). As was stated previously, tax evasion involves a variety of real social costs, such as time, efforts, and money wasted as the taxpayer attempts to ensure he will get away with cheating. These costs include, but are not limited to, for instance, foregone interest payments, more frequent contacts with tax experts in an attempt to discover smarter concealment technologies. Chen (2003) assumes that these costs also encompass private expenditures, such as bribes paid to government tax collectors. Our view of this, however, is more consistent with that of Cremer and Gahvari (1994), who state: "Expenditures to conceal tax evasion may entail "bribes" as well as real resource costs. Bribes are income transfers with no efficiency loss, unless their existence results in rent-seeking activities." In addition, we would like to emphasize another type of costs relevant for Australia, where it is traditionally a common practice for some local businesses to accept only cash transactions. The cash transactions would discourage cash-constrained consumers who might be reluctant to withdraw even a small amount of cash from their credit card accounts because of immediately charged high interests. Hence, such failed transactions would lead to additional excess burden shared by everyone in the economy.

The taxpayer’s flow budget constraint becomes

\[ dk(t) = (1 - \tau - \tau e(t) - \xi e^2(t)) \left( y(t) - c(t) \right) dt + [\sigma \tau e(t) y(t)] dz(t), \]

where \( c(t) \) is real per capita consumption at time \( t \), \( \sigma > 0 \) is the instantaneous standard deviation parameter, and \( z(t) \) obeys a Wiener process. Clearly, the second term on the right-hand side of

15The strictly convex cost structure which we assume here is not arbitrarily chosen. In fact, Cowell (1990) proves within a simple expected utility framework that the taxpayer’s willingness to invest in concealment technologies, enabling him to get away with tax evasion, must be a strictly convex function of the evasion rate. Chen (2003) also assumes the same cost structure as we do. Furthermore, in a developed country like Australia with relatively well-developed third party reporting system and advanced monitoring technologies, it is only reasonable to assume a typical taxpayer will find it excessively costly to conceal his income from the government tax authorities.

16Of course, this effect might be mitigated by the fact that these businesses are usually offering a cheaper service for cash.

17This Merton-type budget equation here closely follows Lin and Yang’s (2001) extension but with a correction offered by Dzhumashev and Gahramanov (2009). In fact, Lin and Yang (2001) were first to analyze a simple stochastic growth model with tax evasion, but only with \( Ak(t) \) production function. Dzhumashev and Gahramanov (2009) also argued that Lin and Yang failed to reconcile Inconsistency 1.
expression (9) involves both the tax and evasion rate, which jointly contribute to the variability to the path followed by \( k(t) \).

Obviously, within a sustainable growth setting in the context of tax evasion we have two control variables: \( c(t) \) and \( e(t) \) to be optimally chosen. Hence, the taxpayer’s problem leads to

\[
\max_{\{c(t), e(t)\}} \mathbb{E}_0 \left[ \int_0^{+\infty} \exp(-\rho t) \ln c(t) dt \right],
\]

subject to:

\[
(8), \ (9) \text{ and } k(0) = k_0,
\]

where \( \mathbb{E}_0 \) is the conditional expectation operator given \( k_0 \), and \( \rho > 0 \) is the discount rate. In a large economy the level of public services is negligibly affected by the taxpayer’s actions and \( g = g(t)/y(t) \) is given in a steady-state. The optimization problem leads to the stochastic Bellman equation

\[
\rho J(k) = \max_{\{c(t), e(t)\}} \left\{ \ln (c(t)) + J'(k) \left( (1 - \tau + \varpi e(t) - \xi^2(t)) y(t) - c(t) \right) + \frac{1}{2} J''(k) (\sigma \tau e(t) y(t))^2 \right\}.
\]

The first-order conditions of the right-hand side of (12) imply

\[
c(t) = \frac{1}{J'(k)},
\]

and

\[
e(t) = -\frac{J'(k) \varpi}{J''(k) (\sigma \tau)^2 y(t) - 2 \xi J'(k)}.
\]

Inserting (13) and (14) into the right-hand side of (12), leads to

\[
\rho J(k) = \ln \left( \frac{1}{J'(k)} \right) - 1 + J'(k) (1 - \tau) y(t) + \left( -\frac{J'(k) \varpi}{J''(k) (\sigma \tau)^2 y(t) - 2 \xi J'(k)} \right)^2 \times \left( \frac{J''(k) (\sigma \tau y(t))^2}{2} - \xi J'(k) y(t) \right) - \frac{\left( J'(k) \varpi \right)^2 y(t)}{J''(k) (\sigma \tau)^2 y(t) - 2 \xi J'(k)}.
\]

The solution to (15) is given by

\[
J(k) = \frac{1}{\rho} \left[ \ln k + \ln \rho - 1 + \frac{(1 - \tau) A_{1\frac{1}{\alpha}} \frac{1 - \alpha}{\alpha}}{\rho} + \frac{A_{1\frac{1}{\alpha}} \frac{1 - \alpha}{\alpha} (\varpi)^2}{\rho \left( A_{1\frac{1}{\alpha}} \frac{1 - \alpha}{\alpha} (\sigma \tau)^2 + 2 \xi \right)} \right] - \frac{1}{\rho} \left( \frac{\varpi}{A_{1\frac{1}{\alpha}} \frac{1 - \alpha}{\alpha} (\sigma \tau)^2 + 2 \xi} \right)^2 \left( \frac{A_{1\frac{1}{\alpha}} \frac{1 - \alpha}{\alpha} (\sigma \tau)^2}{2} + \xi A_{1\frac{1}{\alpha}} \frac{1 - \alpha}{\alpha} \right) \right]\]

\[\]

\[18\]The representative taxpayer’s problem is equivalent to the second-best welfare maximization excercise with the social planner ignoring the externalities (see Barro (1990) and Chen (2003)).
Therefore, utilizing (13) and (14), the taxpayer can derive his control variables:

\[ c(t) = \rho k(t), \tag{17} \]

and

\[ e(t) = \frac{\tau k(t)}{(\sigma \tau)^2 y(t) + 2\xi k(t)}. \tag{18} \]

Now let us temporarily relax the assumption of costly evasion to be consistent with a static environment. When \( \xi = 0 \), expression (18) becomes

\[ e(t) = \frac{\tau k(t)}{\sigma^2 \tau y(t)}. \tag{19} \]

**Remark 2.** In the stochastic dynamic setting, the evasion rate is proportional to the capital-output ratio. Two different economic environments with different role of the government spending should generate different capital-output ratios, and hence, ceteris paribus, the noncompliance rate would be different in general-equilibrium.

Remark 2 cannot follow from a static model as we have seen the evasion rate there is constant at all times and no matter how public spending augments private production. Further note, it is not sufficient to simply develop a stochastic growth model to come up with Remark 2. If one considers \( y(t) = Ak(t) \) technology even in our model, expression (19) will always be constant like in the static case.

Now, assume again \( \xi > 0 \). Then what about an impact of the tax rate on the economy-wide evasion rate? The answer is given in the following proposition.

**Proposition 1** A change in the tax rate ambiguously affects the general-equilibrium evasion rate.

**Proof.** Rewrite (18) as

\[ e(t) \left( A^\frac{1}{\alpha} \sigma^2 \tau^\frac{1}{\alpha} (1 - \tau e(t))^{1-\alpha} + \frac{2\xi}{\tau} \right) - \tau = 0. \tag{20} \]

Apply implicit differentiation to (20) and simplify, to find that

\[ \frac{\partial e(t)}{\partial \tau} = \frac{2\xi \alpha e(t)}{2\xi \alpha \tau + A^\frac{1}{\alpha} \sigma^2 \tau^{\frac{1+2\alpha}{\alpha}} (1 - \tau e(t))^{\frac{1-2\alpha}{\alpha}} (\alpha - \tau e(t)) - e(t) (1 - \tau e(t)) - \frac{2\xi}{\sigma^2 \tau (\alpha - \tau e(t)) + A^\frac{1}{\alpha} \sigma^2 \tau^{\frac{1+2\alpha}{\alpha}} (1 - \tau e(t))^{\frac{1-2\alpha}{\alpha}}} \leq 0, \tag{21} \]

which confirms the proposition.  

Hence, our model does not guarantee that we will obtain a positive relationship between taxes and noncompliance incentives as (21) is impossible to sign. That is, we still can potentially end up running into Yitzhaki’s puzzle. In fact, once a general-equilibrium circular-flow effect is allowed for, little is clear about the signs of other comparative statics. To see this let us derive the comparative statics for the change in other important enforcement parameters, \( \pi \) (the probability of detection),
and \( \theta \) (the penalty rate). Theoretical studies typically confirm that tougher penalties and higher probability of detection discourage noncompliance. It is, of course, the case in a basic static model (just differentiate expression (3) with respect to \( \pi \) and \( \theta \)). We state here the following proposition.

**Proposition 2** A change in the penalty rate, \( \theta \), and the probability of detection, \( \pi \), can increase or decrease the general-equilibrium evasion rate.

**Proof.** Applying implicit differentiation again and performing some straightforward algebra, we find

\[
\frac{\partial e(t)}{\partial \pi} = -\theta \frac{\left( \frac{1-\alpha}{\alpha} A^{1} \sigma^{2} \tau^{1+\alpha} e^{2}(t) (1 - \tau e(t)) \frac{1-2\alpha}{\alpha} + \tau \right)}{A^{1} \sigma^{2} \tau^{1+\alpha} (1 - \tau e(t)) \frac{1-2\alpha}{\alpha}} \leq 0, \tag{22}
\]

and

\[
\frac{\partial e(t)}{\partial \theta} = -\pi \frac{\left( \frac{1-\alpha}{\alpha} A^{1} \sigma^{2} \tau^{1+\alpha} e^{2}(t) (1 - \tau e(t)) \frac{1-2\alpha}{\alpha} + \tau \right)}{A^{1} \sigma^{2} \tau^{1+\alpha} (1 - \tau e(t)) \frac{1-2\alpha}{\alpha}} \leq 0, \tag{23}
\]

which confirms the proposition.

At most we can see if the above two expressions happen to be negative (e.g., when the starting value of \( \alpha \) is large enough), then \( \partial e(t)/\partial \theta < \partial e(t)/\partial \pi \) in absolute terms, which shows that a change in the probability of being caught is more effective in deterring noncompliance. This is in conformity with many empirical findings. But to make exact inferences, we need to calibrate the model fully. We render that in the next section.

### 3 Calibration

#### 3.1 Benchmark scenario and sensitivity analysis

In this section we describe the calibration of our extended model, and report our quantitative findings to confirm some of the salient features of Australian economy. It is typically assumed that the random return on a unit of tax evaded is around 0.90 (Dhami and al-Nowaihi (2007)). Hepburn (1992) reports that in Australia the probability of detection and the penalty rate is 0.11 and 1.39, respectively, and we do likewise. This leads to the expected return rate, \( \pi \), equal to 0.8471, which happens to be very close to that in Chen (2003). We follow the Hepburn’s tax rate variable, and set it to 0.3734 since this number is close to the marginal tax rate of an average Australian worker for the past few decades as reported by the Treasury.

We set the rate of time preference, \( \rho \), to 2\% which is consistent with the range normally assumed in this line of literature. The standard deviation of the normalized process of the random return on tax evasion, \( \sigma \), can be approximated via the expression \( \theta \sqrt{\pi (1-\pi)} \) (see Dzhumashev and Gahramanov (2009) for details), which becomes 0.1892. We set the degree of the government externality, \( 1-\alpha \), to 0.30 but we do a sensitivity analysis in regards with this parameter.\(^{19}\) With

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\(^{19}\)In fact, \( \alpha \) is a parameter which is rather difficult to pin down precisely, and there is much disagreement about its size among economists (e.g., Costa et al. (1987), Aschauer (1989), Munnell (1990), Ford and Poret (1991), Garcia-Mila and McGuire (1992), Tatom (1991), Bajo-Rubio and Sosvilla-Rivero (1993), Sturm and de Haan (1995), Lau and Sin (1997)). Also there is a theoretical possibility that causality might run not from public investment to output and productivity, but the other way around (Fernald 1999), which further adds to mixed results. However, Otto and Voss (1994b) showed that in Australia shocks to private output have no significant feedback effects on public capital. In addition, available empirical studies often employ alternative specifications and different production functions, so
logarithmic preferences, all remains now is to pick the values for $A$ and $\xi$. In order to produce the evasion rate of about 7.11% (as in Hepburn (1992)) and the growth rate of economy (to be elaborated on later), $\gamma$, of about 2.1%, we set $A$ equal to 0.2003 and $\xi$ equal to 2.225.\(^{20}\) The parameters are summarized in Table 1.

Table 1.

<table>
<thead>
<tr>
<th>Parameter name</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability of detection</td>
<td>$\pi$</td>
<td>0.11</td>
</tr>
<tr>
<td>Penalty rate</td>
<td>$\theta$</td>
<td>1.39</td>
</tr>
<tr>
<td>Tax rate</td>
<td>$\tau$</td>
<td>0.3734</td>
</tr>
<tr>
<td>Rate of time preference</td>
<td>$\rho$</td>
<td>0.02</td>
</tr>
<tr>
<td>Coefficient of productivity</td>
<td>$A$</td>
<td>0.2003</td>
</tr>
<tr>
<td>Degree of government externality</td>
<td>$1 - \alpha$</td>
<td>0.30</td>
</tr>
<tr>
<td>Coefficient of evasion costs</td>
<td>$\xi$</td>
<td>2.225</td>
</tr>
<tr>
<td>Variance</td>
<td>$\sigma^2$</td>
<td>0.1892</td>
</tr>
<tr>
<td>Return rate on evasion</td>
<td>$r$</td>
<td>0.8471</td>
</tr>
</tbody>
</table>

The magnitude of the evasion cost parameter can seriously mitigate the evasion incentives. In general, doubling the cost parameter, $\xi$, decreases the evasion rate almost proportionally. Even with an original value of $\xi = 2.225$, assume an extremely attractive tax evasion gamble when $\tau = 100\%$, and $r$ is close to unity. Then the equilibrium evasion rate will be close to 22.5%. That is, 22.5% can be interpreted as an upper bound for the evasion rate of a typical Australian taxpayer, which is intuitively sensible as in a developed country with a sound tax system dodging more and more taxes can become increasingly transparent to government tax officials, who traditionally have little tolerance for such an illegal activity. With high evasion rates the taxpayer should require then significant investments in concealment technologies.\(^{21}\)

The parameter values are summarized in Table 1. All the values are available from literature, except the technology parameter, $A$, and evasion cost parameter, $\xi$, which we calibrate within the model. Our benchmark parameterization suggests the share of evasion costs in income, $\xi e^2(t)$, becomes about 1.12%, which is only slightly higher than that in Chen (2003), and is much more conservative than what is assumed in the experimental study of Bayer and Sutter (2008). Our it might be less clear-cut how the estimates they provide refer to the $\alpha$ parameter we use in this study. Our reading of the available literature concerning Australia generally suggests optimistic results about the role of productive public investments in enhancing the real output. According to Otto and Voss (1994a), the elasticity of the ratio of output to private capital with respect to the ratio of public to private capital exceeds 0.40 in Australia, while Kam and Wang (2008) find it to be equal roughly to 0.35. Kamps (2005) reports $1 - \alpha$ as roughly 0.30. We decide to use the lower bound of the estimate since in our model $g(t)$ is rather total taxes collected, not all of which are productive in reality. Furthermore, our guess for $1 - \alpha$ falls comfortably in the midpoint of the conflicting range typically reported for the estimates on the private output elasticity of public capital (Otto and Voss (1995)).

\(^{20}\)We deliberately choose Hepburn’s estimate of the cash-based income tax evasion in Australia as it is close to the mid-point of the numbers reported by different sources. Bajada (1999), for example, estimates that income underreporting in Australia is as high as 15% of GDP, while Breusch (2005) argues that it is much smaller in fact, roughly about 2% of GDP. We should mention that adjusting the values for the compliance and cost parameters along their feasible and reasonable values is capable of hitting both latter figures within our model, but in so doing, we verified that the results hereafter would not be affected significantly, and hence we move on with our existing parameterization.

\(^{21}\)In addition, we should emphasize that $\xi$ is an unobservable parameter, and the present value of it implies rather steep evasion cost function. The value of $\xi$ should not necessarily be bound to be that large. For instance, if we assume the fraction of income hidden in Australia is as high as that reported in Bajada (1999), we can even consider the value of $\xi$ slightly below 1.
benchmark calibration results in the steady-state share of government tax revenues in output being equal to 35.09%, which is a few percentage points larger than that reported in Galí (1994).  

3.1.1 Comparative statics revisited

We can now turn our attention to Yitzhaki's puzzle and investigate the compliance behavior for different tax rates. Substituting the above values into equation (21) we find \( \frac{\partial e(t)}{\partial \tau} \approx 0.19 \). At the existing evasion and tax rates, a unit increase in the latter increases noncompliance rate by less than a unity. To see if such a pattern holds within a sufficiently large vicinity of the current state of the Australian economy, we fix all the parameter values in Table 1 except the tax rate, which now is allowed to take discrete jumps from the closed interval [0%, 100%] (step size is chosen to be 0.005). The results of the simulation are presented in Figure 1 below.

![Figure 1](image)

Figure 1: Blue line reflects the relationship between the tax rate and the evasion rate for the benchmark parameterization. The present state of Australian economy with the tax 37.34% and the evasion rate 7.11% is labelled by the black dot.

We see a very clear positive association between taxes and noncompliance when the tax rate varies from zero all the way up to 100%. Repeating our exercise for other reasonable values of \( \alpha, \pi \) and \( \theta \) we get a similar positive association between taxes and evasion, and Thus, we do not report those results here. In the economy with tax evasion the government therefore would be unable to render the private income zero even by imposing the heaviest statutory tax burden.

We should point out here that in Figure 1 when taxes are nil, so are the evasion incentives. However, one should not conclude that such an extreme fiscal freedom is going to render the magnitude of hidden economy zero. Our model refers to tax evasion only as a proxy for the hidden economy, but does not capture a variety of potential country-specific social considerations, which would support a minimum level of underground economy even in the absence of any taxes. Such a "natural rate of hidden economy" might in fact be non-trivial in size, and Giles and Caragata (2001) report it to be between 4 to 4.5 percent of GDP in New Zealand, where the long-run estimate of the hidden economy ratio exceeds 8 percent of measured output (Giles (1999)).

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22 For simplicity, of course, we assume that the government budget is balanced.
Turning our attention to the enforcement parameters, \( \pi \) and \( \theta \), we can evaluate the signs of (22) and (23) by using our benchmark calibration parameters. Consequently, \( \partial e (t) / \partial \pi \) and \( \partial e (t) / \partial \theta \) become -0.12 and -0.01, respectively, which shows indeed that a change in the probability of being caught is more effective in deterring evasion than a change in the penalty rate.\(^{23}\) Importantly, the numbers we get conform empirical findings. The detection probability tend to suppress noncompliance, albeit not too aggressively (see, e.g., Fischer et al. (1992) and references therein), while tougher penalties do not significantly impact tax compliance behavior (see, e.g., Baldry (1984), Fortin et al. (2007)). On the contrary, considering the static case and using our benchmark parameterization results in \( \partial e / \partial \pi \approx -5.98 \) and \( \partial e / \partial \theta \approx -9.82 \). Hence, unlike our dynamic model, the static model predicts a much more optimistic effect of traditional enforcement parameters on tax evasion, and asserts that penalties are more effective in combatting tax evasion incentives than detection probabilities. In both static and dynamic models, a change in the probability of detection lowers the return rate, \( r = 1 - \pi \theta \), by a larger magnitude than a change in the penalty rate does. However, in the dynamic model with productive government expenditures, \( g (t) \) increases when \( \pi \) falls at any given level of income, and the rise in \( g (t) \) is larger when \( \pi \) goes up. Hence, the general equilibrium effect of an increase of the probability of detection on the the evasion rate is larger.

### 3.2 Computation of the optimal tax rate

In the above scenario for arguments’ sake we used benchmark parameterization to replicate some observed features of the Australian economy. Hence, the tax rate was exogenously determined and other parameters conveniently produced the observed noncompliance and the growth rates. Now we turn our attention to finding the optimal value for the statutory tax rate, maximizing the utility of a representative household, given the government-taxpayer interaction. It is well-known that such a welfare maximization boils down to the maximization of per capita consumption. Our aim is to see how close the present state of the Australian economy comes to that characterized by the theoretical model. We adopt the following definition.

**Definition** The optimal equilibrium tax rate, \( \tau^* \), is defined as

\[
\tau^* \equiv \arg \max \{ \gamma (\tau) \},
\]

with

\[
\gamma (\tau) \equiv \mathbb{E} \left[ \frac{dk (t)}{dt} / k (t) \right] = \left( 1 - \tau + \pi e (t) - \xi e^2 (t) \right) A^{1/\alpha} \left( 1 - \tau e (t) \right) \frac{1-\alpha}{\alpha} - \rho,
\]

and when the following conditions are simultaneously satisfied: (i) the individual optimization conditions, (17), and (18), ii) the individual budget equation, (9), and iii) the government’s budget equation, (6).

The function \( \gamma \) represents the mean growth rate of capital and consumption (and therefore, of the economy since there are no transitional dynamics), which can be found by substituting (17) into (9), and taking the expected value of the right-hand side, while ensuring (18) always holds. Since we do not have an explicit expression for the equilibrium evasion rate, the maximization of the \( \gamma \)-function with respect to \( \tau \) does not permit an analytical solution. Thus, we conduct the numerical computation of the optimal statutory tax rate.

\(^{23}\)Since the value of \( \alpha \) is greater than \( \pi e (t) \), expressions (22) and (23) will both be negative, and thus we are not illustrating them graphically.
Figure 2 reflects our computation of the optimal tax rate and resulting equilibrium evasion rate for three different levels of the government externality, $1 - \alpha$, with $\alpha = 0.70$ as in the benchmark scenario, as well as with $\alpha$ equal to 0.80 and 0.90.

**Figure 2**

Figure 2: Blue line is for the economy with $\alpha = 0.70$ as in the benchmark scenario. Broken line is for the economy with $\alpha = 0.80$. Solid black line is for the economy with $\alpha = 0.90$. All three lines meet at the black dot, which represents the present state of Australian economy, with the resulting tax rate and evasion rate equal to 37.34% and 7.11%, respectively, and with the resulting growth rate of 2.1%. Red rhombs represent the optimal states of the economy for different values of $\alpha$.

When we consider $\alpha$ equal to 0.90 and 0.80, our evasion rate is very close to 7.1%, and Thus, we just adjust the technology parameter to 0.0940 and 0.1377, respectively, to hit the target growth rate of 2.1% (and the evasion rate then rises to 7.11%). This ensures all three curves in Figure 2 intersect at the black dot, which has the same interpretation as in Figure 1. With a relatively low degree of the government expenditure externality ($\alpha = 0.90$), solid black line reaches its peak at the statutory tax rate equal to only 10 percent, with the corresponding growth rate of 3 percent, and evasion rate 1.9 percent. With higher public expenditure externalities ($\alpha$ equal to 0.80 or 0.70 as in the benchmark) the optimal tax rate becomes 20 and 30 percents, respectively, with the corresponding evasion rates of 3.81 and 5.71 percents. With $\alpha$ equal to 0.80 and 0.70, the optimal growth rate of the economy is about 2.5 and 2.2 percents, respectively. Other things being equal, the greater the productivity of the government is, the more tax revenues are required to capitalize on those benefits. Thus, the optimal marginal tax rate in the benchmark scenario is 30% (and the excess burden of evasion as a fraction of income, $\xi e^2(t)$, would only be 0.73 percent of income). It might, therefore, be tempting to say that our model essentially replicates Barro’s (1990) optimality condition, where the degree of the government spending externality, $1 - \alpha$, equals to the welfare-maximizing statutory tax rate. However, this is not the case. If we decrease $1 - \alpha$ parameter to 0.50, then the optimal tax rate becomes slightly higher, namely 51%. If we set $1 - \alpha$ to 0.80, the optimal tax rate would become 85%. If $1 - \alpha$ further rises to 0.90, the optimal tax rate is 91%.

As was previously mentioned, the presence of tax evasion induces the welfare-maximizing government to increase the tax rate in order to achieve the optimal level of public revenues. But higher taxes stimulate evasion, which adds to the excess burden driven by noncompliance, inducing the government to lower the tax rate. On the other hand, large resource wastes mean lower tax base, tempting the government to increase taxes to finance given expenditure levels. Finally, recall that
taxes (and evasion) in our model further distort the taxpayer’s behavior by adding noise to the path of individual capital profile.

We see the interplay of different forces can move $\tau^*$ to either direction, and there is no a priori reason to expect higher or lower tax rates at various evasion-externality combinations. This becomes clear in Figure 3 below, where on the horizontal axis we plot the evasion rate, and on the vertical axis we label the optimal tax rates and the degree of public sector externality. We allow evasion rate to take initial values of 2 and 15 percents, which is consistent with the range empirically pinned down for Australia. We then, for the sake of illustration, assume higher evasion rates, up to 35 percent.

![Figure 3](image-url)

**Figure 3**

For instance, blue solid line represents the economy where the degree of government externality is 60%. Corresponding blue stars are the optimal general-equilibrium statutory tax rates for different target noncompliance rates. When, for example, the evasion rate is 35% and $1 - \alpha$ is 0.60, $\tau^*$ is only 50.5%. Consider now our benchmark value (black line with $1 - \alpha = 30\%$). We see black stars remarkably closely overlap with the straight line at various evasion rates. Hence, for reasonable values of evasion and public sector externality degrees, we find that the statutory tax rate can be set according to the natural condition for productive efficiency proposed by Barro (1990). That is, the mere existence of noncompliance cannot serve as an efficiency argument to justify fiscal expansion.

### 4 Conclusion

In this paper we analyze the income tax evasion phenomenon within an endogenous stochastic growth model with costly compliance. The calibration of our model to some of the salient features of Australian economy predicts a realistic evasion rate. The resource costs wasted in the process of tax noncompliance are calculated to be around one percent of income. We also were able to avoid a non-sensible positive relationship between taxes and compliance incentives present in traditional models. The numerical estimation shows that a cut in the marginal income tax rate by a few percentage points will be both growth-enhancing, and evasion-reducing.

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24 Again, we adjust $\xi$ and $A$ values accordingly to get the target 2.1% growth rate of the economy.
Since two economic environments with different roles of the government spending in private production should in principle lead to different capital-output ratio, the non-compliance rate will vary across the economies. This is very important as when the government collects taxes it also gives something back in return. The latter activity surely should exert some influence on noncompliance. Static models, unlike ours, find no such influence (unless the utility function is augmented with public spending in a special way like in Cowell and Gordon (1988)).

Finally, it has been argued by Chen (2003) that income taxation must be more aggressive when tax evasion is present. That is, the optimal tax rate must be higher than Barro’s natural efficiency condition. A political implication would be that the government can always call for a heavier taxation as a necessary reaction for noncompliance. Our model shows, however, for a wide range of reasonable simulations, the income tax rate should not exceed the degree of the government expenditure externality for private production. That is, fiscal expansion should be limited by the usefulness of government revenues in the first place.

To finalize, we wish to consider some caveats. We would like to stress that in our paper we purely focus on efficiency issues of personal income tax and disregard distributional aspects and other non-income taxation mechanisms. However, one might argue that lowering marginal income tax rates and correspondingly adjusting consumption taxation to ensure a revenue-neutral tax mix change might in fact contribute to the equity and compliance of the Australian personal tax system (Mathews (1980), Swan (1984), Groenewegen (1984), (1985)). However, according to Kesselman’s (1993) counter-argument, such a shift between two taxation schemes can be equity and compliance-detrimental as industry sectors evading previously on their income taxes, would switch to evading more on indirect taxes on their corresponding value added. Furthermore, besides individual responses to changes in tax structure, the response of trade unions is important as well. It has been argued that a simple revenue-neutral reduction in the marginal rate of income tax and a simultaneous increase in consumption tax would increase the wage demands of all trade unions, provided that unions are willing to trade employment for wages (Creedy (1992)). Then a rise in the income tax threshold might be needed to curtail the unions’ wage aspirations (Creedy and McDonald (1990)).

Even if the focus were on an income taxation scheme alone, the extent of tax evasion would be capable of affecting the redistributive effect of a progressive tax schedule, which we ignored in order not lose analytical tractability. Freire-Serén and Panadés (2008) proved that for a given progressive tax function, tax evasion modifies the distribution of the after-tax income depending upon the attitude of the taxpayer toward absolute risk aversion. In addition, another worthwhile extension would be to decompose the tax rate variable into personal, indirect and corporate segments and analyze the implications for the hidden economy and growth maximization. An interesting methodology regarding this argument is presented in Scully (1996) and Caragata and Giles (2000). We believe extending our modelling technique with appropriate maximization arguments for tax policy to account for the above arguments should open interesting avenues for further debates.

25Though, in Australia the overall power of unions has been notably decreasing since past decades.
References


