Financial Econometrics Series

SWP 2012/10

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State Dependent Asymmetric Loss and the Consensus Forecast of Real U.S. GDP Growth

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We would like to thank C. James Hueng and Kevin Corder for their insightful comments and suggestions. Any remaining errors are ours.
Abstract

It has been well documented that the consensus forecast from surveys of professional forecasters show a bias that varies over time. In this paper, we examine whether this bias may be due to forecasters having an asymmetric loss function. In contrast to previous research, we account for the time variation in the bias by making the loss function depend on the state of the economy. The asymmetry parameter in the loss function is specified to depend on set state variables which may cause forecasters to intentionally bias their forecasts. We consider both the Lin-Ex and asymmetric power loss functions. For the commonly used Lin-Ex and Lin-Lin loss functions, we show the model can be easily estimated by least squares. We apply our methodology to the consensus forecast of real U.S. GDP growth from the Survey of Professional Forecasters. We find that forecast uncertainty has an asymmetric effect on the asymmetry parameter in the loss function dependent upon whether the economy is in expansion or contraction. When the economy is in expansion, forecaster uncertainty is related to a negative bias in the median forecast of real GDP growth. In contrast, when the economy is in contraction, forecaster uncertainty is related to a positive bias in the median forecast of real GDP growth. Our results are robust to the particular loss function that is employed in the analysis.

Key Words: Survey forecasts, Asymmetric loss, Time-varying bias

JEL Classifications: C53; D83
1. Introduction

Significant resources are spent each year by businesses and governments for obtaining forecasts of a variety of economic variables. Numerous surveys of professional forecasters repeated over time provide researchers with rich data sets to evaluate the forecasting accuracy of the forecasting profession. In spite of significant improvements in economic data collection and public availability, and advances in linear and nonlinear forecasting techniques, research over the last thirty years has consistently demonstrated that professional forecasters produce biased forecasts. Among others, Carlson (1977), Urich and Wachtel (1984), Caskey (1985), Zarnowitz (1985), Frankel and Froot (1987), Croushore (1993, 1997), Jeong and Maddala (1996) and Souleles (2002) found that forecasts of the key variables such as inflation, interest rates and GDP growth reported by professional forecasters are biased. Apparently, there are episodes where forecasters are systematically overly optimistic or overly pessimistic. These findings seem to call into question the value of professional forecasting services.

Recently, researchers have argued that biased forecasts may be the result of rational behavior if forecasters face asymmetric loss functions. Extending the work of Granger (1969), Christoffersen and Diebold (1997) showed that under asymmetric loss, the optimal forecast is the conditional mean plus a bias that depends on the parameters in the loss function and the second and higher order moments of the conditional distribution of the variable being forecasted. Assuming that forecasters have an asymmetric exponential loss function and that interest rates follow a GARCH process, Bachelor and Peel (1998) showed that rationality cannot be rejected for forecasts of three month treasury bill yields. Elliot, Komunjer and Timmermann (2005) proposed a generalized method of moments test for forecast rationality.
assuming forecasters have an asymmetric power loss function. Using their test, they showed that the bias in the IMF and OECD forecasts of the budget deficits of G7 countries is consistent with the forecasters having asymmetric loss functions. Capistran (2006) also used their test to show the U.S. Federal Reserve's Green Book forecasts of inflation are biased and consistent with the Fed forecasters having an asymmetric power loss function.

The above studies all assume that the asymmetric loss functions of forecasters remain constant over time. Time variation in the optimal bias is assumed to be due to time variation in the higher order moments of the variable being forecasted. However recent studies by Krane (2003), Bachelor (2007) and Patton and Timmermann (2007) suggested that the loss function not only depends on the forecast error but also on the state of the economy. In that case, the bias in agents' forecasts can be explained either by the time-varying asymmetry parameter or by time-varying higher order moments or by both. Although there is extensive evidence for conditional heteroskedasticity in inflation and interest rates, there is little evidence of time-varying conditional variances or higher order moments in quarterly real economic variables such as GDP or its individual components. If this is the case, time variation in the optimal forecast bias of such variables must be due to time variation in the loss function itself. To our knowledge, the only paper which formally tests state dependence in the loss function of forecasters is the paper by Patton and Timmermann (2007). They estimated a quadratic spline approximation to the implied loss function of the U.S. Fed Green Book forecasts of real GDP growth by the generalized method of moments. The estimated loss function was allowed to depend on the current level of real GDP growth. They found that the level of GDP growth significantly affected the shape of the loss function. When GDP growth was high, the loss function appeared to be symmetric. When GDP growth was
low, the loss function placed greater loss on over predicting growth than on under predicting growth.

In this paper we test whether forecasters have time-varying asymmetric loss in a parametric framework. Unlike Patton and Timmermann (2007), our model can easily incorporate multiple state variables and is easy to estimate. We assume that forecasters have an asymmetric loss function where the parameter that determines the asymmetry is a function of a linear combination of variables that represent the state of the economy. For two of the most commonly used loss functions, we show that the asymmetry parameter can be estimated by least squares. We use our model to test whether one quarter ahead forecasts of real GDP growth from the Survey of Professional Forecasters have a time-varying bias that depends on the state of the economy. We consider factors such as duration of the business cycle, uncertainty of forecasts and type of government, which may cause agents to intentionally bias their forecasts. The paper proceeds as follows. In section 2 we present the theoretical results. In section 3 we describe the data and variables. In section 4 we analyze the empirical results. Finally, section 5 concludes the paper.

2. State Dependence in Forecast Bias

Suppose $y_t$ is the series to be forecasted. For simplicity, we assume the series is to be forecasted with one period horizon. Our results can be generalized to any horizon. The series in period $t + 1$ can be decomposed as $y_{t+1} = \mu_{t+1,t} + \epsilon_{t+1,t}$, where $\mu_{t+1,t}$ is the mean in period $t + 1$ conditional on the information set available in time period $t$ and $\epsilon_{t+1}$ is an innovation which has mean zero and is uncorrelated with elements in the information set. Let $\hat{y}_{t+1,t}$ be the one period ahead forecast of $y_t$ using all information available in time period $t$. 
When forecasters have quadratic loss the optimal predictor is $\hat{y}_{t+1,t} = \mu_{t+1,t}$ and the forecast error $e_{t+1} = y_{t+1,t} - y_{t+1,t} = \epsilon_{t+1}$ is the innovation in period $t + 1$. As a result, under quadratic loss forecast errors have mean zero and are orthogonal to elements in the information set. Standard forecast efficiency tests, which assume quadratic loss, are based on regressing the forecast error $e_{t+1}$ on variables in the information set and testing for significance.

If forecasters have asymmetric loss, the properties of forecast errors are very different. Granger (1969) and Christoffersen and Diebold (1997) showed that if agents have asymmetric loss functions, the optimal predictor is the conditional mean plus a bias $\hat{y}_{t+1,t} = \mu_{t+1,t} + \lambda_{t+1,t}$, where the bias $\lambda_{t+1,t}$ depends on the loss function and the second and higher order conditional moments of $y_{t+1}$. The forecast error becomes

$$e_{t+1} = -\lambda_{t+1,t} + \epsilon_{t+1,t} \quad (1)$$

The analytic expression for the bias $\lambda_{t+1,t}$ depends on the assumed parametric forms of the loss function and the conditional distribution of $y_{t+1}$. If the loss function or the second or higher moments of $y_{t+1}$ are functions of the information set, the bias is a time-varying function of the information set. As a consequence, standard efficiency tests will reject forecast rationality.

### 2.1 Lin-Ex Loss

In this section we introduce a time-varying asymmetry parameter into the commonly used Lin-Ex loss function introduced by Varian (1974). If the asymmetry parameter in the loss function is time-varying, the Lin-Ex loss function can be written as
\[ L(y_{t+1} - \hat{y}_{t+1,t}) = \exp\{\alpha_t (y_{t+1} - \hat{y}_{t+1,t})\} - \alpha_t (y_{t+1} - \hat{y}_{t+1,t}) - 1 \]

where \( \alpha_t \) can be any real number. When \( \alpha_t > 0 \), the loss is approximately linear for over prediction and approximately exponential for under prediction. On the other hand, if \( \alpha_t < 0 \), the loss is approximately linear for under prediction and approximately exponential for over prediction. We specify the asymmetry parameter \( \alpha_t \) as a linear combination of a set of state variables \( z_t \) so that \( \alpha_t = z_t' \gamma \). The state variables are assumed to be part of the forecasters information set and represent information that alters the forecaster’s relative loss in over predicting versus under predicting. As shown by Christoffersen and Diebold (1997), assuming that the forecast errors are conditionally normal, the optimal predictor under Lin-Ex loss is \( \hat{y}_{t+1,t} = \mu_{t+1,t} + \sigma_{t+1,t}^2 \alpha_t / 2 \). The forecast error becomes

\[
e_{t+1} = -\frac{\sigma_{t+1,t}^2}{2} \alpha_t + \epsilon_t
\]

\[
= -\frac{\sigma_{t+1,t}^2}{2} (z_t' \gamma) + \epsilon_t
\]

Under Lin-Ex loss the time-varying bias in the forecast can be due to the time-varying asymmetry parameter \( \alpha_t \) or due to the conditional variance \( \sigma_{t+1,t}^2 \) of the series being forecasted or due to both. In the absence of time-varying higher order moments, the bias can only be explained by the time-varying asymmetry parameter. As we described above, empirically it is found that the one quarter ahead GDP growth rate forecasts do not have higher order moment dynamics. In this case, the bias can only be explained by the time-varying asymmetry parameter \( \alpha_t = z_t' \gamma \). If we assume a constant conditional variance \( \sigma_{t+1,t}^2 = \sigma^2 \), the forecast error becomes

\[
e_{t+1} = -\frac{\sigma^2}{2} (z_t' \gamma) + \epsilon_{t+1}
\]

\[
= z_t' \gamma' + \epsilon_{t+1} \quad (2)
\]
Given a sequence of observed forecast errors, the maximum likelihood estimator of $\gamma^*$ can be computed by the least squares regression of $e_{t+1}$ on $z_t$. The significance of the state variables can be tested using standard t-tests. We can estimate the time-varying bias with the negative of the predictions from this regression $\hat{\lambda}_{t+1,t} = -z_t' \hat{\gamma}^*$. We can estimate the time-varying asymmetric parameter by $\hat{\alpha}_t = 2 \hat{\lambda}_{t+1,t} / \hat{\sigma}_\epsilon^2$, where $\hat{\sigma}_\epsilon^2$ is the LS estimator of the variance of $\epsilon_{t+1}$ in (2). The regression (2) takes the form of the standard regression test for forecast efficiency. From the above analysis, such a test can be interpreted as a test for time-varying asymmetry under Lin-Ex loss.

Because $\alpha_t$ represents a behavioral parameter in the forecaster's loss function, it would be useful to construct a confidence interval for $\alpha_t$ in each time period $t$. It is straightforward to present an asymptotic sampling theory for $\hat{\alpha}_t = -2z_t' \hat{\gamma}^* / \hat{\sigma}^2$. From standard linear regression theory, $\hat{\gamma}^*$ and $\hat{\sigma}^2$ are jointly asymptotically normal with elements of their variance-covariance matrix $V$ given by $\text{var}(\hat{\gamma}^*) = \sigma^2(Z'Z)^{-1}$, $\text{var}(\hat{\sigma}^2) = 2 \sigma^4 / (T - 1)$ and $\text{cov}(\hat{\gamma}^*, \hat{\sigma}^2) = 0$, where $Z$ is a $T \times k$ matrix whose $t$th row is $z_t' \gamma$. Let $\theta = (\hat{\gamma}^*, \hat{\sigma}^2)'$ and $f_t = d \hat{\alpha}_t / d \theta = [-2 \hat{\sigma}^{-2} z_t, 2 \hat{\sigma}^{-4} z_t' \hat{\gamma}^*]'$. By the delta method [see, for example, Green (2008)], $\hat{\alpha}_t$ is asymptotically normal and a consistent estimator of its asymptotic variance is $\text{avvar}(\hat{\alpha}_t) = f_t' \hat{V} f_t$, where $\hat{V}$ is the usual estimator of the variance-covariance matrix of the LS estimators of $\hat{\gamma}^*$ and $\hat{\sigma}^2$ in the regression (2). Then, a 95 percent confidence interval for $\alpha_t$ in each period $t$ can be calculated as $\hat{\alpha}_t \pm 1.96 \sqrt{f_t' \hat{V} f_t}$. 
2.2 Asymmetric Power Loss

In this section we introduce a time-varying asymmetry parameter into the asymmetric power loss function used by Christoffersen and Diebold (1996, 1997) and Elliot, Komunjer and Timmermann (2005, 2008). When \( \alpha \) is time-varying, the loss function \( L \) can be written as,

\[
L(\hat{y}_{t+1,t}, \alpha_t) = \left[ \alpha_t + (1 - 2\alpha_t)I(y_{t+1,t} - \hat{y}_{t+1,t}, 0) \right] |y_{t+1,t} - \hat{y}_{t+1,t}|^p
\]

where \( 0 < \alpha_t < 1 \) is the asymmetry parameter and \( I(.) \) is the indicator function that takes the value 1 when the condition in the argument is true. The loss function is asymmetric for \( \alpha_t \neq 0.5 \). In applications, the power \( p \) is usually assumed to be known. This family of loss functions includes many of the loss functions commonly used in the literature. When \( \alpha_t \neq 0.5 \) and \( p = 1 \), the loss function is asymmetric Lin-Lin. When \( \alpha_t \neq 0.5 \) and \( p = 2 \), the loss function is asymmetric Quad-Quad. Because the asymmetry parameter is required to be in the interval \((0, 1)\), we employ a probit transformation and assume \( \alpha_t = \Phi(z_t' \gamma) \), where \( \Phi(.) \) is the standard normal cumulative distribution function. Assuming conditional normality, the optimal predictor is

\[
\hat{y}_{t+1,t} = \mu_{t+1,t} + \omega_t \sigma_{t+1,t}.
\]

where the time varying parameter \( \omega_t \) solves the equation

\[
(1 - \alpha_t) \int_{-\infty}^{\omega_t} |z - \omega_t|^{p-1} \phi(z)dz - \alpha_t \int_{\omega_t}^{\infty} |z - \omega_t|^{p-1} \phi(z)dz = 0
\]

and \( \phi(z) \) is the standard normal density [see Christoffersen and Diebold (1996)].

If we assume that \( p \) is known and that the conditional standard deviation is constant, the coefficients on the state variables can be estimated from the observed forecast errors by the method of maximum likelihood. The forecast error is

\[
e_{t+1} = -\omega_t \sigma + \epsilon_{t+1}.
\]

The log-likelihood function of the forecast errors is
\[
l_T(y, \sigma) = -\frac{T}{2} \log(2\pi) - \frac{T}{2} \log(\sigma^2) - \frac{1}{2} \frac{\sum_{t=1}^{T} e_{t+1} + \omega_t \sigma}{\sigma^2},
\]
where \(\omega_t\) is a function of \(\gamma\) through (4). For any given value of \(\gamma\), (4) can be solved numerically for each \(\omega_t\) and the log-likelihood function can, in turn, be evaluated. The log-likelihood function can then be numerically maximized using the standard methods. Given the MLE's of \(\gamma\) and \(\sigma\), we can estimate the time-varying asymmetry parameter by \(\hat{\alpha}_t = \Phi(z_t' \hat{\gamma})\) and the time-varying bias by \(\hat{\lambda}_{t+1,t} = \hat{\omega}_t \hat{\sigma}\). The above estimation is greatly simplified when \(p = 1\) and the loss function is Lin-Lin. The optimal predictor is easily seen to reduce to \(\hat{\gamma}_{t+1,t} = \mu_{t+1,t} + \Phi^{-1}(\alpha_t) \sigma\). Because we assume \(\alpha_t = \Phi(z_t' \gamma)\) to insure that \(\alpha_t\) in \((0,1)\), the optimal predictor becomes simply \(\hat{\gamma}_{t+1,t} = \mu_{t+1,t} + \sigma z_t' \gamma\). The forecast error is
\[
e_{t+1} = -\sigma z_t' \gamma + \varepsilon_{t+1} = z_t' \gamma^* + \varepsilon_{t+1}
\]
In this case maximum likelihood estimation is equivalent to linear regression. The regression (5) is identical to the Lin-Ex regression (2). The only difference lies in the parameterizations \(\gamma^* = -\sigma^2 \gamma/2\) and \(\gamma^+ = -\sigma \gamma\). Therefore, the standard regression test for forecast efficiency can be interpreted as a test for time-varying asymmetry under either Lin-Ex or Lin-Lin loss. As with Lin-Ex loss, we can estimate the time-varying bias with the negative of the predictions from this regression \(\hat{\alpha}_{t+1,t} = -z_t' \gamma^+\). A consistent estimator of the time-varying asymmetry parameter is \(\alpha_t = \Phi(\hat{\alpha}_{t+1,t} / \hat{\sigma})\), where \(\hat{\sigma}\) is the LS estimator of the standard deviation of \(\varepsilon_{t+1}\) in (5).

Using the MLE's, we can construct confidence intervals for the time-varying asymmetry parameter \(\alpha_t\) in the asymmetric power loss function. By the asymptotic normality of the MLE and the delta method, \(\alpha_t = \Phi(z_t' \gamma)\) is asymptotically normal and a consistent estimator of it's asymptotic variance is \(\text{var} \Phi(z_t' \gamma) = \phi(z_t' \gamma)^2 z_t' \Sigma z_t\), where \(\phi(\cdot)\) is the standard normal density.
function and \( \hat{\Sigma} \) is any of the conventional estimators of the asymptotic variance-covariance matrix of the MLE \( \hat{\gamma} \). A 95 percent confidence interval for \( \alpha_t \) is \( \hat{\alpha}_t \pm 1.96\phi(z_t'\hat{\gamma}) \sqrt{z_t'\hat{\Sigma}z_t} \). If it is assumed that \( p = 1 \) and the regression (5) is estimated by LS, the estimator of \( \alpha_t \) is 
\[ \hat{\alpha}_t = \Phi(-z_t'\hat{\gamma}^+ / \hat{\sigma}). \]
In this case, let \( \theta = (\hat{\gamma}^+ \hat{\sigma}^2) \) and 
\[ \hat{\theta}_t = d\hat{\alpha}_t/d\theta = [-\phi\left(-\frac{z_t'\hat{\gamma}^+}{\hat{\sigma}}\right) \sigma^{-1} z'_t, -\phi\left(-\frac{z_t'\hat{\gamma}^+}{\hat{\sigma}}\right) z'_t \hat{\gamma}^+/2\hat{\sigma}^3]' \]. Then, again by the delta method, \( \alpha_t \) is asymptotically normal and a consistent estimator of its asymptotic variance is 
\[ \text{avvar}(\hat{\alpha}_t) = \hat{g}_t'\hat{V}g_t \]  where \( \hat{V} \) is the usual estimator of the variance-covariance matrix of the LS estimators of \( \hat{\gamma}^+ \) and \( \hat{\sigma}^2 \) in (5). Then a 95 percent confidence interval for \( \alpha_t \) is 
\[ \hat{\alpha}_t \pm 1.96\sqrt{\hat{g}_t'\hat{V}g_t}. \]

### 3. Data and Variables that may Explain Bias

In this section we test for state dependence bias in the one quarter ahead real U.S. GDP growth rate forecasts from the Survey of Professional Forecasters (SPF). Starting with the first quarter of 1968, the National Bureau of Economic Research, together with the the American Statistical Association, began conducting surveys of forecasts of important economic variables produced by private sector economists. The forecasted variables included measures of output, inflation, unemployment and interest rates. In the early years the survey averaged about 50 participants in each quarter. The number of respondents had dwindled to about 20 participants by the late 1980’s. The Federal Reserve Bank of Philadelphia revived the survey in 1990 after it was discontinued by the ASA and the NBER. The survey once again averages about 50 participants. Croushore (1993) provides a very detailed description of the SPF. In the third quarter of 1981, the scope of the survey was expanded to include, among other forecasts, forecasts of the one quarter ahead real GDP growth rate. These are the
forecasts we use in this paper. We use the median consensus forecast. We replicated all of our work using the mean consensus forecast and the conclusions were the same. Our sample period begins with the third quarter of 1981 and ends with the fourth quarter of 2007.

There are many reasons why forecasters may have asymmetric loss functions and intentionally report biased forecasts. Some authors have suggested that agents may bias their forecasts based on whether the economy is in expansion or contraction. The ability to predict turning points in economic activity is very valuable in economic planning. Forecasters may seek to build a reputation for their ability to forecast turning-points. Failure to predict an actual turning-point may be more costly than incorrectly predicting ones that do not occur. This suggests that when the economy is in expansion, forecasters may intentionally bias their forecast of GDP growth downward. Similarly, when the economy is in recession, they may intentionally bias their forecast upward [see Loungni and Trehan (2002) and Zarnowitz and Braun (1993)]. We test whether expansion/recession in the economy can cause bias in agents' forecasts by using a expansion/recession dummy in our regression. We construct an expansion/recession indicator from the NBER's U.S business cycle chronology. We define our dummy variable (RECDM) to take the value 1 if the economy is in recession and zero otherwise. The NBER dates the beginning of a recession as a significant decline in economic activity lasting more than few months. The NBER considers not only a decline in real GDP, but also a broad decline in real income, employment, industrial production and wholesale-retail-sales. The NBER usually announces the turning-points several quarters after they occur. We use the date of the turning-point and not the announcement date because professional forecasters are likely to be aware of the general economic decline shortly after they occur and prior to the official announcement by the NBER.
In addition to the current phase of the business cycle, duration of the current phase may also cause forecasters to bias their forecasts. As the length of the expansion becomes longer, agents may become increasingly optimistic and bias their forecasts upward. Similarly, as the length of a contraction becomes longer, agents may become increasingly pessimistic and bias their forecast downward. Bachelor and Dua (1990) suggested that forecasters may, in fact, find it beneficial to develop a reputation as optimists or pessimists. To test this source of bias, we include duration (DURATION) measured as the number of quarters since the last turning point as a variable in our regression. We also include an interaction between the expansion/recession dummy and duration (REC\_DUR) to allow for asymmetric effects between the phase of the business cycle and the business cycle duration.

We also test whether the political parties of the current administration produces a bias in the forecasts of real GDP growth. There has been a long literature that documents the difference in the two U.S. political parties emphasis on the inflation/output trade-off in the economy. Traditionally, the Democrats are viewed as more concerned with stimulating growth and reducing unemployment in the short-run. Whereas, Republicans are more concerned with keeping inflation low to promote growth and stability in financial markets. Hibbs (1977, 1986) and Alesina, Londregan and Rosenthal (1993) gave time series evidence for these stylized notions. More recently, Snowbergs, Wolfers and Zitewitz (2007) used election prediction market data to show that the market's expectations concerning equity returns, interest rates and oil prices were directly impacted by the probabilities of which party would be elected. Hence the political party in office might explain the bias so we introduce a political party dummy (POLIDM) variable. The dummy variable for type of government takes the value 1 if a Republican is in office and zero if a Democrat is in office.
In addition to the above variables, we consider whether forecaster uncertainty affects forecaster bias. Fildes and Stekler (2002) suggested that the relationship between forecaster bias and uncertainty is important, but the effect may be ambiguous. When there is a lack of consensus among forecasters, forecasters may have a stronger incentive to try and build a reputation as an optimist or a pessimist. It is also possible that in the presence of high uncertainty, forecasters may place less reliance on their formal or informal models and bias their forecast towards the long-run historical average of real GDP growth. Our measure of uncertainty (UNCERT) is the standard deviation of the individual one quarter ahead forecasts of real GDP growth from the individual level SPF. As with business cycle duration, we also include an interaction term between our uncertainty measure and the business cycle dummy (REC_UNCER) to allow uncertainty to have an asymmetric effect over the business cycle.

4. Empirical Results

In this empirical section, we show that time varying bias in the SPF forecasts of one quarter ahead real GDP growth may be explained by certain state variables. As we argue above, under asymmetric loss a time-varying forecast bias could be due to either the time variation in the conditional variance or time variation in the asymmetry parameter. We begin our empirical analysis by testing for time-varying second order moments in the forecast error of GDP growth rate. We first conduct LM tests for ARCH through lag eight. All of the tests fail to reject the null hypothesis of no ARCH. We also conduct a variety of tests to see if the conditional variance of the forecast error is determined by lagged levels or first differences of real GDP growth. These tests were also insignificant, further indicating a constant conditional variance for the forecast error. The previous tests are conducted assuming a
constant conditional mean. As described in Bachelor and Peel (1998), if the conditional variance of the forecast error is time-varying, but the asymmetry parameter is constant, the forecast errors should follow a GARCH-M process under Lin-Ex and Lin-Lin loss. Therefore, we also estimate GARCH(1,1)-M models and GARCH(1,1)-M models that included lagged levels and first differences of real GDP growth. For all of the models, the variables in the conditional variances are insignificant. Also, the GARCH-M coefficients on the conditional variances are themselves insignificant. The above tests suggest that the forecast errors for GDP growth rate do not have time-varying variances nor biases that depend on them.

Specifying the conditional variance as constant, we estimate models which allows the asymmetry parameter to depend on RECDM, POLIDM, UNCERT and DURATION. To allow for possible asymmetric effects over the business cycle, we also include the interactions between the recession dummy RECDM and the continuously varying variables UNCERT and DURATION. We estimate the models assuming agents forecast under both Lin-Ex and asymmetric power loss. For asymmetric power loss, we consider both a Lin-Lin specification with $p=1$ and a Quad-Quad specification with $p=2$. Recall that in Lin-Ex and Lin-Lin loss, the linear regressions used to estimate the asymmetry parameter are identical. Therefore, we present two sets of models. One for Lin-Ex/Lin-Lin and one for Quad-Quad. The Quad-Quad model is estimated by maximum likelihood.

The estimated models and accompanying diagnostic statistics are shown in Table 1. In the first column of Table 1 we present the general model for Lin-Ex and Lin-Lin loss. We find the intercept and the coefficients on RECDM, POLIDM, DURATION and REC_DUR to be insignificant. A zero intercept indicates that there is no time-invariate systematic bias in the
forecasts. Insignificance of the recession dummy indicates forecasters have no propensity to over or under forecast real GDP growth depending on whether the economy is in expansion or contraction. This is contrary to most findings in the literature. The insignificance of POLIDM suggests that agents do not appear to be optimistic nor pessimistic based on which party is in office. The coefficients on UNCERT and REC_UNCER are significant. The sign of the coefficient on UNCERT is negative and the sign of the coefficient on REC_UNCER is positive. Furthermore, the coefficient on UNCERT is less in absolute value than the coefficient on REC_UNCER. This implies that when the economy is in recession, increasing uncertainty causes forecasters to introduce a positive bias. Whereas, when the economy is in expansion, increasing uncertainty causes forecasters to introduce a negative bias. Uncertainty causes forecasters to make conservative forecasts. As uncertainty increases, forecasters bias the forecast towards its historical average. Unlike the finding by McNees (1976, 1988, 1997), McNees and Reis (1983), and Zarnowitz and Braun (1993) that during recession agents over predict, our results suggest that in the presence of recession, it is increasing uncertainty causes agents to over predict. The results for the Quad-Quad model are very similar. The magnitude, signs and level of significance of the estimated parameters are similar between the Lin-Ex/Lin-Lin and Quad-Quad specifications. The finding that forecaster uncertainty and the current phase of the business cycle determine the time-varying forecast bias does not appear to depend on the specification of the parametric form of the forecaster loss function.

We drop the insignificant variables and re-estimate the models to obtain final specifications. These results are also shown in Table 1. The magnitude, signs and level of significance of the coefficients in the final models are similar to the general models. In both specifications, the SIC is minimized by the final model. There is no evidence of mis-specification in the models. Q-statistics through lags four and eight based on the residuals
and squared residuals are insignificant and indicate that there is no dependence in the errors through the first or second moments. The parametric form of the estimated models depend highly on the forecast errors being normally distributed. The Jarque-Bera tests for normality of the errors are also insignificant, suggesting that the normality assumption is plausible.

For each of the final models, we estimate the time-varying asymmetry parameter \( \alpha_t \) for each time period. In Figures 1, 2 and 3 we show the estimated asymmetry parameters with accompanying 95 percent confidence bands under Lin-Ex, Lin-Lin and Quad-Quad loss. Considering Figure 1, the asymmetry parameter shows considerable variation over time under Lin-Ex loss. The parameter is predominantly negative, being associated with periods when the economy is in expansion. During periods of economic expansion, forecasters place a greater loss on over predicting real GDP growth in the presence of forecaster uncertainty. During the 1982, 1990 and 2000 recessions, the asymmetry parameter is positive and much larger in magnitude. During these periods of contraction, forecasters place a larger loss on under predicting real GDP growth in the presence of forecaster uncertainty. Recall that for Lin-Ex loss, the loss function is symmetric when \( \alpha_t = 0 \). For the majority of the time periods, the confidence bands exclude 0, indicating that the asymmetry is statistically significant. In Figures 2 and 3, the pattern of variation over time of the asymmetry parameter under Lin-Lin and Quad-Quad loss is similar to that under Lin-Ex loss shown in Figure 1. For Lin-Lin and Quad-Quad, the loss function is symmetric when \( \alpha_t = 0.5 \). For the majority of the time periods, the confidence bands exclude 0.5, again indicating that the asymmetry is statistically significant. Under Lin-Lin loss, the asymmetry parameter is slightly less in magnitude than the parameter under Quad-Quad loss. The figures clearly convey that the pattern of variation in the asymmetry of the loss function does not appear to depend on the specification of the parametric form of the loss function.
In Figure 4, we show the estimated time-varying bias implied by each of the models. Recall that the estimated bias under Lin-Ex and Lin-Lin are identical. The magnitude of the estimated biases is slightly less under Quad-Quad loss than under Lin-Ex and Lin-Lin loss. The over pattern of the biases are similar. During periods of economic expansion, forecasters bias their growth forecasts downward in the presence of forecaster uncertainty with the biases ranging between -.5 and -2 percent at an annualized rate. During the three recessions in our sample period, the biases become positive. During the 1982 recession, the biases reach 2.6 percent for Quad-Quad and 5.2 percent for Lin-Ex and Lin-Lin loss. During the 1990 and 2000 recessions, the biases are smaller in magnitude and range between .8 and 1.9 percent.

Conclusion

Although there have been important advances in time series modeling, professional forecasters still produce forecasts that have a bias that varies over time. We examine whether the bias in the SPF forecast of real U.S. GDP growth can be explained by a time-varying asymmetric loss function. We propose a method for estimating the time-varying degree of asymmetry and forecast bias for the asymmetric loss functions most commonly considered in the forecasting literature. For the important Lin-Ex and Lin-Lin loss functions, all of the inference can be based on the LS estimator of a linear regression. Contrary to previous work, our empirical results show that the direction and magnitude of the bias in forecasts of real GDP growth are not simply dependent on the phase of the business cycle. Rather, the direction and magnitude of the bias depends on an interaction between forecaster uncertainty and the phase of the business cycle.
Bibliography


Table 1
Determinants of Time-varying Asymmetry Parameter in the One Quarter Ahead GDP Growth Rate, 1981:3-2007:4

<table>
<thead>
<tr>
<th>Variables</th>
<th>Lin-Ex/Lin-Lin Loss</th>
<th>Quad-Quad Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>General</td>
<td>Final</td>
</tr>
<tr>
<td>Intercept</td>
<td>-0.49 (0.78)</td>
<td>-0.03 (0.36)</td>
</tr>
<tr>
<td>RECDM</td>
<td>1.27 (1.50)</td>
<td>0.69 (1.28)</td>
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<tr>
<td>POLIDM</td>
<td>0.61 (0.46)</td>
<td>0.52 (0.38)</td>
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<tr>
<td>UNCERT</td>
<td>-0.63** (0.33)</td>
<td>-0.57** (0.29)</td>
</tr>
<tr>
<td>DURATION</td>
<td>0.006 (0.02)</td>
<td>0.003 (0.01)</td>
</tr>
<tr>
<td>REC_UNCER</td>
<td>3.43** (1.79)</td>
<td>1.70** (0.40)</td>
</tr>
<tr>
<td>REC_DUR</td>
<td>-1.61 (1.30)</td>
<td>-0.91 (0.58)</td>
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<tr>
<td>Adj Rsq</td>
<td>0.21</td>
<td>0.20</td>
</tr>
<tr>
<td>SIC</td>
<td>4.43</td>
<td>4.30</td>
</tr>
<tr>
<td>Q(4)</td>
<td>0.161</td>
<td>0.123</td>
</tr>
<tr>
<td>Q(8)</td>
<td>0.443</td>
<td>0.264</td>
</tr>
<tr>
<td>Q² (4)</td>
<td>0.899</td>
<td>0.852</td>
</tr>
<tr>
<td>Q² (8)</td>
<td>0.950</td>
<td>0.676</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>0.499</td>
<td>0.854</td>
</tr>
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</table>

Standard errors are in parenthesis. ** denotes significant at 5% level and * denotes significant at 10% level. For residuals, squared residuals and normality test p-values are given.
Figure 1. Estimated time-varying asymmetry parameter and 95 percent confidence bands under Lin-Ex loss.
Figure 2. Estimated time-varying asymmetry parameter and 95 percent confidence bands under Lin-Lin loss.
Figure 3. Estimated time-varying asymmetry parameter and 95 percent confidence bands under Quad-Quad loss.
Figure 4: Estimated time-varying forecast biases under Lin-Ex, Lin-Lin and Quad-Quad.