

# Maths Eligibility Assessment

## Instructions

This exam is to be completed without a calculator. You will have 1.5 hours to complete this exam. Some questions will have part marks, it is highly recommended to show all of your working to obtain as many marks as possible.

**Question 1 (4 marks):** Put each of the following into the form  $y = mx + c$

i)  $15x = y + 2(3 - 3x + y)$       iii)  $9x^2 = y^2 + 6x - 1$

ii)  $1 = \frac{\frac{x}{2}(5y-1)-2xy}{x^2}$       iv)  $x = \frac{x^2 - 12 - 3y}{y + 1}$

**Question 2 (4 marks):** State the domain and range of the following:

i)  $y = 3x + 14$       iii)  $y = \frac{x + 3}{x^2 - 9}$

ii)  $y = \frac{2x}{x^2 - 4}$       iv)  $y = \sqrt{4x^2 - 16}$

**Question 3 (8 marks):** Graph the following and label any key points (e.g. intercepts, turning points, asymptotes):

i)  $y = x^2 + 2x - 15$       iii)  $y = \frac{1}{x^2 - 9}$

ii)  $y = \frac{1}{x - 4}$       iv)  $y = \sqrt{x^2 - 4}$

**Question 4 (8 marks):** Differentiate the following with respect to  $x$ :

i)  $f(x) = e^{2x}(2x^2 + x - 1)$       iii)  $f(x) = (x^2 - x)\ln(4x)$

ii)  $f(x) = \frac{\sqrt{x^2 - 1}}{x}$       iv)  $f(x) = \frac{x^2 + 4}{(x^3 - x)^3}$

**Question 5 (8 marks):** Evaluate the following:

i)  $\int_0^2 x^3 - x^2 + 2dx$       iii)  $\int_{-2}^2 4x^2 + x dx$

ii)  $\int_1^3 \frac{1}{x} dx$       iv)  $\int_5^{10} \sqrt{x + 5} dx$

**Question 6 (2 marks):** If O is the origin and P is a point on the line  $y = 3(x + 1) - 2$ , find the length of OP which is a minimum.

**Question 7 (5 marks):** Consider the following equations:

$$15 = 2x_1 + 3x_2 - x_3$$

$$6 = -x_1 + x_2 - 4x_3$$

$$-4 = x_1 + 3x_2 - 6x_3$$

i) (1 marks) Put this into a matrix form,  $Ax = b$

ii) (2 marks) Use Gaussian elimination to solve for  $x$ .

iii) (2 marks) Find the determinant of  $A$

**Question 8 (4 marks):** There are 2 bags of marbles. In the first bag there are 6 marbles: 4 green and 2 red. In the other bag there are 5 marbles: 3 green and 2 red. If a bag is chosen with equal probability and a marble selected at random, what is the probability that a randomly drawn marble is red? If a green marble is drawn, what was the probability that it was drawn from the first bag?

**Question 9 (3 marks):** The following relationship describes the growth of bacteria in a particular environment:

$$B(t) = B_0 e^{\frac{\ln(2)T}{6000}t}$$

where,  $t$  is time in minutes and  $T$  is temperature in Kelvin (0 degrees Celsius = 273.15 Kelvins).

- i) (1 marks)** If bacteria double every 20 minutes, what is the temperature?
- ii) (2 marks)** What is the rate of change of bacteria after 1 hour as a function of temperature?
- iii) (2 marks)** Given that:

$$\begin{aligned}\sin(2\theta) &= 2\sin(\theta)\cos(\theta), \text{ and} \\ \cos(2\theta) &= 1 - 2\sin^2(\theta)\end{aligned}$$

simplify:

$$\frac{\sin(2x)}{1 + \cos(2x)}$$

and then find the exact values of  $\theta$  if  $\tan(\theta) = 2 - \sqrt{3}$  for  $\theta \in [0, 2\pi]$

END OF EXAM